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Analysis for Lateral Collision Risk Assessment of RNAV Parallel Routes

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Abstract: With the development of performance based navigation technologies, the plan of area navigation parallel routes is promoted now. For the safety of the newly implemented RNAV parallel routes, the quantitative risk analysis must be performed. In this study, the lateral collision risk assessment of RNAV parallel routes has been carried out. Firstly, the improved Reich Model is introduced. Then the methodology of lateral and vertical overlap probability are focused on. Different from the most of previous research, the non-core distribution is modeled by a "Separated Double Exponential" distribution which is more realistic than Double Exponential distribution and the vertical overlap probability is calculated through the tolerance value of maintained level according to CCAR-93TM-R4. Finally, the Collision Risk of Beijing-Shanghai parallel routes based on RNAV2 is evaluated. Results shows that the parallel routes can satisfy safety requirement and indicates that the method is feasible.

Key words: Collision risk, parallel route, RNAV, probability of lateral overlap, probability of vertical overlap

INTRODUCTION

According to China's PBN implementation roadmap, PBN based on the instrument flight procedures will be implemented to all ATS routes by 2017. At present, several area navigation parallel routes are designed and implemented. For the safety assessment of the newly implemented RNAV parallel routes, the quantitative risk analysis must be performed.

Reich (1966a, b, c) established the Reich Model which had been widely used for safety assessment to analyze the long-range air traffic systems separation standards. Davis and Sharpe (1993) estimated collision risk of the 60 n mile separation of parallel routes of North Atlantic by using the Reich Model and assessed the safety taking into account the traffic sample data of this airspace. Moek *et al.* (2001) used the similar method to analyze the risk assessment of RNP10 and RVSM in the South Atlantic flight identification regions. Then, the regional airspace safety assessment for separation were conducted. Mito (2003) investigated the effect of lateral offsets on the lateral collision risk for the existing NOPAC routes. The qualitative risk assessment for the RNP 10 ATS routes L510, N571, P628 and P762 for the longitudinal collision risk had been carried out using Reich Model by ICAO (2011).

Domestic research on separation standard started relatively later. Wang and Xu (2001) analyzed the lateral collision risk by probability theory (Ying and Xu, 2002)

did initially research about Reich Model on the lateral collision risk between aircraft in the airspace. Han *et al.* (2006) assessed the safety of parallel routes based on area navigation. Zhang *et al.* (2007) did an analysis of collision risk on parallel routes based on VOR navigation using improved Reich Model; Zhang *et al.* (2009) researched lateral collision risk model based on CNS position error.

In this study, we do an analysis for lateral collision risk assessment of RNAV parallel routes on the basis of our research achievement.

We first get the improved Reich lateral collision Model. And then the latest achievement of the affected factors of lateral collision risk is introduced which are focused on methodology of probability of lateral overlap and the probability of vertical overlap. Next, combining the above aspects, the lateral collision risk of RNAV parallel routes is assessed. Finally, an example is analyzed. The result shows that the model is feasible.

IMPROVED REICH MODEL

Reich model: The Reich model is the most widely used for analyzing collision risk. The key of the model is that each aircraft is considered as a rectangle box which has the mean sizes of λ_x , λ_y , λ_z . These sizes represent the mean length, the width and the height of each aircraft group, respectively. The collision risk between two boxes is equal to the collision risk between a point and a box having the sizes of $2\lambda_x$, $2\lambda_y$, $2\lambda_z$ in mathematics which is called

