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Simulation Analysis and Comparison of Effects of Various Materials on Elastic Wave Propagation Velocity

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Abstract: According to elastic theory, the motional differential function is established based on accurate boundary conditions, aiming at propagation characteristics of elastic wave in the elastic body. With analytic method, effects of various parameters of same materials and effect of various materials on elastic wave are completely simulated and analyzed. Finally, the accurate analytic solutions of diverse transverse and longitudinal waves are obtained. Moreover, effects of different materials on propagation velocity are compared.

Key words: Elastic wave, transversal wave, longitudinal wave, propagation velocity, simulating calculation

INTRODUCTION

The static problems of objects are considered in the engineering practice as well as dynamic problems. For example, certain parts of vehicle and airplane structure, bridge decks of bridge structure, some components of mechanical structure and equipment etc. Displacement, deformation, stress of structure like this always change with time. It is not only function of position coordinates, but also function of time. Elastic body doesn't cause displacement, deformation, stress in all position of the elastic body when get the load in static equilibrium condition. At the beginning of loading, the part far away from load application point is not disturbed. After the lord begins, displacement, deformation, stress caused by load propagate to other parts with a certain speed in waves. This kind of wave is elastic wave that investigated in this study.

Research shows that, propagation velocity of elastic wave is influenced by material quality of elastic body itself (Fang *et al*., 1998; Mavko *et al*., 1995). Types of elastic wave, methods and means of propagation velocity analysis are largely enriched through a large number of research results (Wang *et al*., 2000; Shi and Yang, 2001) Literature (Du *et al*., 1999) analyze two propagation velocity problems of plane wave with transfer operator method. And lots of useful conclusions are obtained as a result. Finite element method and boundary element method are used on the solution of elastic wave propagation (Liu and Wei, 2003; Liu *et al*., 1999; Han *et al*., 2005). Analysis of effects of various materials parameters on elastic wave propagation velocity are not enough.

For study this issue deeply, the establishment of an effective method to analyze this issue is necessary. Based on elastic theory, with dynamic problem of elasticity analysis, by using motional differential function, geometric equation and physical equation, effects of various parameters of same materials and effect of various materials on elastic wave are completely simulated and analyzed in this article. Finally, two basic propagation forms of elastic wave in elastic body which under different conditions are obtained, that is the accurate analytic solutions of transverse and longitudinal waves. And comparison of the effects of various materials on propagation velocity is obtained.

MECHANICAL MODEL OF ELASTIC WAVE PROPAGATION

The assumptions of ideal elastic body and small displacement are used for dynamic problem discussed in this study, thus, the geometric and physical equations established for the static problem are still applicable to some point in the dynamic problem. Only the equilibrium equations of elastic body will be replaced by motional differential function for now (Nayfeh, 1991). In the establishment of elastic body's motional differential function, in addition to the stress, strength when establishing equilibrium equations, the inertial force on the elastic body should be considered:

$$\rho\frac{\partial^2 u}{\partial t^2}, -\rho\frac{\partial^2 v}{\partial t^2}, -\rho\frac{\partial^2 w}{\partial t^2}$$

where, u, v, w are displacement components of any point in elastic body, ρ is density of elastic body.

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Like the establishment of equilibrium equations of elastic body, but inertial force is taken into consideration at this time, then motional differential function of elastic body is established as follows:

$$\begin{aligned}&\frac{\partial \sigma_x}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}+F_x-\rho\frac{\partial^2 u}{\partial t^2}=0\\&\frac{\partial \sigma_y}{\partial y}+\frac{\partial \tau_{yz}}{\partial z}+\frac{\partial \tau_{xy}}{\partial x}+F_y-\rho\frac{\partial^2 v}{\partial t^2}=0\\&\frac{\partial \sigma_z}{\partial z}+\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+F_z-\rho\frac{\partial^2 w}{\partial t^2}=0\end{aligned} \tag{1}$$

Among the equations, F_x, F_y, F_z are volume force components.

