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Optimization Research of Urban Public Transport Network from the Perspective of Risk Attitudes of Decision Makers

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Abstract: Considering the risk attitude of decision makers on the impact of public transportation network optimization decision, public transportation network grey relevancy optimization model is put up on the base of cumulative prospect theory after elaborating objective function and constraints of public transportation network. Firstly, the [-1,1] linear transformation operator is used to standardize the original decision-making information and get the positive and negative ideal schemes which are taken as the reference points of priority by the way of TOPSIS. Secondly, the positive and negative grey relevancy matrixes are established for taking the positive and negative ideal schemes as reference sequences and taking decision-making programs as compared sequence on the base of grey system theory. Thirdly, the positive and negative prospect value matrixes and the nonlinear planning model for the maximum of comprehensive prospect value are put up on the base of cumulative prospect theory and its prospect value functions, the optimum weight vector is solved and the best program is determined. Finally, Case of Xi'an shows that the method can adjust the public transportation network better and the optimization solution is more reasonable to meet the actual needs.

Key words: Traffic engineering, public transport network, cumulative prospect theory, grey relational, optimization

INTRODUCTION

City public traffic system is the important part of city traffic system. Along with our country city changing quickly and motor vehicle increasing sharply, giving priority to the development of public transport has become the basic outlet to solve city traffic problem. Public traffic line network optimization is the key issue of the development of public transport and scientific and reasonable design of the public traffic line network and optimization method plays a decisive significance for emplacing the efficient transportation network (Liu, 2008). Domestic and foreign scholars have done a lot of studies about the city public traffic line network optimization. One class is a pure research which try to solve simple quantitative model of public transport network optimization problems, such as 0-1 nonlinear planning model (Hu *et al*., 2005) and bi-level planning model (Jin, 2007). Another is the combination of qualitative and quantitative, in the process of optimization in part by a quantitative model to solve, such as "step by step design, optimizing network" method (Wang *et al*., 2002) and the city public traffic line network optimization PSO algorithm (Han *et al*., 1997).

Prospect theory is put forward by Kahneman and Tversky (1979) which can't be realized by rational decision making behavior. Kahneman thinks that the individual assign non probability weight for different results through taking the probability into decision weighting function which is one of the most-cited research results in the field of economics. Subsequently, they put forward cumulative prospect (Tversky, 1992) which introduces capacity probability, solves the problem of strong dominant and multiple outcomes of treatment. In recent years, prospect theory is realized and researched by domestic scholars (Zhou and Wang, 2005) in the field of transportation on the path choice behavior. For example Zhao and Zhang (2007) research the traveler's route choice behavior model under the condition of finite discrete distributions based on prospect theory Xu *et al*. (2007) put forward the path choice behavior analysis method through traveler route choice survey data based on prospect theory. Yang and Yan (2008) research the influence of route choice problem under the basic frame of real-time traffic information based on prospect theory.

In the real work about city public traffic line network optimization, decision makers often consider policy, city space development and many other factors which leads to subjective risk preference of decision makers for planning Schemes which will directly affect the final decision. Therefore, it is necessary to consider decision-makers risk factors in public traffic line network optimization. However, the traditional optimization methods are provided that the decision maker is the absolute reason and has complete information and the same preferences, ignores the risk attitude of decision makers for the public traffic line network optimization. In view of this, this study combines cumulative prospect theory with grey relational analysis, build up public traffic line network grey relational optimization model considering decision-makers risk attitude on the base of cumulative prospect theory which is more consistent with human thinking. In last, the paper takes Xi'an city public traffic line network optimization as example, illustrate the optimization method is scientific and effective.

BASIC PROBLEMS PUBLIC TRAFFIC LINE NETWORK OPTIMIZATION

Optimization principle: The core content about city public traffic network optimization is to apply scientific methods to determine the reasonable city public traffic lines layout which consider the social and economic development and road network layout on the base of public traffic demand and traffic temporal and spatial distribution characteristics. In order to make the city public line network meet the traffic flow assignment of the actual operation situation, improve the convenience of customers and the public transportation enterprise benefit, promote the sustainable development of the city traffic, the public transit network optimization should insist on the following principles.