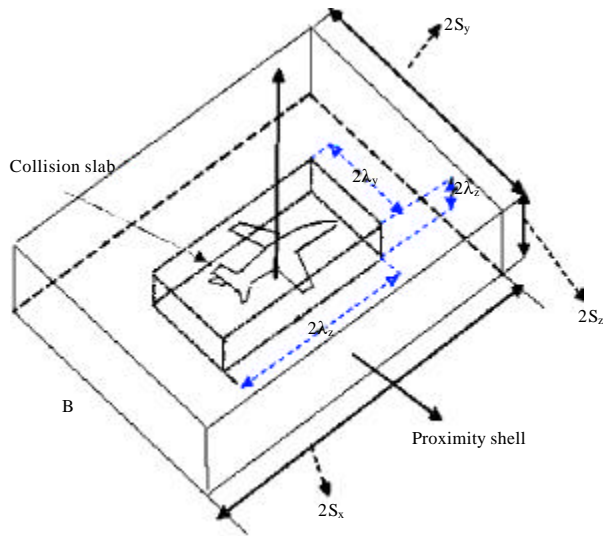


Fig. 1: Collision slab and proximity shell

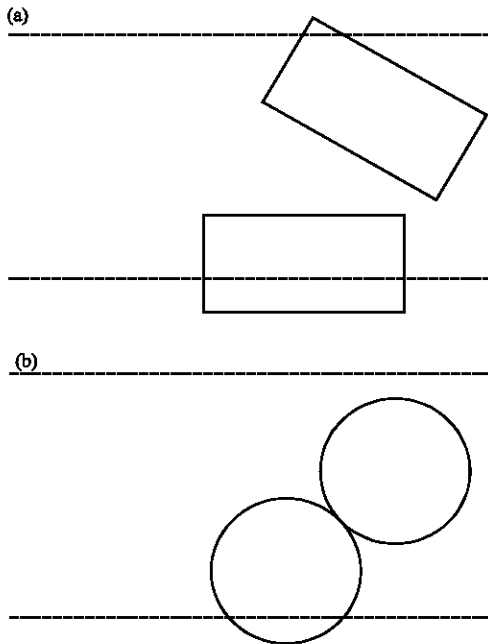


Fig. 2: Sketch map for aircraft yawing movement

collision slab. The other concept is proximity shell which is rectangle box with the sizes of $2S_x$, $2S_y$, $2S_x$, represent lateral, longitudinal and vertical separation standard. They are shown in Fig. 1.

Improved reich model: When aircraft deviate from parallel route, cylinder collision slab is more rational than rectangle collision slab and the collision risk is more accurate. The difference is shown in Fig. 2.

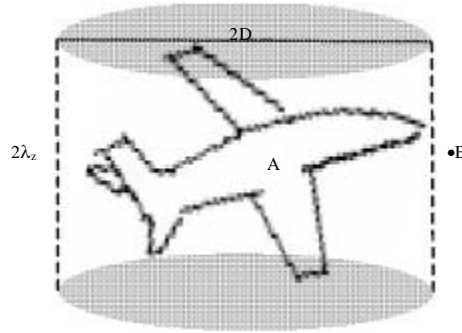


Fig. 3: Collision slab

So, the improved Reich Model is defined as follows: the collision slab is a cylinder of $2D$ radius and height $2\lambda_z$, wherein D represents average length of an aircraft flying on parallel routes. As shown in Fig. 3.

LATERAL COLLISION RISK ASSESSMENT OF PARALLEL ROUTES

In order to make the complex problem simple, some assumptions need to be given:

- It is assumed that the number of aircraft involved in one collision is two and only two
- It is assumed that there is no corrective action by pilots or ATC when two aircraft are about to collide
- It is assumed that aircraft flies in area control center with constant cruising speed

Considering lateral collision risk, the vertical separation must be zero. The improved Reich lateral collision risk model was used to estimate the level of safety for lateral collision of the parallel routes. The model is as follows:

$$N_{ay} = P_y(S_y)P_z(0)\frac{D}{S_x} \left[E_y(\text{same}) \left\{ \frac{|\Delta v|}{2D} + \frac{|\bar{y}|}{2D} + \frac{|\bar{z}|}{2\lambda_z} \right\} + E_y(\text{opp}) \left\{ \frac{2|\bar{v}|}{2D} + \frac{|\bar{y}|}{2D} + \frac{|\bar{z}|}{2\lambda_z} \right\} \right] \quad (1)$$

The parameters in Eq. 1 are defined as follows:

- N_{ay} = Expected number of accidents (two for every collision) per flight hour due to the loss of lateral separation between co-altitude aircraft flying on tracks with planned S_y mile lateral separation
- S_y = Minimum planned lateral separation

- $P_y(S_y)$ = Probability that two aircraft assigned to two parallel routes with S_y mile lateral separation will lose all planned lateral separation
- $P_x(0)$ = Probability that two aircraft assigned to same flight level are at same geometric height
- $E_y(\text{same})$ = Same direction lateral occupancy at same assigned flight level
- $E_y(\text{opp})$ = Opposite direction lateral occupancy at same assigned flight level
- S_x = Length of half the interval in n mile used to count proximate aircraft at adjacent routes
- λ_z = Average height of an aircraft flying on parallel routes
- $|\overline{\Delta v}|$ = Average relative speed of two aircraft flying on parallel routes in same direction
- $|\overline{v}|$ = Average ground speed on an aircraft
- $|\overline{y}|$ = Average relative lateral speed of aircraft pair at loss of planned lateral separation of S_y
- $|\overline{z}|$ = Average relative vertical speed of a co-altitude aircraft pair assigned to the same route

Some parameters in Eq. 1 change slightly and make a little effect on collision risk, so they are been got from reference or related experience. In this study, $P_y(s_y)$, $p_z(0)$ and D are mainly focused on.

