



# Journal of Applied Sciences

ISSN 1812-5654

**science**  
alert

**ANSI***net*  
an open access publisher  
<http://ansinet.com>

## A Price-discount Sharing Contract with Novel Return Policy for the Dominant Retailer

<sup>1</sup>Kewen Pan, <sup>2</sup>Chang Su, <sup>3</sup>S.C.H. Leung and <sup>4</sup>Guoqing Zhang

<sup>1</sup>School of Business Administration and Contemporary Business and Trade Research Center,  
Zhejiang Gongshang University, Hangzhou, China

<sup>2</sup>Bank of China, Anhui Branch, Hefei, China

<sup>3</sup>Department of Management Sciences, City University of Hong Kong, Hong Kong

<sup>4</sup>Faculty of Engineering, University of Windsor, Windsor, Canada

---

**Abstract:** Retailing channels are increasingly being controlled by dominant retailers who have the >power= to dictate pricing and ordering policies to manufacturers. We construct a price-sensitive newsvendor model to determine optimal pricing and ordering policies where a linear Price-Discount Sharing (PDS) contract and a novel integrative return policy are used to help a dominant retailer obtain maximal profit.

**Key words:** Supply chain management, price-discount sharing, return policy

---

### INTRODUCTION

In December 2010, Master Kong (a famous Chinese company which mainly produces instant noodles) asked Carrefour (a dominant retailer who has many supermarkets around the world) to increase retail prices of its instant noodles. Carrefour demanded that if the retail prices were increased, it should receive one half of the incremental prices. There has been another similar case. In November 2010, Lianhua Supermarket (a dominant supermarket in China) refused to sell biscuits of Kraft Foods? (a globally famous food company) because they did not reach agreement on rebates. Conflict of interest and opinion between supplier and retailer is common in the market. A dominant retailer such as Wal-Mart has a large retailing network comprising many retail stores and it is a large distributor for a very large number of manufacturers. So it has the >power = to control pricing and ordering policies of products to manufacturers.

In this study, we focus on how a dominant retailer can make optimal pricing and ordering decisions with a linear Price-Discount Sharing (PDS) contract and a novel integrative return policy under demand uncertainty.

Cachon and Lariviere (2005) pointed out that for any revenue-sharing contract there exists a unique price-discount contract that generates the same profits for the manufacturer and the retailer. Bernstein and Federgruen (2005) showed that a linear PDS scheme achieves coordination for price-setting in the newsvendor model. Consistent with Pan *et al.* (2009), we also assume

that the linear PDS contract is used by the dominant retailer and the manufacturer for coordination:

$$w = \varphi p + (1 - \varphi)c \quad 0 < \varphi < 1 \quad (1)$$

where,  $p$  and  $c$  are the retail price and manufacturing cost of the product, respectively and  $\varphi$  is contract parameter which reflects the channel >power = structure.

We introduce a novel integrative return policy in our model where return price is  $\theta w$  ( $0 \leq \theta \leq 1$ ), which is correlated with wholesale price. When  $\theta = 1$ , it means the retailer is so dominant that it can require the manufacturer to buy back all unsold products at the wholesale price, i.e., the manufacturer bears the entire risk for unsold products; when  $\theta = 0$ , it means there is no return policy and the manufacturer refuses to buy back any unsold products; and when  $0 < \theta < 1$ , it is the general form similar to models of other researchers.

### PRICING AND ORDERING POLICY

We consider the multiplicative demand case, market demand for the product at retail price  $p$  is:

$$X(p, \varepsilon) = D(p)\varepsilon \quad (2)$$

where,  $D(p)$  is price dependent part and is a random variable defined in the range  $[0, \infty]$ , where

$$D(p) = Kp^{-\gamma} \quad K > 0, \gamma > 0 \quad (3)$$

$$\varepsilon \sim N(\mu, \sigma^2) \tag{4}$$

Consistent with Petruzzi and Dada (1999), we define:

$$z = Q/D(p) \tag{5}$$

Similarly, the retailer = s profit function is:

$$\begin{aligned} \pi_r &= pX(p, \varepsilon) - (w + \frac{h}{2})Q - w^b[X(p, \varepsilon) - Q]^+ + (\theta w - h/2)[Q - X(p, \varepsilon)]^+ \\ &= pD(p)\varepsilon - (w + \frac{h}{2})D(p)z - (w + c^b)D(p)[\varepsilon - z]^+ \\ &\quad + (\theta w - h/2)D(p)[z - \varepsilon]^+ \end{aligned} \tag{6}$$

and the manufacturer = s profit function is:

$$\begin{aligned} \pi_m &= (w - c)Q + (w - c)[X(p, \varepsilon) - Q]^+ - [\theta w - s][Q - X(p, \varepsilon)]^+ \\ &= (w - c)D(p)z + (w - c)D(p)[\varepsilon - z]^+ \\ &\quad - [\theta w - s]D(p)[z - \varepsilon]^+ \end{aligned} \tag{7}$$