Firstly, the lines should conform to the main traffic flow for more passengers with bus service. Lines are laid by the shortest distance in order to shorten passenger total travel time and to improve the convenience of residents travel. Secondly, Public traffic line network optimization should insist on improving the bus enterprise efficiency principle, whose object is to improve line network coverage and reduce the repetition routes using existing roads. Thirdly, Public traffic line network optimization should consider the principles of sustainable development. The lines laying not only conforms to the objective laws of the current urban passenger flow and distribution, but also reflect the future development of the city traffic change to match the overall urban planning and lead to a reasonable direction for the development of urban space.

Objective function: Transit network optimization target is to minimize the generalized cost and maximize service to make the public transport network social costs minimization and social benefits maximization. Considering the testability of surveying data and the actual situation of xi'an, the study select total travel time of passengers, passengers direct rate, user fees and operating expenses and line network coverage from the spatial dimension, time dimension and value dimension in order to establish public transportation network optimization objective function on the basis of summing up the results of previous studies.

Equation of the total travel time of passengers trips:

$$T=\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}T_{ij} \tag{1}$$

where, T is Total time of bus travel (h), a_{ij} is Bus passengers from node i to node j, T is Total time of bus travel from node i to node j, n-Number of traffic zones.

Equation of the highest rate of passenger direct:

$$V=\sum_{i=1}^{m}\sum_{j=1}^{m}K_{ij}/\sum_{\mu=1}^{n}\sum_{v=1}^{n}D_{\mu v} \tag{2}$$

where, V is Direct rate of passengers, K_{ij} is Direct passenger volume from origination i to terminal j, $D_{\mu v}$ is Quantity of OD from traffic zone μ to zone v, m is Nodes number in bus operation network, n is Total number of traffic zones.

Equation of the least cost for user costs and operator:

$$P=\lambda_1\sum_{s}\sum_{t}f_{st}T_{st}(x)+\lambda_2\sum_{s}\sum_{t}cB_{st}I_{st}\left(\sum_{k}x_{st}^{k}\right) \tag{3}$$

where, P is User costs and operation costs, λ_1 is The hour trip costs, f_{st} is Passenger volume from public transport matching node s to node t, T_{st} is Travel time from pub transport matching node s to node t in a given public transport network vector (X) (h), λ_2 is Weight between the time expense and investment of the public transport system, c_{st} is Cost of single bus from public matching node s to node t, B_{st} is Number of buses from public matching node s to node t:

$$I_{st}\left(\sum_{k}x_{st}^{k}\right)$$

is Capital cost from public matching node s to node t.

Equation of maximal rates of network coverage:

$$\partial=\sum_{\phi=1}^{m}l_{R\phi}\Big/\sum_{\phi=1}^{m}l_{A\phi} \tag{4}$$

where, ∂ is Network coverage, $l_{R\phi}$ is Line length of the zone ϕ, l_{Ai} is Network length of the zone ϕ.

Set of constraints: On the base of city road traffic planning design specification (GB50220-95), combing the

Table 1: Set of constraints and ranges

No.	Set of Constrains	Symbols	Ranges	Units
1	Length of the line	l	[5, 15]	km
2	Non-line coefficient of lines	ρ	90, 1.40	%
3	Uneven coefficient of passenger volume	o	[0, 1.5]	%
4	Average No. of conversions of passengers	r	[0, 2]	%
5	Road' load efficiency	η	[0.6, 1]	%
6	Lines' overlap coefficient	e	[0.3]	%

multiple factors of city transit system planning and actual situation of Xi'an, the study determines six main constraint indexes and its ranges.

OPTIMIZATION PRINCIPLE AND SPECIFIC STEPS

Optimization theory: Prospect theory (Kahneman and Tversky, 1979) analyze the problem mainly from the perspective of gains and losses which think that people's attitude treat the gains and losses is asymmetric. When people face with income, they tend to "risk averse"; when people face with loss, they tend to "risk seeking". Evaluation basis of gains and losses is the reference point. The risk preference of people under the condition of uncertainty is nonlinear relation which is consistent with public traffic line network optimization decision-making principles. The optimization result based on prospect theory tally with the actual situation of public traffic line network optimization decision making behavior. Correlation analysis (Du and Pang, 2006) is a method descripting similar degree among factors in the system which is wildly applied in Grey Theory. The city public traffic line network system is a the complex gray system including multiple factor and levels and the optimization goal in system are not independent of each other, so the study use the gray correlation analysis method to judge the schemes and reference point correlation degree.

