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## Minimum Effort Reliability Allocation Method Considering Fuzzy Cost of Punching Machine Tools

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**Abstract:** Punching machine tools is divided into 6 serial subsystems, including transmission system, electrical system, hydraulic system, pneumatic system, sliding system and lubrication system. For the purpose of minimizing the fuzzy cost, under the constraint of MTBF of punching machine tools in random failure period, a reliability allocation model based on a new effort function is constructed by adopting analysis hierarchy process used to ascertain cost weights of reliability factors. In an application case a simulated annealing algorithm is used for solving the minimum effort reliability allocation model which is transformed into unconstraint optimization model by an external penalty function method. A reliability allocation scheme with minimum fuzzy cost of improved punching machine tools could be got, reliability indexes such as average failure rate, MTBF, failure frequency of punching machine tools could be obtained.

**Key words:** Reliability of punching machine tools, fuzzy cost, reliability allocation, effort function

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### INTRODUCTION

Reliability is one of the key indicators to evaluate the performance of machine tools. At present, reliability of chinese punching machine tools is at a low level compared to others. Reliability design is the key to improve the reliability of punching machine tools. Reliability allocation is an important part in the design process. Commonly reliability allocation methods include equal allocation method, AGREE method (Ping, 1997), fuzzy allocation method (Yang *et al.*, 2011), minimum effort method (Zhang *et al.*, 2009) and so on. Equal allocation method is usually used at the early design stage in the case of unclear machine description and functional division. The method is simple, but not for complex system with different reliability levels of subsystems. AGREE method is proposed by U.S. electronic equipment reliability advisors group, the importance of subsystems for whole machine and the complexity of subsystems are mainly considered in this method. Fuzzy allocation method is based on the experience of experts or engineers who would score those indicators which affect reliability of subsystems and according to the scoring results reliability indexes of all the subsystem are allocated. The method is directly dependent on the experience of experts to allocate reliability indexes which is often considered to be subjective and lack of objective basis.

Cost is one of the main factors affecting reliability level of machine tools. Costs of punching machine tools not only include direct cost but also include

indirect cost, such as technology cost, maintenance cost, environmental cost, repair cost and failure hazard loss. There is certain fuzziness for indirect cost. Direct cost and indirect cost are collectively referred as fuzzy costs in this paper which include all the costs for improving product reliability. Because of the fuzziness of total cost of machine tools, it is very hard to get detailed statistics relationship between cost and reliability which is hard to establish empirical relationship. Effort function (Pabst and Muller, 2006; He *et al.*, 2012) is used to reflect the relationship between cost and reliability of product. Dale and Winter bottom proposed a number of rules for effort function to overcome the problem of lack of statistical data (Zhang *et al.*, 1990). According to the conditions, a new effort function would be constructed based on average failure rate. Based on the function, a minimum effort reliability allocation model (Liu *et al.*, 2002) would be constructed for the purpose of minimizing the fuzzy cost under the constraint of MTBF. Finally, the minimum effort reliability allocation model is applied to actual reliability allocation of a certain type of chinese punching machine tools in random failure period.

### PUNCHING MACHINE TOOLS RELIABILITY BLOCK DIAGRAM

In accordance with the principle of functional independence, the punching machine tools is divided into 6 serial subsystems, including transmission system, electrical system, hydraulic system, pneumatic system,

sliding system and lubrication system. The reliability block diagram of punching machine tools is shown in Fig. 1.

For the punching machine tools, failure rate varies with different stages of the life cycle. In general, the failure frequency and failure rate are constant in random failure period. According to the reliability logic diagram of punching machine tools, the average failure rate  $\lambda$ , Mean Time Between Failures (MTBF) and failure frequency  $f$  of the machine tools satisfies the following theoretical relationship shown in Eq. 1:

$$\begin{cases} \lambda_j = \frac{1}{m_j} \\ \lambda = \sum_{j=1}^s \lambda_j \\ \lambda_j = \frac{f_j}{M} \end{cases} \quad (1)$$

In Eq. 1,  $\lambda_j$  represents the average failure rate of the  $j$ th subsystem;  $\lambda$  represents the average failure rate of the whole machine, the unit is times/hour;  $s$  represents the total number of subsystems,  $s = 6$ ;  $f_j$  represents the failure