The displacement components are contained in the motional differential function and the mass of the dynamic problems of elasticity are worked out by the displacement. In order to help to understand, the geometric equations and physical equation are taken into the Eq. 1 and the motional differential function which expressed by the displacement is obtained. In addition, because the body force is ignored during the elastic wave propagation, the motional differential function is expressed as:

$$\begin{aligned}&\frac{E}{2(1+\mu)\rho}\left(\frac{1}{1-2\mu}\frac{\partial g}{\partial x}+\nabla^2 u\right)-\frac{\partial^2 u}{\partial t^2}=0\\&\frac{E}{2(1+\mu)\rho}\left(\frac{1}{1-2\mu}\frac{\partial g}{\partial y}+\nabla^2 v\right)-\frac{\partial^2 v}{\partial t^2}=0\\&\frac{E}{2(1+\mu)\rho}\left(\frac{1}{1-2\mu}\frac{\partial g}{\partial z}+\nabla^2 w\right)-\frac{\partial^2 w}{\partial t^2}=0\end{aligned} \tag{2}$$

Of which:

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

$$g=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}$$

E is Young's modulus of material, μ is Poisson's ratio of material.

The Eq. 2 is basic differential equation which is needed for solving the dynamic problems by displacement. Based on the initial condition and the boundary condition of the dynamic problems, the displacement component is obtained by the equation. Then the stress component is obtained by geometric equation and physical equation. The initial conditions of dynamic problems is the function that using elastic displacement components u, v, w as the coordinates at the time t = 0 and the known function using elastic speed components $\partial u/\partial t$, $\partial v/\partial t$, $\partial w/\partial t$ as the coordinates at the time t = 0. The processing method of the boundary condition is the same with that of displacement boundary condition and the stress boundary condition of the static force problems, as they are applicable at any time in the dynamic problem.

SIMULATION ANALYSIS OF TWO KINDS OF ELASTIC WAVE PROPAGATION VELOCITY

Two basic forms of elastic wave propagation will be discussed in the study, the longitudinal and transversal waves. Longitudinal wave is analyzed first, the displacement potential function is as follows:

$$\varphi=\varphi(x, y, z, t) \tag{3}$$

The displacement components can be expressed as follows:

$$u=\frac{\partial\varphi}{\partial x},\ v=\frac{\partial\varphi}{\partial y},\ w=\frac{\partial\varphi}{\partial z} \tag{4}$$

Therefore, g in motional differential function (2) can be written as:

$$g=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=\nabla^2\varphi \tag{5}$$

The equation above is taken to motional differential function (2), after simplification, elastic wave propagation velocity of longitudinal wave can be obtained:

$$s_1=\sqrt{\frac{E(1-\mu)}{(1+\mu)(1-2\mu)\rho}} \tag{6}$$

It can clearly be seen from the above equation, when different material parameter E, μ, ρ are taken, different propagation velocity of longitudinal wave will be obtained.

Accurate analytic solution of the propagation velocity under different conditions can be obtained by using the method described above. The specific calculation process won't be described here for the limited space. All the calculation and simulation in the paper are completed in the software Mathematica, because Mathematica has a powerful function processing function. For the impact of different factors, comprehensive simulation calculation can be conduct about the propagation velocity by the analytic expression.