So the retailer=s expected profit function is:

$$\begin{aligned} \Pi_r(p, z) &= pD(p)\mu - (w + h/2)D(p)z - (w + c^b)D(p)\Theta(z) \\ &\quad + (\theta w - h/2)D(p)\Lambda(z) \\ &= \{[(1 - \varphi)(p - c) - c^b]\mu + (c^b - h/2)z \\ &\quad - \{(1 - \theta)[\varphi p + (1 - \varphi)c] + c^b + h/2\}\Lambda(z)\}Kp^{-\gamma} \end{aligned} \tag{8}$$

Because the retailer is dominant, the manufacturer desires to sell its product through the retailer; we assume that additional cost  $c^b$  per unit the manufacturer charges for backorders is equal to the additional cost of production on an emergency basis. So the manufacturer=s expected profit function is:

$$\begin{aligned} \Pi_m(p, z) &= (w - c)D(p)z + (w - c)D(p)\Theta(z) - [\theta w - s]D(p)\Lambda(z) \\ &= \{\varphi(p - c)\mu + \{(1 - \theta)[\varphi p + (1 - \varphi)c] \\ &\quad - c + s\}\Lambda(z)\}D(p) \end{aligned} \tag{9}$$

Where:

$$\Theta(z) = \int_z^\infty (y - z)f(y)dy, \Lambda(z) = \int_0^z (z - y)f(y)dy$$

Thus the retailer = s maximization problem becomes:

$$\text{Maximize}_{p,z} \Pi_r(p, z) \tag{10}$$

To solve the problem, we take the first and second partial derivatives:

$$\Pi_r(p, z)$$

of with respect to p and z:

$$\begin{aligned} \frac{\partial \Pi_r(p, z)}{\partial p} &= [(1 - \varphi)\mu - (1 - \theta)\varphi\Lambda(z)]Kp^{-\gamma} \\ &\quad - \gamma Kp^{-\gamma-1} \{[(1 - \varphi)(p - c) - c^b]\mu + (c^b - h/2)z \\ &\quad - \{(1 - \theta)[\varphi p + (1 - \varphi)c] + c^b + h/2\}\Lambda(z)\} = 0 \end{aligned} \tag{11}$$

$$\begin{aligned} \frac{\partial^2 \Pi_r(p, z)}{\partial p^2} &= -2[(1 - \varphi)\mu - (1 - \theta)\varphi\Lambda(z)]\gamma Kp^{-\gamma-1} \\ &\quad + \gamma(\gamma + 1)Kp^{-\gamma-2} \{[(1 - \varphi)(p - c) - c^b]\mu + (c^b - h/2)z \\ &\quad - \{(1 - \theta)[\varphi p + (1 - \varphi)c] + c^b + h/2\}\Lambda(z)\} \\ &= -(\gamma - 1)[(1 - \varphi)\mu - (1 - \theta)\varphi\Lambda(z)]Kp^{-\gamma-1} \end{aligned} \tag{12}$$

$$\frac{\partial \Pi_r(p, z)}{\partial z} = \{(c^b - \frac{h}{2}) - \{(1 - \theta)[\varphi p + (1 - \varphi)c] + c^b + h/2\}F(z)\}Kp^{-\gamma} = 0 \tag{13}$$

$$\frac{\partial^2 \Pi_r(p, z)}{\partial z^2} = -\{(1 - \theta)[\varphi p + (1 - \varphi)c] + c^b + h/2\}f(z) < 0 \tag{14}$$

From Eq. 12, when  $-(\gamma - 1)[(1 - \varphi)\mu - (1 - \theta)\varphi\Lambda(z)] < 0$  :

$$\Pi_r(p, z)$$

is concave in p for a given z. Thus it is possible to reduce (10) to an optimization problem over the single variable z by first solving for an optimal value of p as a function of z and then substituting the result back into:

$$\Pi_r(p, z)$$

Searching over the resulting optimal trajectory to maximize:

$$\Pi_r(p^*, z)$$

we can obtain  $z^*$ .