Concrete steps: Supposed there are $\bar{n}$ alternative decision-making schemes to be selected in the public transport network optimization which can be noted $A = \{A_1, A_2,\ldots, A_{\bar{n}}\}$, four objective functions were taken as index set which can be noted as $G = \{G_1, G_2, G_3, G_4\}$ = (total travel time, passengers direct rate, user fees and operators cost, rate of network daily average load, rate of network coverage). Furthermore $Y=(y_{\bar{i}\bar{j}})_{\bar{n}\times 5}$ shows the decision-making matrix of schemes to the index set, whose weight can be noted as $w = (w_1, w_2, w_3, w_4)$ $\bar{i}\in N, \bar{j}\in M, N=\{1,2,3\cdots\bar{n}\}, M=\{1,2,3,4\}$:

$$Y=(y_{\bar{i}\bar{j}})_{\bar{n}\times 4}=\begin{bmatrix} y_{11} & y_{12} & y_{13} & y_{14} \\ y_{21} & y_{22} & y_{23} & y_{24} \\ \cdots & \cdots & \cdots & \cdots \\ y_{\bar{n}1} & y_{\bar{n}2} & y_{\bar{n}3} & y_{\bar{n}4} \end{bmatrix} \tag{5}$$

Standardized decision matrix: Because five optimal objectives have different units, it is necessary to standardize the objective functions. the total travel time G_1 and user costs and operator fees G_3 can be determined as cost indexes, user costs and operating costs G_2 and passengers direct access rate G_4 are beneficial indexes based on analyzing the method and meaning of optimal objectives.

For cost indexes $G_{\bar{j}}, y_{\bar{i}\bar{j}}$ can be standardized as:

$$r_{ij}=\frac{\max(y_{\bar{j}})-y_{\bar{i}\bar{j}}}{\max(y_{\bar{j}})-\min(y_{\bar{j}})}\ (\bar{i}=1,2,\cdots,\bar{n};\bar{j}=1,3) \tag{6}$$

For beneficial indexes, $y_{\bar{i}\bar{j}}$ can be standardized as:

$$r_{ij}=\frac{y_{\bar{i}\bar{j}}-\min(y_{\bar{j}})}{\max(y_{\bar{j}})-\min(y_{\bar{j}})}\ (\bar{i}=1,2,\cdots,\bar{n};j=2,4) \tag{7}$$

In above equation:

$$\max(y_{\bar{j}})=\max\{y_{\bar{i}\bar{j}}/1<\bar{i}<\bar{n}\}$$

$$\min(y_{\bar{j}})=\min\{y_{\bar{i}\bar{j}}/1<\bar{i}<\bar{n}\}$$

So, the decision-making matrix Y after standardized manipulation can be denoted as following matrix:

$$R=(r_{\bar{i}\bar{j}})_{\bar{n}\times 4}=\begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ r_{21} & r_{22} & r_{23} & r_{24} \\ \cdots & \cdots & \cdots & \cdots \\ r_{\bar{n}1} & r_{\bar{n}2} & r_{\bar{n}3} & r_{\bar{n}4} \end{bmatrix} \tag{8}$$

Determining the ideal solution: When decision-makers make a decision, they often measure the gains and losses of the decision-making according to reference point. The positive and negative ideal solutions were taken as reference points on the thinking of TOPSIS (Olson, 2004). The positive ideal solution can be noted as $A^+ = (r_1^+, r_2^+, r_3^+, r_4^+)$ and the negative ideal solutions can be noted as $A^- = (r_1^-, r_2^-, r_3^-, r_4^-)$.

Calculate the positive and negative correlation coefficient matrix: Based on the gray relational analysis method (Du and Pang, 2006), the ideal solution A^+ and the negative ideal solution A^- can be taken as the reference sequence, the schemes can be taken as compared columns.

The correlation coefficient of the positive ideal scheme about optimization goals can be expressed as:

$$\xi_{\bar{i}\bar{j}}^+=\frac{\min_i\min_j|r_{\bar{i}\bar{j}}-r_{\bar{j}}^+|+\rho\max_i\max_j|r_{\bar{i}\bar{j}}-r_{\bar{j}}^+|}{|r_{\bar{i}\bar{j}}-r_{\bar{j}}^+|+\rho\max_i\max_j|r_{\bar{i}\bar{j}}-r_{\bar{j}}^+|} \tag{9}$$

The correlation coefficient of the negative ideal scheme about optimization goals can be expressed as:

$$\xi_{ij}^{-} = \frac{\min_i \min_j \left|r_{ij}^{-} - r_j^{-}\right| + \rho \max_i \max_j \left|r_{ij}^{-} - r_j^{-}\right|}{\left|r_{ij}^{-} - r_j^{-}\right| + \rho \max_i \max_j \left|r_{ij}^{-} - r_j^{-}\right|} \quad (10)$$

In above equation, the distinguishing coefficient $\rho \in [0,1]$ and the study makes its value $\rho = 0.5$.