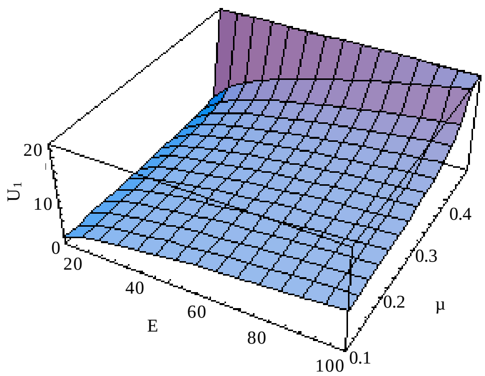

Fig. 1: Relationship of longitudinal wave speed between Young's modulus and Poisson's ratio ($\rho = 0.05$)

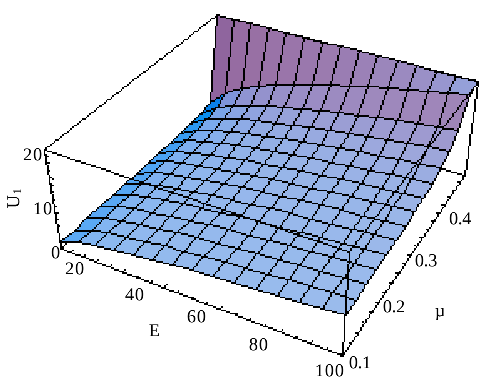

Fig. 2: Relationship of longitudinal wave speed between Young's modulus and Poisson's ratio ($\rho = 0.09$)

For the situation of the various parameters of same material and various materials, the calculation results are given in the following figures.

Effect of Young's modulus and Poisson's ratio on elastic body longitudinal wave propagation velocity while various material density ($\rho = 0.05$, $\rho = 0.09$) are described by Fig. 1 and 2, respectively. It can be seen from the figure that as Young's modulus and Poisson's ratio increase, the longitudinal wave propagation velocity increases too. To Poisson's ratio, the increase of propagation velocity is gentle at the beginning, while Poisson's ratio reached about 0.47, propagation velocity increase rapidly. Besides, it can be seen clearly through the comparison of Fig. 1 and 2 that, propagation velocity decreases as the density increases.

By the same analysis, transversal wave propagation velocity is obtained:

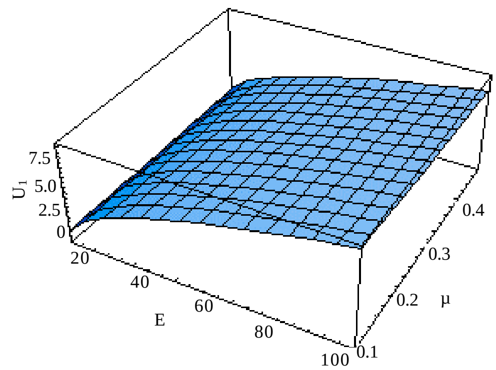

Fig. 3: Relationship of transversal wave speed between Young's modulus and Poisson's ratio ($\rho = 0.05$)

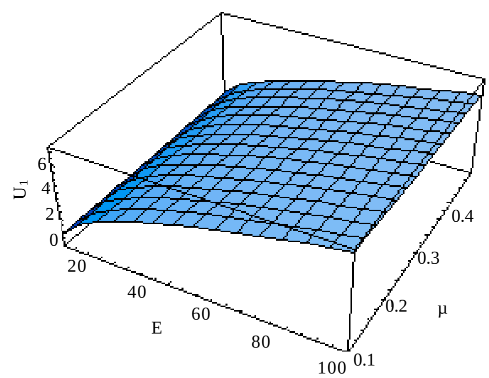

Fig. 4: Relationship of transversal wave speed between Young's modulus and Poisson's ratio ($\rho = 0.09$)

$$s_2 = \sqrt{\frac{E}{2(1+\mu)\rho}} \tag{7}$$

Based on the equation above, the transversal wave propagation velocity is completely simulated. The calculation results of the various parameters of same material and various materials are given in the following figures.