From Eq. 11, we directly obtain:

$$p^* = p(z) = \frac{\gamma \{[(1 - \varphi)c + c^b]\mu - (c^b - \frac{h}{2})z + \{(1 - \theta)(1 - \varphi)c + c^b + \frac{h}{2}\}\Lambda(z)\}}{(\gamma - 1)[(1 - \varphi)\mu - (1 - \theta)\varphi\Lambda(z)]} \tag{15}$$

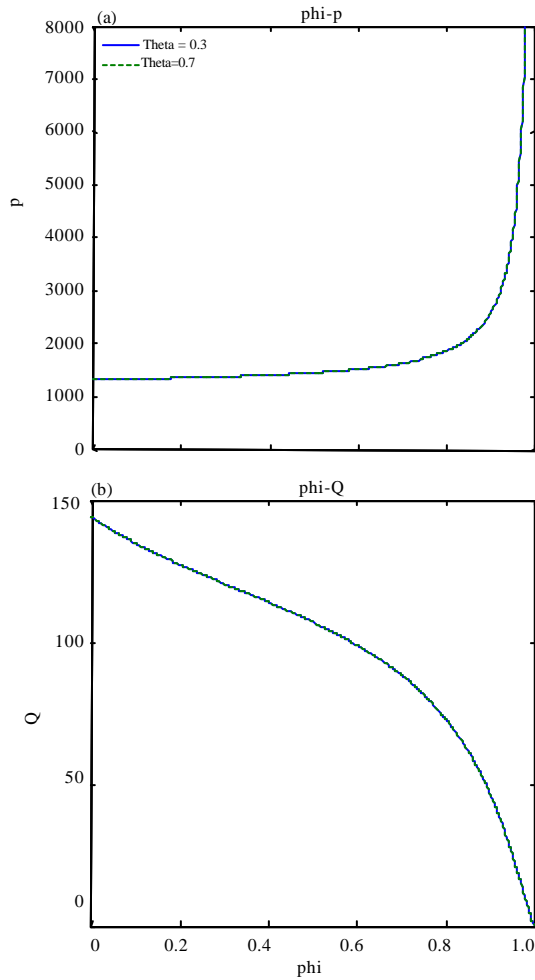


Fig. 1(a-b): Trends  $p^*$  of  $Q^*$  with  $\phi$

Substituting  $p^* = p(z)$  into 10, the optimization problem becomes maximization over the single variable  $z$ :

$$\text{Maximize}_z \Pi_r(p, z) \tag{16}$$

Objective function 16 is unimodal in  $z$  when  $F(\cdot)$  is a distribution function satisfying  $2r(\cdot)^2 + r'(\cdot) > 0$ , where,  $r(\cdot) = f(\cdot) / [1 - F(\cdot)]$  denotes the failure (hazard) rate.

We use numerical solutions to analyze the characteristics of the model in Section 3.

**NUMERICAL EXAMPLE**

**Trends of  $p^*$ ,  $Q^*$ ,  $\Pi_r^*(p, z)$  and  $\Pi_m(p, z)$  with  $\phi$ :** Some parameters are given as follows:

$c = \$200, h = \$20, c^b = \$40, s = \$100, K = 10^6, \gamma = 1.2, \mu = 1, \sigma = 0.2.$