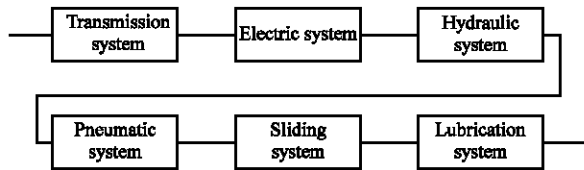


Fig. 1: Punching machine reliability logic diagram

frequency of the  $j$ th subsystem;  $m_j$  represents the MTBF of the  $j$ th subsystem, the unit is hour;  $M$  represents the MTBF of the whole machine.

### PUNCHING MACHINE TOOLS FUZZY COST FACTORS

There are so many factors affecting the reliability of punching machine tools. Analyzing from improving reliability of machine tools, main factors leading to increased fuzzy cost include direct cost, but also include indirect cost, such as technology cost, maintenance cost, environmental cost, repair cost and failure hazard loss. The direct cost of the machine is the sum of the costs of all machine parts, mainly including the costs used for the purchase of materials, outsourcing and spare parts, etc. Technology cost is the fees used for improving design level, manufacture, assembly and other technologies of machine tools. Maintenance cost is the fees used for routine maintenance of machine tools. Environmental cost is the fees used for protecting the environment for the normal operation of machine tools. Because punching machine tools is repairable system, repair cost is the fees used for repairing machine tools. Failure hazard loss is the fees which is lost when serious failures happen except repair cost.

Because of the fuzziness of the cost, expert fuzzy scoring (Peng *et al.*, 2005) and AHP method are adopted to quantify all the costs spent on different factors which are used for achieving quantitative reliability allocation of punching machine tools. As is shown in Fig. 2, fuzzy cost

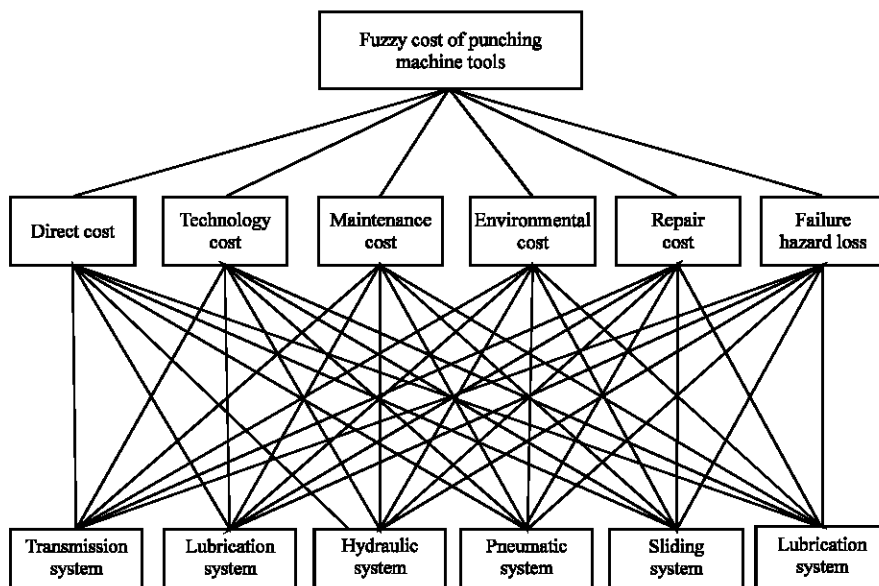


Fig. 2: Hierarchy chart of fuzzy cost of punching machine tools

of punching machine tools includes 3 layers, the first layer is target layer which is the total cost of punching machine tools. The second layer is criterion layer which is all the costs spent on different factors. The third layer is program layer which is the fuzzy costs of all subsystems.

AHP method is applied to determine weights of all the costs in different factors. Firstly, judgment matrix D should be constructed, as is shown in Eq. 2:

$$D = \begin{bmatrix} u_{11} & u_{12} & \dots & u_{1n} \\ u_{21} & u_{22} & \dots & u_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ u_{n1} & u_{n2} & \dots & u_{nn} \end{bmatrix} \quad (2)$$

In Eq. 2,  $u_{ij}$  represents the ratio between the cost spent on the  $i$ th factor and the cost spent on the  $j$ th factor.  $u_{ji}$  is the inverse of  $u_{ij}$ .  $n$  is the number of all the factors, its value is 6. 1-9 scale is used for expert fuzzy scoring. The scoring standard refers to Table 1.