Effect of Young's modulus and Poisson's ratio on elastic body Transversal wave propagation velocity while various material density ($\rho = 0.05$, $\rho = 0.09$) is described by Fig. 3 and 4, respectively. It can be seen from the figure that, the difference from longitudinal wave is that, while Young's modulus increases, propagation velocity increases as well. But while Poisson's ratio increases, propagation velocity decreases on the contrary. By the comparison between Fig. 3 and 4, it can be seen that the

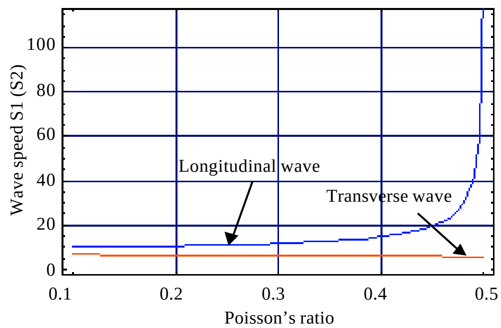

Fig. 5: Speed comparison between transversal wave and longitudinal wave

speed decreases with the increasing of density. Besides, longitudinal wave is compared with transversal wave and the propagation velocity of the former is greater than the latter obviously at the same condition. To illustrate this point intuitive, the effects of the Poisson's ratio on longitudinal wave and transversal wave while the Young's modulus are fixed value are given at this following figure.

It is clearly observed from the graph that the velocity of the transversal wave is less than the longitudinal wave. As a result, signal of longitudinal wave was always received firstly at the measurement of elastic wave (Fig. 5).

COMPARISON OF EFFECTS OF YOUNG'S MODULUS ON ELASTIC WAVE PROPAGATION VELOCITY

The effect of materials on elastic wave propagation velocity is discussed with the example of two kinds of Young's modulus. The effects of Young's modulus on transversal and longitudinal wave propagation velocity on the condition of certain Poisson's ratio and certain density are studied here:

- $\mu = 0.25$, $\rho = 0.67$
- $\mu = 0.35$, $\rho = 0.79$

In this two different cases above, propagation velocity can be calculated specifically by using the method described above. The results are given in the following two figures.

In the two figures above, blue line and red line represent the longitudinal and transversal wave propagation velocity in the first and second case. It can be observed from the two figures above that with the increasing of Young's modulus, propagation velocity of

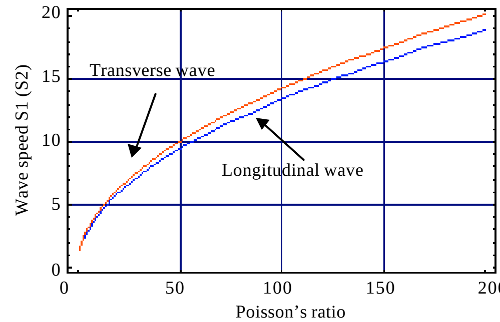

Fig. 6: Speed comparison of longitudinal wave under two kinds of conditions

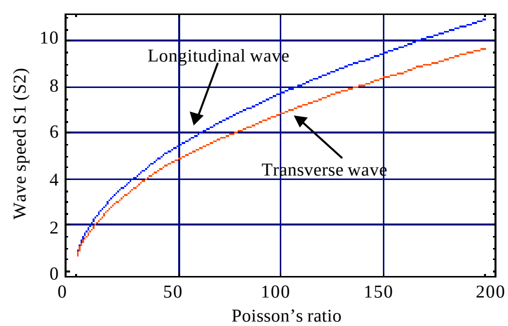

Fig. 7: Speed comparison of transversal wave under two kinds of conditions

longitudinal and transversal wave increase. The propagation velocity of longitudinal wave (Fig. 6) in second case is greater than the first case. It is the opposite for the transversal wave (Fig. 7). And once again these two figures show the conclusion that the propagation velocity of longitudinal wave is greater than that of transversal wave.

CONCLUSION

According to elastic theory, calculation models of elastic wave propagation were established in this study. Based on analytic method, with dynamic problem of elasticity analysis, by using motional differential function, geometric equation and physical equation, effects of various parameters of same material and effects of various materials on elastic wave are completely simulated and analyzed. Finally, the various accurate analytic solutions of various transverse and longitudinal waves are obtained. Moreover, effects of various materials on propagation velocity are compared.

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