Therefore, the positive and negative correlation coefficient matrix can be expressed as:

$$\xi^{+} = \begin{bmatrix} \xi_{11}^{+} & \xi_{12}^{+} & \xi_{13}^{+} & \xi_{14}^{+} \\ \xi_{21}^{+} & \xi_{22}^{+} & \xi_{23}^{+} & \xi_{24}^{+} \\ \cdots & \cdots & \cdots & \cdots \\ \xi_{n1}^{+} & \xi_{n2}^{+} & \xi_{n3}^{+} & \xi_{n4}^{+} \end{bmatrix} \quad (11)$$

$$\xi^{-} = \begin{bmatrix} \xi_{11}^{-} & \xi_{12}^{-} & \xi_{13}^{-} & \xi_{14}^{-} \\ \xi_{21}^{-} & \xi_{22}^{-} & \xi_{23}^{-} & \xi_{24}^{-} \\ \cdots & \cdots & \cdots & \cdots \\ \xi_{n1}^{-} & \xi_{n2}^{-} & \xi_{n3}^{-} & \xi_{n4}^{-} \end{bmatrix} \quad (12)$$

Determining the positive and negative prospects matrix: According to research of Wang *et al.* (2010), the positive prospect value of scheme can be quantified as:

$$v_{ij}^{+} = \left(1 - \xi_{ij}^{-}\right)^{0.88}$$

and the negative prospect value of scheme can be quantified as $v_{ij}^{-} = -2.25\left[-(\xi_{ij}^{+} - 1)\right]^{0.88}$.

Therefore, the positive prospect value matrix can be expressed as:

$$V^{+} = \begin{bmatrix} v_{11}^{+} & v_{12}^{+} & v_{13}^{+} & v_{14}^{+} \\ v_{21}^{+} & v_{22}^{+} & v_{23}^{+} & v_{24}^{+} \\ \cdots & \cdots & \cdots & \cdots \\ v_{n1}^{+} & v_{n2}^{+} & v_{n3}^{+} & v_{n4}^{+} \end{bmatrix} \quad (13)$$

Therefore, the negative prospect value matrix can be expressed as:

$$V^{-} = \begin{bmatrix} v_{11}^{-} & v_{12}^{-} & \cdots & v_{14}^{-} \\ v_{21}^{-} & v_{22}^{-} & \cdots & v_{24}^{-} \\ \cdots & \cdots & \cdots & \cdots \\ v_{n1}^{-} & v_{n2}^{-} & \cdots & v_{n4}^{-} \end{bmatrix} \quad (14)$$

Constructing comprehensive prospect: The bigger comprehensive prospect value is the better for each scheme (Kahneman and Tversky, 1979). If prospect weight can be noted as $\pi^{+}(w_j)$ when decision makers face earnings and prospect weight can be noted as $\pi^{-}(w_j)$ when decision makers face loss, comprehensive prospect value for each scheme can be expressed as:

$$V_i = \sum_{j=1}^{m} v_{ij}^{+}\pi^{+}(w_j) + \sum_{j=1}^{m} v_{ij}^{-}\pi^{-}(w_j)$$

In above equation:

$$\pi^{+}(w_j) = \frac{w_j^{\gamma^{+}}}{\left[w_j^{\gamma^{+}} + (1 - w_j)^{\gamma^{+}}\right]^{1/\gamma^{+}}}$$

$$\pi^{-}(w_j) = \frac{w_j^{\gamma^{-}}}{\left[w_j^{\gamma^{-}} + (1 - w_j)^{\gamma^{-}}\right]^{1/\gamma^{-}}}$$

$$\gamma^{+} = 0.61,\ \gamma^{-}0.69$$

So optimization model can be established and the objective function can be expressed as max V = (V_1, V_2, V_3, V_4, V_5). Because each scheme are fair competed, optimization model can be expressed as M_1:

$$\max V = \sum_{i=1}^{n}\sum_{j=1}^{m} v_{ij}^{+}\pi^{+}(w_j) + \sum_{i=1}^{n}\sum_{j=1}^{m} v_{ij}^{-}\pi^{-}(w_j)$$

$$\text{s.t.} \sum_{j=1}^{m} w_j = 1, w_j \geq 0, w \subset H$$

Optimal solutions $w^* = (w_1^*, w_2^*, w_3^*, w_4^*, w_5^*)$ can be obtained to solve the above model and the optimal comprehensive prospect value for: Scheme can be expressed as:

$$(M_2): V_i^{*} = \sum_{j=1}^{m} v_{ij}^{+}\pi^{+}(w_j^{*}) + \sum_{j=1}^{m} v_{ij}^{-}\pi^{-}(w_j^{*})$$

Therefore, the sequence of schemes can be determined by comprehensive prospect value.

EXAMPLE

Xian city public traffic planning draws up the five kinds of solutions. Optimization target value of each scheme were calculated by the investigation data and showed in Table 2.

Table 2: Objective function values of programs

A_i	G_1	G_2	G_3	G_4
A_1	52	52.0	4730	47.3
A_2	101	67.0	15990	57.1
A_3	53	42.0	4730	38.2
A_4	97	52.5	13130	55.4
A_5	66	37.5	8030	33.0

Table 3: Values of constraint indexes

A_i	l	ρ	o	r	η	e
A_1	11.50	1.40	1.43	1.5	0.85	2.0
A_2	13.60	1.32	1.48	1.8	0.88	1.5
A_3	12.00	1.20	1.36	2.0	0.79	1.8
A_4	10.80	1.14	1.42	1.3	0.68	2.3
A_5	14.00	1.26	1.30	1.9	0.74	2.1

The objective weight information is:

$$\begin{Bmatrix} 0.1 \le w_1 \le 0.2; 0.25 \le w_2 \le 0.4; \\ 0.25 \le w_3 \le 0.4; 0.1 \le w_4 \le 0.2 \end{Bmatrix}$$

Five kinds of scheme constraint index (as shown in Table 3) were compared with the constraint set which expresses that the scheme can satisfy the urban transit network optimization of the constraint condition.

According to the Eq. 5-14, the optimization choice can be determined.

Step 1: Standardization of the decision-making matrix

The decision matrix can be built as:

$$Y = \begin{bmatrix} 52 & 52 & 4730 & 47.3 \\ 101 & 67 & 15990 & 57.1 \\ 53 & 42 & 4730 & 38.2 \\ 97 & 52.5 & 13130 & 55.4 \\ 66 & 37.5 & 8030 & 33.0 \end{bmatrix}$$

The decision-making matrix can be standardization as:

$$R = \begin{bmatrix} 1 & 0.4915 & 1 & 0.5934 \\ 0 & 1 & 0 & 1 \\ 0.9796 & 0.1525 & 1 & 0.2158 \\ 0.0816 & 0.5085 & 0.2540 & 0.9294 \\ 0.7143 & 0 & 0.7069 & 0 \end{bmatrix}$$

Step 2: Determination of the positive and negative ideal schemes:

$$A^+ = (1, 1, 1, 1),\ A^- = (0, 0, 0, 0)$$

Step 3: Determination of correlation coefficient matrix:

$$\wp^+ = \begin{bmatrix} 1 & 0.4958 & 1 & 0.5515 \\ 0.3333 & 1 & 0.3333 & 1 \\ 0.9608 & 0.3711 & 1 & 0.3893 \\ 0.3525 & 0.5043 & 0.4013 & 0.8764 \\ 0.6364 & 0.3333 & 0.6305 & 0.3333 \end{bmatrix}$$

$$\wp^- = \begin{bmatrix} 0.3333 & 0.5043 & 0.3333 & 0.4573 \\ 1 & 0.3333 & 1 & 0.3333 \\ 0.3379 & 0.7662 & 0.3333 & 0.6986 \\ 0.8596 & 0.4958 & 0.6631 & 0.3498 \\ 0.4118 & 1 & 0.4143 & 1 \end{bmatrix}$$