Then follow the steps below to calculate the weights of fuzzy costs. Homogeneous linear equation, that is Eq. 3, is a general way to calculate the eigenvector corresponding to eigenvalue of judgment matrix D (Wang *et al.*, 2012).  $\lambda_{max}$  is the largest eigenvalue of judgment matrix D. According to Eq. 3 the eigenvector corresponding to  $\lambda_{max}$  could be got and then normalize the eigenvector, cost weight vector  $W = [w_1, w_2, \dots, w_1, \dots, w_n]$  of all factors is got.  $w_i$  represents the cost weight of the  $i$ th factor:

$$\begin{cases} (u_{11} - \lambda)w_1 + u_{12}w_2 + \dots + u_{1n}w_n = 0 \\ u_{21}w_1 + (u_{22} - \lambda)w_2 + \dots + u_{2n}w_n = 0 \\ \dots \\ u_{n1}w_1 + u_{n2}w_2 + \dots + (u_{nn} - \lambda)w_n = 0 \end{cases} \quad (3)$$

$$CR = \frac{CI}{RI} \quad (4)$$

Finally, consistency test should be conducted according to Eq. 4. In Eq. 4, CR is called random consistency ratio of judgment matrix. CI is called the general consistency index of judgment matrix. The value of CI satisfies Eq. 5. RI is called average random consistency index of judgment matrix. For the judgment matrix from 5 to 9 orders, the value of RI refers to Table 2. When  $CR < 0.1$ , it is considered that the judgment matrix passes consistency test shown that cost weights are reasonable. Otherwise updates expert fuzzy scoring until the judgment matrix passes consistency test:

$$CI = \frac{\lambda_{max} - n}{n - 1} \quad (5)$$

According to AHP method above, all the cost weights of different factors are listed in Table 3. As is shown in Table 3, the weight of direct cost is maximum that is consistent with the actual situation of enterprises. Technology cost is same as repair cost ranked second, showing that repair cost is very high and it is significant to reduce failures and fuzzy costs by improving reliability of punching machine tools. Based on the above analysis and method, the cost weights of each subsystem of all factors are listed in Table 4.

**SUBSYSTEM RELIABILITY EFFORT FUNCTION**

Effort function has the generality for describing the relationship between cost and reliability of most systems. When the actual cost and reliability data are known, exact function could be obtained by curve fitting. In many cases, however, it is hard to get specific function because of the lack of objective

Table 1: Scoring scales for comparison of multi-factors costs of punching machine tools

Scoring scales	Meaning
1	Equivalence
3	A little expensive
5	Quite expensive
7	Very expensive
9	Extremely expensive
2, 4, 6, 8	Between above scales

Table 2: Values of RI for consistency test

n	5	6	7	8	9
RI	1.12	1.24	1.32	1.41	1.45

Table 3: Factor cost weights of punching machine tools

Factor cost	Direct cost	Technology cost	Maintenance cost	Environmental cost	Repair cost	Failure hazard loss
weight $w_i$	0.466	0.1757	0.0904	0.0461	0.1757	0.0461

Table 4: Factor cost weights of all subsystems of punching machine tools

Factor loss	Transmission system	Electrical system	Hydraulic system	Pneumatic system	Sliding system	Lubrication system
Direct cost	0.3791	0.1989	0.1410	0.0736	0.1368	0.0705
Technology cost	0.3127	0.2047	0.1798	0.0651	0.1379	0.0998
Maintenance cost	0.2016	0.1611	0.1473	0.1319	0.1568	0.2014
Environmental cost	0.1202	0.1726	0.2029	0.1469	0.0909	0.2665
Repair cost	0.4157	0.1083	0.0855	0.0438	0.2158	0.1310
Failure hazard loss	0.4058	0.0950	0.1382	0.0483	0.2367	0.0760
Total cost	1.8351	0.9406	0.8947	0.5096	0.9749	0.8452

statistical data. The following rules of effort function are proposed by Dale and Winie bottom for solving the problem.