Probability of lateral overlap based on RNAV: The probability of lateral overlap of aircraft nominally flying on adjacent flight paths, separated by S_y , is denoted by $p_y(s_y)$ and is defined as:

$$P_y(S_y) = P(|S_y + (A-B)| \leq D) \tag{2}$$

where, A and B are assumed to be the lateral deviations of two aircraft which are nominally separated by S_y . We assume that A and B are identically distributed but statistically independent with a distribution F_y .

When a distinction is made between normal deviations and gross deviations, the overall probability density F_y may be taken as mixture of two component densities, a core distribution $F_{\text{nom}, N_y}(y)$ and a non-core distribution $F_{\text{gross}, N_y}(y)$:

- The core distribution $F_{\text{nom}, N_y}(y)$, represents errors that derive from standard navigation system deviations related to normal deviations. These errors are always present, as navigation systems are not perfect and they have a certain precision

- The non-core distribution $F_{\text{gross}, N_y}(y)$, represents Gross Navigation Errors (GNE) related to gross deviations that corresponds to what may be viewed as non-nominal performance

Therefore, the overall lateral deviation distribution is modeled as:

$$F_y(y) = (1-\alpha)F_{\text{nom}, N_y}(y) + \alpha F_{\text{gross}, N_y}(y) \tag{3}$$

The mixing parameter α is the probability of a gross navigational error. The mixing parameter $\alpha = 2.9957 \times 10^{-5}$ according to technical requirement and standard of Boeing company by Han *et al.* (2006).

We assume that $f(y)$, $F_{\text{gross}, N_y}(y)$, $F_{\text{nom}, N_y}(y)$, represent the distribution density of F_y , $F_{\text{gross}, N_y}(y)$, $F_{\text{nom}, N_y}(y)$, respectively.

Next, the two component densities are to be specified.

The core distribution density $F_{\text{nom}, N_y}(y)$.

As the double exponential is more conservative with regard to the occurrence of deviations larger than the RNAV value, The core lateral deviation distribution density $F_{\text{nom}, N_y}(y)$ is modeled by a Double Exponential distribution with a parameter $a_1 > 0$ as the rate that is:

$$f_{\text{nom}, N_y}(y) = \frac{1}{2a_1} \exp\left(-\left|\frac{y}{a_1}\right|\right) \tag{4}$$

$F_{\text{nom}, N_y}(y)$ may be specified on the basis of the RNAV type of the airspace. The core density is determined by n-NM/95% containment under RNAVn:

$$\int_{-n}^n \frac{1}{2a_1} \exp\left(-\left|\frac{y}{a_1}\right|\right) = 0.95 \tag{5}$$

The parameter a_1 can be estimated directly from the RNAV value for the airspace. It follows from Eq. 4 and 5 that the parameter a_1 is given by:

$$a_1 = -\frac{n}{\log 0.05} \tag{6}$$

The non-core distribution density $F_{\text{gross}, N_y}(y)$:

The non-core distribution $F_{\text{gross}, N_y}(y)$ is modeled by a (Separated Double Exponential) distribution with parameters $\mu > 0$, representing the separation and $a_2 > 0$ the rate parameter that is, if $B \sim F_{\text{gross}, N_y}(y)$:

$$P(B > u + y) = \frac{1}{2} e^{-\frac{y}{a_2}} \text{ and } P(B < -u - y) = \frac{1}{2} e^{-\frac{y}{a_2}}$$

It means that the non-core distribution $F_{gross, N_y}(y)$ gives no mass in $[-u, u]$ and outside it decays as a Double Exponential distribution with rate parameter a_2 .

The density of this distribution is given by:

$$f_{gross, N_y}(y) = \begin{cases} \frac{1}{2a_2} e^{-\frac{|y+u|}{a_2}}, & \text{if } y < -u \\ 0, & \text{if } -u \leq y \leq u \\ \frac{1}{2a_2} e^{-\frac{|y-u|}{a_2}}, & \text{if } y > u \end{cases} \quad (7)$$

This modeling is similar but more realistic than Double Exponential distribution which has been used by Moek *et al.* (2001).