Step 4: Determination of outlook matrix:

$$V^+ = \begin{bmatrix} 0.6999 & 0.5393 & 0.6999 & 0.5840 \\ 0 & 0.6999 & 0 & 0.6999 \\ 0.6957 & 0.2783 & 0.6999 & 0.3481 \\ 0.1776 & 0.5474 & 0.3839 & 0.6847 \\ 0.6269 & 0 & 0.6246 & 0 \end{bmatrix}$$

$$V^- = \begin{bmatrix} 0 & -1.2316 & 0 & -1.1111 \\ -1.5748 & 0 & -1.5748 & 0 \\ -1.1301 & -1.4961 & 0 & -1.4578 \\ -1.5348 & -1.2134 & -1.4326 & -0.3575 \\ -0.9238 & -1.5748 & -0.9370 & -1.5748 \end{bmatrix}$$

Step 5: Establishment of optimization model:

$$\max V = \sum_{i=1}^{5}\sum_{j=1}^{5} v_{ij}^{+}\pi^{+}(w_j) + \sum_{i=1}^{5}\sum_{j=1}^{5} v_{ij}^{-}\pi^{-}(w_j)$$

$$\begin{cases} 0.1 \le w_1 \le 0.2, 0.25 \le w_2 \le 0.4, \\ 0.25 \le w_3 \le 0.4; 0.1 \le w_4 \le 0.2, \\ w_1 + w_2 + w_3 + w_4 = 1 \end{cases}$$

Solving the model and get the optimal solution $w^* = (0.2, 0.25, 0.4, 0.15)$.

Then we can calculate V_i^* on the base of the equation M2:

$$V_1^* = 0.1285;\ V_2^* = 0.6592;\ V_3^* = -0.1882;$$
$$V_4^* = -0.8864;\ V_5^* = -1.0134$$

The result shows that $A_1 > A_3 > A_2 > A_4 > A_5$. Therefore, the first scheme is the best one.

CONCLUSION

Prospect theory is more suitable for the decision maker's actual decision-making behavior under the uncertainty cases which provides a more effective tool for the analysis of transit network optimization decision making problems. On the basis of previous studies, this study gives the transit network optimization objective function and determines the constraint conditions and builds up the comprehensive prospect value maximization model based on the cumulative prospect theory and grey system theory. Finally, Xi'an transit network optimization is taken as an example to indicate effectiveness and practicality of the method. Due to the emphasis of lines optimization is different in the different city, it is necessary to adjust parameter properly in practicality to scientifically describe the influence of the decision-making behavior.

REFERENCES

Du, D. and Q.H. Pang, 2006. Modern Comprehensive Evaluation Methods and Featured Case. Tsinghua University press, Beijing.

Han, Y., W.B. Li and X.F. Li, 1997. Algorithm PSO of adjustment and optimization for traffic network. China J. Highway Transport, 12: 100-105.

Hu, Q.H, Q. Shi and W.H. Zhang, 2005. Ideal Decision-making method of urban public traffic line network optimization. J. Traffic Transportation Eng., 5: 82-85.

Jin, W.L., 2007. A dynamical model of the traffic assignment problem. Transportation Res. Part B, 41: 32-48.

Kahneman, D. and A. Tversky, 1979. Prospect theory: An analysis of decision under risk. Econometrica, 47: 263-291.

Liu, H.D., 2008. Design theory on public transport network and implement method. Tongji University, Shanghai.

Olson, D.L., 2004. Comparison of weights in TOPSIS models. Mathematical Comput. Modeling, 40: 82-85.

Tversky, B., 1992. Distortions in cognitive maps. Geoforum., 23: 131-138.

Wang, W., X.M. Yang and X.W. Chen, 2002. Plan Method of Urban Public Transport System and Management. Science Press, Beijing.

Wang, Z.X., Y.G. Dang, L.I. Pei and Y.Y. Wang, 2010. Multi-index grey relational decision-making based on cumulative prospect theory. Control. Decis., 25: 232-236.

Xu, H.L., J. Zhou and X.G. Chen, 2007. Analysis and demonstration of the traveler's route choice behavior rule based on the prospect theory. J. Transportation Syst. Eng. Inform. Technol., 7: 95-101.

Yang, Z.Y. and G.Y. Yan, 2008. Model research of route choice under real-time information based on prospect theory. J. Shijiazhuang Railway Instit.: Nat. Sci., 21: 47-83.

Zhao, L. and X.C. Zhang, 2007. A traveler route choice model based on prospect theory and case. China Civil Eng. J., 40: 82-86.

Zhou, W. and M.Z. Wang, 2005. Weighting risk and uncertainty based on prospeet theor. Syst. Eng. Theory Practice, 25: 74-78.