Common effort function can be divided into two categories: Discrete effort function and continuous effort function. Discrete effort function can be obtained by interpolation and fitting based on the actual statistical data from similar systems. In fact it is hard to get the actual statistical data from similar systems. To solve the problem of lack of data, 6 rules above of continuous effort function are proposed by engineers based on long-term experience. Common continuous effort function includes lagrange effort function (Liu and Liang, 2004), two-parameter logarithmic effort function (Liu and Zhang, 2005), etc. Some empirical parameters with unclear physical meaning are usually contained in the common continuous effort functions. According to the rules of continuous effort function, a new continuous effort function is defined based on average failure rate of system in this study. As is shown in Eq. 6:

$$C(\lambda_0, \lambda) = ac \ln \frac{\lambda_0}{\lambda} \tag{6}$$

In Eq. 6,  $\lambda_0$  is the current average failure rate of system,  $\lambda$  is the reduced average failure rate of system,  $C(\lambda_0, \lambda)$  represents the cost required to reduce average failure rate from  $\lambda_0$  to  $\lambda$ ,  $\ln(\lambda_0/\lambda)$  represents the effort required to reduce average failure rate from  $\lambda_0$  to  $\lambda$ ,  $c$  is the current total cost of system,  $a$  is an undetermined empirical parameter which is a constant. According to the rules of continuous effort function and the negative correlation between reliability and failure rate, corresponding rules of continuous effort function based on average failure rate can be clearly obtained as below:

$$C(\lambda_0, \lambda) = ac \ln \frac{\lambda_0}{\lambda} \geq 0$$

Always holds,  $0 < \lambda < \lambda_0$ :

$$C(\lambda_0, \lambda_1) + C(\lambda_1, \lambda_2) = ac \ln \frac{\lambda_0}{\lambda_2} = C(\lambda_0, \lambda_2)$$

and  $0 < \lambda_2 < \lambda_1$ :

- $C(\lambda_0, \lambda)$  is differentiable in the interval  $[\lambda_0 + \infty)$ :

$$\frac{\partial^2}{\partial \lambda^2} C(\lambda_0, \lambda) = \frac{ac}{\lambda^2} > 0$$

always holds in the interval  $[\lambda_0 + \infty)$  and  $c(\lambda)$  is a convex function

- For any fixed  $\lambda$ ,  $0 < \lambda$ , if  $\lambda$  approaches 1,  $C(\lambda_0, \lambda)$  approaches infinity:

$$\frac{dC(\lambda_0, \lambda)}{d\lambda} = -\frac{k}{\lambda} < 0$$

always holds in the interval  $[\lambda_0 + \infty)$ ,  $C(\lambda)$  is a monotonic decreasing function

### MINIMUM EFFORT RELIABILITY ALLOCATION MODEL

Based on the cost weights of all subsystems in a variety of reliability factors, under the constraint of MTBF of punching machine tools in random failure period, a minimum effort reliability allocation model based on the new cost function above is constructed by taking minimum fuzzy cost of punching machine tools as the target.

**Target of reliability allocation model:** Cost weights matrix  $H$  of all subsystems of old machine tools is got by expert fuzzy scoring and AHP. As is shown below:

$$H = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1n} \\ h_{21} & h_{22} & \dots & h_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ h_{n1} & h_{n2} & \dots & h_{nn} \end{bmatrix} \tag{7}$$

In Eq. 7,  $h_{ij}$  represents the  $i$ th cost weight of reliability factor of the  $j$ th subsystem of old machine tools.  $c_j$  represents the cost of the  $j$ th subsystem of old machine tools. As is shown in Eq. 8:

$$c_j = \sum_{i=1}^n w_i h_{ij} \tag{8}$$

Then the total fuzzy cost  $C$  of whole old machine tools is shown in Eq. 9:

$$C = \sum_{j=1}^n \sum_{i=1}^n w_i h_{ij} \tag{9}$$

According to Eq. 6, the additional cost  $\Delta c'_j$  increased after reliability allocation of  $j$ th subsystem is shown in Eq. 10:

$$\Delta c'_j = ac_j \ln \frac{\lambda_j}{\lambda'_j} \tag{10}$$

In Eq. 10,  $\lambda_j$  represents the average failure rate of the  $j$ th subsystem of old machine tools,  $\lambda'_j$  represents the average failure rate of the  $j$ th subsystem of new machine tools. Then the total fuzzy cost  $C'$  of whole new machine tools is shown in Eq. 11:

$$C' = C + \sum_{j=1}^s \Delta c'_j = C + a \sum_{j=1}^s c_j \ln \frac{\lambda_j}{\lambda'_j} \tag{11}$$