According to reference ICAO (2011), the parameter μ is taken to be n based on RNAV n consideration and a_2 is then estimated by maximizing the wingspan overlap probability by selecting a_2 equal to the designated separation minimum. The estimated value of $a_2 = S_y$.

The parameter D:

$$D = 0.0381 \times 0.57 + 0.0203 \times 0.38 = 0.0294$$

According to the statistic of the distribution of flights along Beijing-Shanghai parallel routes, the population is dominated by Boeing type and Airbus type, the percentage of these two types is around 57 and 38%, respectively among the total number of flights. The typical types are B747-400 and A320-100. Their performance data is displayed in Table 1.

Probability of vertical overlap $P_z(0)$: $P_z(0)$ is always a constant value in most of safety assessment. However, it is different in specified airspace. The probability of vertical overlap is estimated by “the tolerance value used to determine that pressure-altitude-derived level information displayed to the controller is accurate shall be ± 60 m (± 200 ft) in RVSM airspace”.

The probability of vertical overlap of aircraft is defined by:

$$P_z(0) = P(|Z_1 - Z_2| \leq \lambda_z) \quad (8)$$

where, Z_1 and Z_2 are the height deviations of two aircraft.

Table 1: Performance data per type

Variable	B747-400	A320-100
Length (nm)	0.0381	0.0203
Height (nm)	0.0105	0.0064

We assume that Z_1 and Z_2 are statistically independent with distribution F_z . Here we only assume that F_z is a Double Exponential distribution with parameter $\beta > 0$ that is, with density function:

$$f(z) = \frac{1}{2\beta} \exp\left(-\frac{|z|}{\beta}\right) \quad (9)$$

Now we consider the aircraft is flying during area control center. According to CCAR-93TM-R4, the tolerance value of height displayed by radar screen shall be ± 60 m (± 200 ft) in RVSM airspace. So a typical aircraft stays within ± 200 ft of its assigned flight level 95% of the time.

The parameter β is given by:

$$\beta = -\frac{0.032915}{\log 0.05} = 0.010987$$

CASE STUDY

We take Beijing-Shanghai parallel routes as an example. The airspace description is as follows: the routes is operated with RNAV2; the minimum lateral separation is 10.8 n mile; the radar separation is 5 n mile which is used on RNAV2 Beijing-Shanghai parallel routes.

The parameters of $E_y(\text{same})$, $E_y(\text{opp})$, $|\overline{\Delta v}|$, $|\overline{v}|$, $|\overline{y}|$ and $|\overline{z}|$ are derived from the safety assessment of ICAO. The related parameters is displayed in Table 2.

The collision risk of the simulated Beijing-Shanghai parallel routes has been evaluated. As stated in reference (ICAO, 1998), the value of the TLS which applies to the lateral dimensions is 5×10^{-9} fatal accidents per flight hour. This lateral collision risk N_{sy} is 541×10^{-10} which meets the agreed TLS value. However, one of the assumptions made in the collision risk model is that there is no allowance made for the value of air traffic control intervention to reduce the risk that a pair of aircraft loses planned separation. Consequently, the risk estimates presented in

Table 2: Values of the related parameters

Parameter	Value	Parameter	Value
S_z/n mile	5.0000	$ \overline{\Delta v} $	20.000
S_y/n mile	10.8000	$ \overline{y} $	42.000
$P_y(S_y)$	-	$P_z(0)$	0.362
-	0.0180	-	0.000
D/n mile	0.0294	$ \overline{v} $	475.000
λ_z/n mile	0.0084	$ \overline{z} $	1.500
N_{sy}	-	RNAV	2.000

the study should be considered conservative that is, higher than is likely the case in the airspace (Moek *et al.*, 2001).

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CONCLUSION

In this study, the lateral collision risk Beijing-Shanghai RNAV2 parallel routes has been analyzed by improved Reich Collision Risk Model. The estimated collision risk meets the agreed TLS value. It means that the safety assessment supports the lateral separation standards on Beijing-Shanghai RNAV2 parallel routes. However, it is necessary to collect and analyze more traffic sample data to ensure the distribution of lateral deviation which is make the effect of collision risk significantly. Meanwhile, the relevant information be provided is of the utmost importance, so the other parameters must be got in detail to specified airspace to ensure safe operation.

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