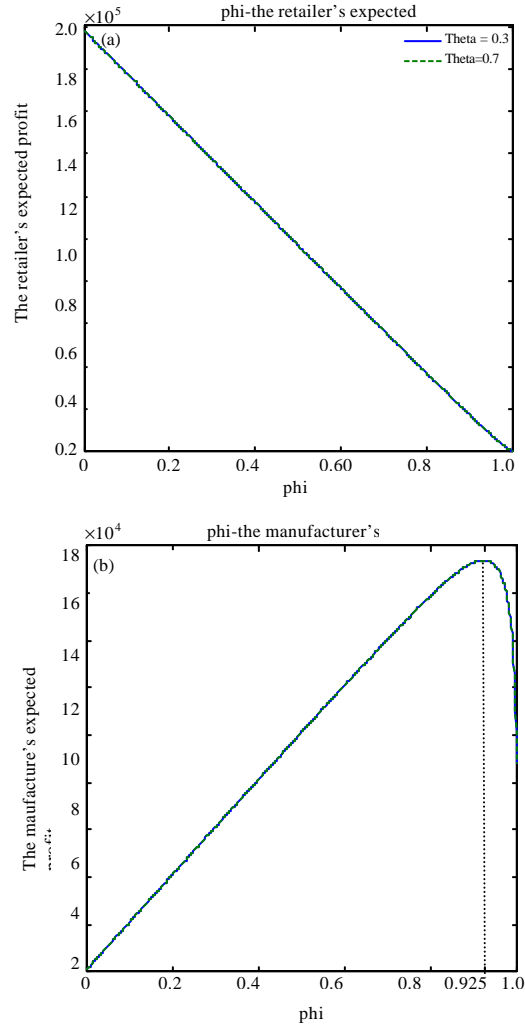


Fig. 2(a-b): Trends of  $\Pi_r^*(p, z)$  with  $\phi$

If  $\theta = 0.3$  or  $\theta = 0.7$ , when  $0 \leq \phi \leq 1$ , we draw the trends of,  $p^*, Q^*$ :

$$\Pi_r^*(p, z)$$

and:

$$\Pi_m(p, z)$$

with  $\phi$  (Fig. 1).

**Trends of  $p^*, Q^*, \Pi_r^*(p, z)$  and  $\Pi_m(p, z)$  with  $\theta$ :** Some parameters are given as follows:

- $c = \$200, h = \$20, c^b = \$40, s = \$100, K = 10^6, \gamma = 1.2, \mu = 1, \sigma = 0.2.$   
If  $\phi = 0.4, \phi = 0.6$  when  $0 \leq \theta \leq 1$ , we draw the trends of  $p^*, Q^*$  (Fig. 2):