In the target function,  $C$  and  $a$  are constants which would not affect the design variables of reliability allocation model. In order to simplify the target function, the minimum cost is converted to the minimum effort shown in Eq. 12:

$$\text{Min} = \sum_{j=1}^s c_j \ln \frac{\lambda_j}{\lambda'_j} = \sum_{j=1}^s \sum_{i=1}^n w_i h_{ij} \ln \frac{\lambda_j}{\lambda'_j} \tag{12}$$

**Constraints of reliability allocation model:** MTBF is the reliability allocation indicator in this study. Let  $M'$  be the value of MTBF of new machine tools. According to theoretical Eq. 1, the following relationship shown in Eq. 13 exists between average failure rate  $\lambda'_j$  of new machine tools and  $M'$ :

$$\sum_{j=1}^s \lambda'_j \leq \frac{1}{M'} \tag{13}$$

Additionally the average failure rate of subsystems of new machines needs to meet the range constraint, as is shown in Eq. 14:

$$0 < \lambda'_j < \lambda_j \tag{14}$$

**Construction of reliability allocation model:** Taking minimum fuzzy cost of punching machine tools as the target, under the constraint of MTBF in random failure period, the minimum effort reliability allocation model is constructed as below:

$$\begin{aligned} \text{Min} &= \sum_{j=1}^s \sum_{i=1}^n w_i h_{ij} \ln \frac{\lambda_j}{\lambda'_j} \\ \text{s.t.} &\begin{cases} \sum_{j=1}^s \lambda'_j \leq \frac{1}{M'} & (j=1,2,\dots,s; i=1,2,\dots,n) \\ 0 < \lambda'_j < \lambda_j \end{cases} \end{aligned} \tag{15}$$

In Eq. 15,  $s$  represents the number of all subsystems of the punching machine tools and  $s = 6$ .  $n$  represents the number of all the reliability factors and  $n = 6$ .  $w_i$  represents the cost weight of  $i$ th reliability factor.  $h_{ij}$  represents the  $i$ th cost weights of reliability factor of the  $j$ th subsystem of old machine tools.  $M'$  is the value of MTBF of new machine tools.  $\lambda'_j$  is the average failure rate of new machine tools.  $\lambda_j$  is the average failure rate of old machine tools.

### RELIABILITY ALLOCATION OF PUNCHING MACHINE TOOLS

Take a certain type of punching machine tools as the research object of which the MTBF is 329.5 hours according to the data collected from machine tools enterprises. And fuzzy cost scores of the punching machine tools in variety of reliability factors are provided by technical staffs in machine tools enterprises. MTBF of improved punching machine tools is set at 500 h in the application case. Minimum effort reliability allocation model is solved by adopting external penalty function method (Chen *et al.*, 1997; Meng *et al.*, 2010) and simulated annealing algorithm (Salcedo *et al.*, 2006). Results of reliability allocation indexes in the application case are listed in Table 5.

Through analyzing the effort order of all subsystems transmission system is unnecessary to improve reliability because of the maximum cost under the constraint: MTBF = 500 h. It is most beneficial to improve reliability of pneumatic system for saving the total cost of punching machine tools. Additionally there is a positive correlation between fuzzy costs and efforts of all the subsystems. In the application case the specific reliability allocation

Table 5: Results of reliability allocation index for punching machine tools

Index	Machine category	Transmission system	Electrical system	Hydraulic system	Pneumatic system	Sliding system	Lubrication system
Average failure rate ( $10^{-4}$ h)	Old	2.993846	8.261538	3.830769	5.747692	2.753846	6.760000
	New	2.993846	5.124339	2.762076	3.098174	2.246620	3.775471
MTBF (h)	Old	3340.180000	1210.420000	2610.440000	1739.820000	3631.280000	1479.280000
	New	3340.180000	2013.610000	3547.230000	3210.200000	4482.740000	2580.290000
Failure frequency	Old	0.098700	0.272200	0.126200	0.189400	0.090700	0.222800
	New	0.149692	0.255584	0.135688	0.154483	0.114560	0.190018
Subsystem effort	0	0.473618	0.327875	0.644445	0.214925	0.559471	

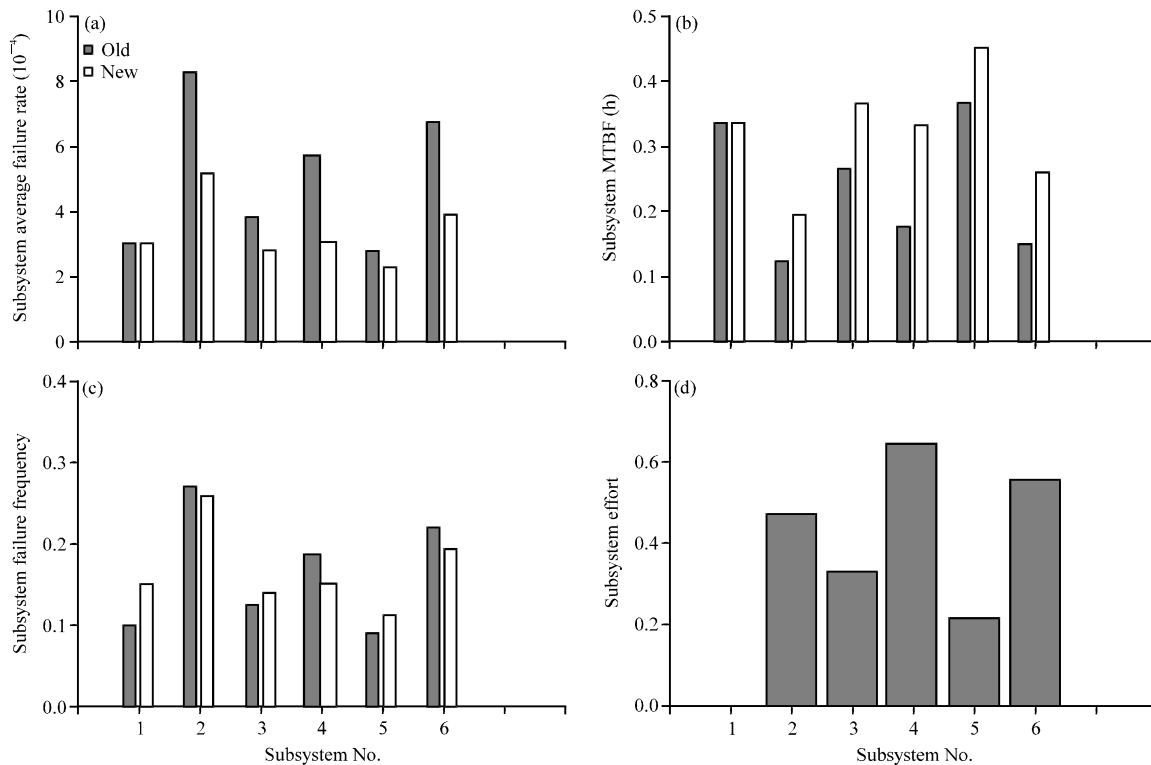


Fig. 3(a-d): Histogram for reliability allocation indexes of punching machine tools

scheme with the minimum fuzzy cost and the floor cost-effectiveness ratio is listed in Table 5. The histogram of reliability allocation indexes is shown below in Fig. 3.

**CONCLUSION**

Quantitative method for evaluating fuzzy cost of punching machine tools is proposed by adopting expert fuzzy scoring and AHP in this study. And cost weights of all the subsystems of punching machine tools in many reliability factors are got by the method above. According to the rules of continuous effort function and engineering experience a new effort function based on failure rate which is used for establishing the statistical relationship between reliability and fuzzy cost of the punching machine tools is proposed. A reliability allocation scheme with minimum fuzzy cost of improved punching machine tools could be got through the minimum reliability allocation model. The results of the application case of punching machine tools show that pneumatic system is the subsystem with the floor cost-effectiveness ratio.

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