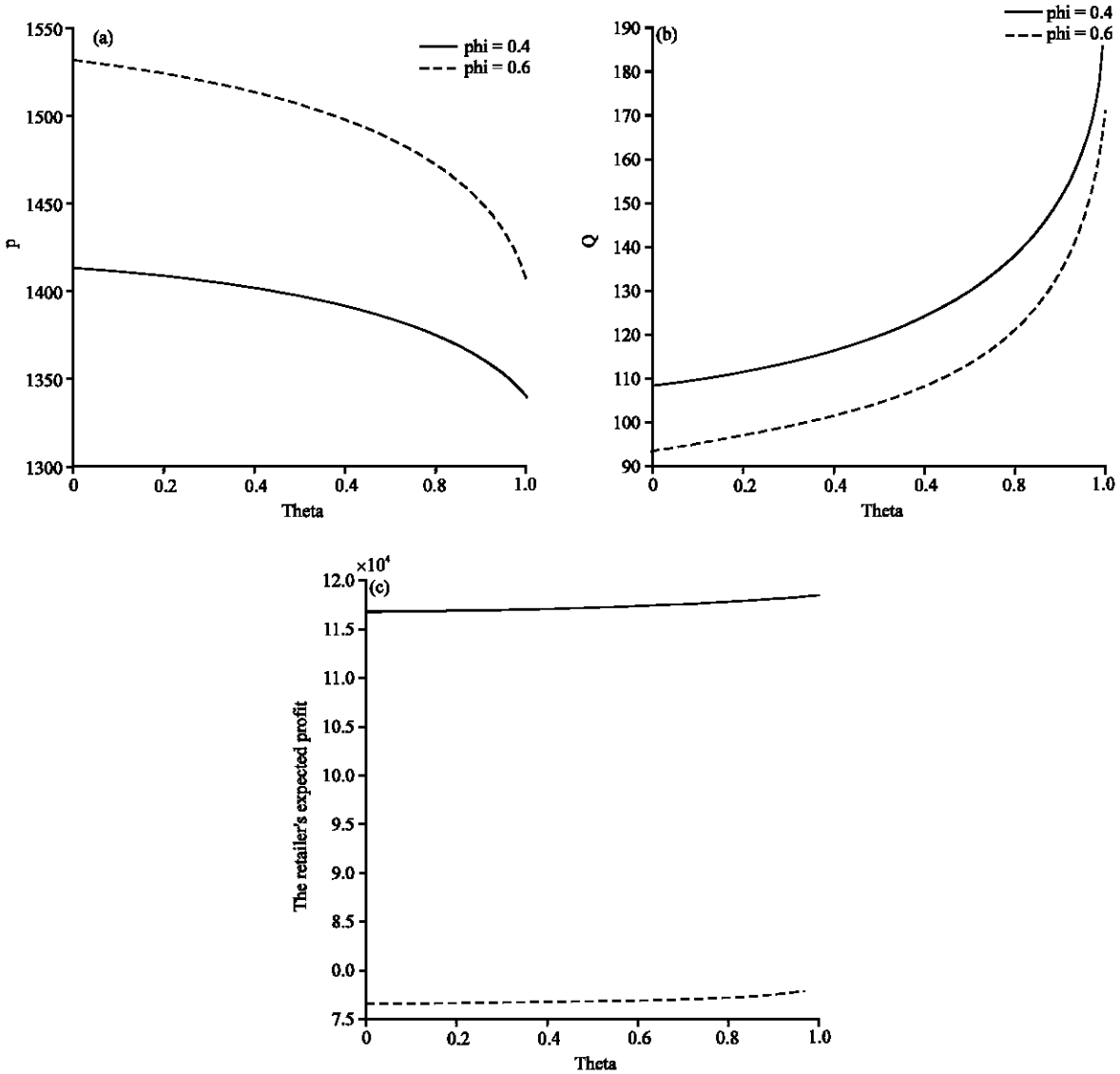


Fig. 3(a-c): Trends of  $p^*$ ,  $Q^*$ ,  $\Pi_r^*(p, z)$  with  $\theta$

$$\Pi_r^*(p, z)$$

and  $\Pi_m(p, z)$  with  $\theta$ . When we tried to draw the trend of  $\Pi_m(p, z)$  with  $\theta$ , we found that it increases first and then decreases with minor change, so we draw the trend of  $\Pi_m(p, z)$  with  $\theta$ , respectively for  $\phi = 0.4$  and  $\phi = 0.6$  to show the trend clearly

**ANALYSIS OF SENSITIVITY OF OPTIMAL SOLUTIONS TO PARAMETERS  $\gamma$  AND  $\sigma$**

Because the trends of  $p^*$ ,  $Q^*$ ,  $\Pi_r^*(p, z)$  and  $\Pi_m(p, z)$  are similar when  $\phi = 0.4$  or  $\phi = 0.6$  and  $\theta = 0.3$  or  $\theta = 0.7$  we

need to discuss sensitivity of  $p^*$ ,  $Q^*$ ,  $\Pi_r^*(p, z)$  and  $\Pi_m(p, z)$  only to parameters  $\gamma$  (market price sensitivity) and  $\sigma$  (standard deviation of random demand  $\epsilon$ ), for certain  $\phi$  and  $\theta$  (Fig. 3 and 4).

Some parameters are given as follows:

- $c = \$200$ ,  $h = \$20$ ,  $c^b = \$40$ ,  $s = \$100$ ,  $K = 10^6$ ,  
 $\mu = 1$ ,  $\phi = 0.6$ ,  $\theta = 0.7$ .

When  $\gamma = 1.1, 1.2$  or  $1.3$  and  $\sigma = 0.1, 0.2$  or  $0.3$ , we obtain the corresponding optimal  $z^*$ ,  $p^*$ ,  $Q^*$ ,  $\Pi_r^*(p, z)$  and  $\Pi_m(p, z)$ , as shown in Table 1

From Table 1, we can see that parameter  $\gamma$  (market price sensitivity) has obvious influence on  $p^*$ ,

**Table 1: Optimal solutions with different  $\gamma$  and  $\sigma$**

$\gamma$	$\sigma$	$z^*$	$p^*$	$Q^*$	$\Pi_r^*(p, z)$	$\Pi_m(p, z)$
1.1	0.1	0.84	2614.3	146.3	165376.8	252422.7
	0.2	0.68	2756.8	111.0	164346.0	252307.0
	0.3	0.51	2900.8	79.0	163386.9	252182.6
1.2	0.1	0.87	1417.4	143.2	77938.2	120849.3
	0.2	0.73	1486.3	113.9	77067.4	120736.7
	0.3	0.59	1555.8	87.0	76251.5	120606.4
1.3	0.1	0.88	1021.0	107.9	38397.4	60437.0
	0.2	0.76	1067.8	87.6	37794.0	60338.1
	0.3	0.63	1115.1	69.0	37228.3	60225.2

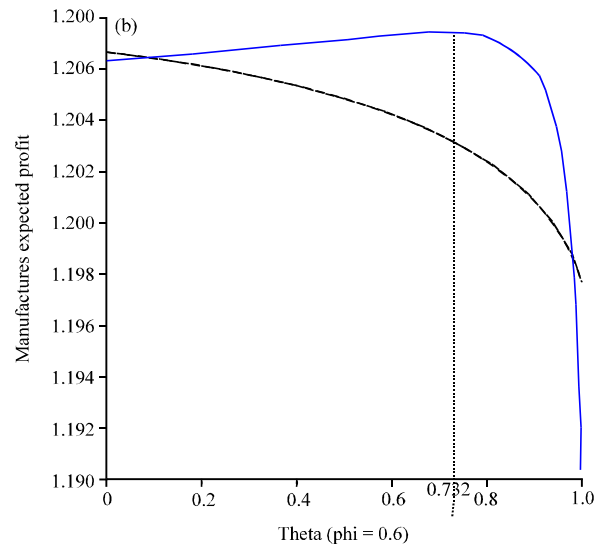
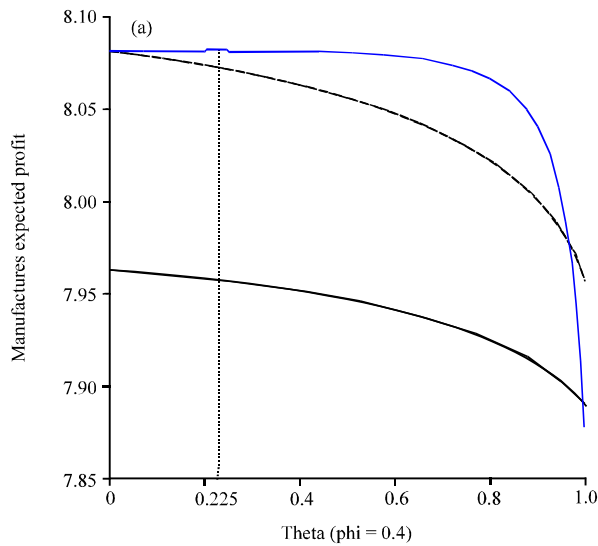


Fig. 4(a-b): Trends of  $\Pi_m(p, z)$  with  $\theta$

$\Pi_r^*(p, z)$  and  $\Pi_m(p, z)$ ; parameter  $\sigma$  (standard deviation of random demand  $\epsilon$ ) has obvious influence on  $p^*$  and  $Q^*$  but minor influence on  $\Pi_r^*(p, z)$  and  $\Pi_m(p, z)$ .

**CONCLUSION**

If the manufacturer and the retailer use our model to make the optimal pricing and ordering policies, when they negotiate  $\varphi$  and  $\theta$ , the retailer will try to set a lower  $\varphi$  and a higher  $\theta$  to obtain a higher profit while the manufacturer will try to make the retailer to set a higher  $\varphi$  and a  $\theta$  as the turning point to obtain a higher profit. The final values of  $\varphi$  and  $\theta$  depend on the channel power they have relative to each other in the supply chain. When the retailer is dominant in the supply chain, it can set  $\varphi$  as low as possible and  $\theta = 1$  to obtain maximal profit and the manufacturer will agree; when the manufacturer is dominant relative to the retailer, it can set  $\varphi$  as high as possible and  $\theta$  as the turning point to obtain maximal profit.

Looking back at the two cases at the beginning of the paper, Carrefour is a dominant retailer but Master Kong is also a powerful manufacturer in China. So if they use our model to determine their contract and solve the contradiction, they can negotiate PDS contract parameter  $\varphi$  and return policy parameter  $\theta$ . The final values for the two parameters depend on the channel power they have relative to each other in the supply chain. Similarly, Lianhua Supermarket is a dominant retailer in China but Craft Foods is also a powerful manufacturer and they also can use our model to solve their contradiction.

**ACKNOWLEDGMENT**

This Research is supported by the National Natural Science Foundation of China (71301145), the Ministry of Education, Humanities and Social Science Research Fund of China (11YJC630159) and the Contemporary Business and Trade Research Center of Zhejiang Gongshang University which is the Key Research Institute of Social Sciences and Humanities Ministry of Education (12JDSM05Z).

**REFERENCES**

Bernstein, F. and A. Federgruen, 2005. Decentralized supply chains with competing retailers under demand uncertainty. *Manage. Sci.*, 51: 18-29.  
 Cachon, G.P. and M.A. Lariviere, 2005. Supply chain coordination with revenue-sharing contracts: Strengths and limitations. *Manage. Sci.*, 51: 30-44.  
 Pan, K., K.K. Lai, L. Liang and S.C.H. Leung, 2009. Two-period pricing and ordering policy for the dominant retailer in a two-echelon supply chain with demand uncertainty. *Omega*, 37: 919-929.  
 Petruzzi, N.C. and M. Dada, 1999. Pricing and the newsvendor problem: A review with extensions. *Operat. Res.*, 47: 183-194.