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Granulation-based Measure for Risk Assessment

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Abstract: The weighted method is the most often used operators to aggregate criteria in decision making problems with the assumption that there are no interactions among criteria while in the real world, most criteria have inter-dependent or interactive characteristics. The discrete Choquet integral is a more suitable aggregation operator as it takes into accounts the interactions and not assuming the additivity and independence. This study proposes a granulation-based method to construct fuzzy measures needed by the discrete Choquet integral and a real data set is analyzed. The advantage of the granulation-based method is that no population probability is to be estimated such that the error of estimating the population probability is reduced. Three methods, including weighted sum method, the discrete Choquet integral with the entropy-based method and our proposed discrete Choquet integral with the granulation-based method, are used in this study to evaluate the enterprise financial risk based on a comprehensive assessment of the financial risk by multiple discriminant analysis. The results show that the discrete Choquet integral with the granulation-based method outperform the weighted sum method and the discrete Choquet integral with the entropy-based method in the evaluation of enterprise financial risk.

Key words: Risk assessment, choquet integral, fuzzy measure, granulation

INTRODUCTION

Under market economy conditions, due to the fierce competition, uncertainty of the business environment, as well as people's limitations of understanding this uncertainty, it inevitably bring financial crisis, resulting in business failure or operational risks which not only brings great loss to stockholders, creditors, managers and other interest parts, but also affects the stability of social economy. The evaluation of enterprise financial risk which can discover the hidden danger in corporate finance can play an important role in preventing companies from running into bankruptcy. So, effectively evaluating corporate financial risk is an important and challenging issue for both listed companies and investors.

Existing methods, such as linear regression, analytic hierarchy process, fuzzy comprehensive evaluation, or artificial neural network, may support financial risk evaluation. But the operator used to aggregate criteria in most of those methods is the Weighted Sum Method (WSM). WSM is a widely used method in multi-criteria decision making problems, whereas the additivity property of the interaction among variables is assumed in WSM (Hu, 2008). But for the issue of evaluation of enterprise financial risk, an assumption of additivity may not be reasonable enough as the financial risk factors are not always independent of each other. According to Ooghe and Waeyaert (2004) and Ooghe and de Prijcker (2008) enterprise financial risk factors have inter-dependent or interactive characteristics, so it is not suitable for us to aggregate them by traditional aggregation operators based on additive measures. In contrast to WSM, the discrete Choquet integral takes into account the interactions among criteria and has been proven to be a useful aggregation operator to model those dependencies. Thus, to approximate the human subjective evaluation level, it would be more suitable to apply Choquet integral, where interactions phenomena among enterprise financial risk factors are considered. Recently, Tzeng *et al*. (2005) applied this approach to hierarchical enterprise intranet web sites assessment. Tsai and Lu (2006) used this method to evaluate the service quality of e-stores. Berrah *et al*. (2008) developed a method for quantifying the causal relationships between the various criteria affecting the service rate performance based on a Choquet integral aggregation. Saad *et al*. (2008) used CI to aggregate several attributes in the objective function of a flexible job-shop scheduling problem. Buyukozkan and Ruan (2010) proposes an integrated multi-criteria evaluation methodology for software development experts and managers to better enable them to position their projects in terms of the associated risks.

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Recently, Narukawa and Torra (2007) reviewed the use of fuzzy measures and integrals (Choquet and Sugeno integrals) and showed their applications. They also considered their computational cost showing that it is not much higher than the straightforward information fusion methods (e.g., the arithmetic mean and the weighted mean) usually used in most applications. In particular, they have shown that the computational cost of the integrals is exactly the same cost that characterizes the OWA operator has. This also provided the motivation for our research.

In addition, there is a key issue unsolved in the application of fuzzy integral with the determination of density values to decide the fuzzy measures in the fusion process. In this study, entropy-based method and our proposed granulation-based method to construct the fuzzy measures in the discrete Choquet integral are discussed.

Generally, the current approach to the evaluation of enterprise financial risk is inclined to follow the traditional method of pairing up failed and non-failed firms using historical data. However, with the radical changing of global economy and customer demand, historical data cannot fully reflect the current market information and the prerequisite of matching up two diverse groups is much more difficult, so it is hard to approach a classification accuracy of 100% using current methodologies (Wu *et al*., 2006). In this situation, it is better to directly evaluate current financial statement using the recent data rather than to investigate numerous pairs of failed and non-failed firms using historical data. This important situation has also motivated this study.

This study aims to contribute to the evaluation of enterprise financial risk by introducing an integrated methodology that addresses some important issues not investigated until now. More precisely:

- We propose an evaluation and rating method that models interdependencies among enterprise financial risk factors based on discrete Choquet integral
- We propose a granulation-based method to construct the fuzzy measures needed by the discrete Choquet integral and a real dataset is analyzed

FUZZY MEASURES AND DISCRETE CHOQUET INTEGRAL

The weighted method scalarizes a set of objectives into a single objective by pre-multiplying each objective with a user-supplied weight. This method is the most commonly used classical approach in multi-criteria decision making problems, whereas the additivity property of the interaction among variables is assumed in weighted method. On the contrary, the discrete Choquet integral is proven to be an effective aggregation operator that extends the WSM. In particular, the discrete Choquet integral are able to model some kind of interaction between features: This is the main motivation of this research.

Choquet integral: The philosophy of the Choquet integral was first introduced in capacity theory (Choquet, 1954) and used as a (fuzzy) integral with respect to a fuzzy measure proposed by Hohle (1982) and then rediscovered later by Murofushi and Sugeno (1989, 1991) Choquet integral is defined to integrate functions with respect to the fuzzy measures. In the weighted sum method, each criteria i is given a weight $w_i \in [0, 1]$ representing the importance of this criteria in the decision making. In the Choquet integral model, where criteria can be dependent, a fuzzy measure is used to define a weight on each combination of criteria, thus making it possible to model the interaction existing among criteria. Fuzzy measure was first introduced by Sugeno (1974) which make a monotonicity instead of additive property. The definition of fuzzy measures and Choquet integral are as follows.

Let $X = \{x_1, x_2 \ldots x_n\}$ be the set of criteria and let P(X) denote the power set of X or set o f all subsets of X.

Definition 1: Let X be a finite set of criteria. A discrete fuzzy measure on X is a set function μ: $P(X) \to [0,1]$, satisfying the following conditions:

- $\mu(\phi) = 0$, $\mu(X) = 1$ (boundary conditions)
- If A, $B \in p(X)$ and $A \subseteq B$ then $\mu(A) \leq \mu(B)$ (monotonicity)

$\mu(s)$ can be viewed as the grade of subjective importance of decision criteria set S. Thus, in addition to the usual weights on criteria taken separately, weights on any combination of criteria are also defined. This makes possible the representation of interaction between criteria.

Definition 2: Let μ be a fuzzy measure on $X = \{1, 2, ..., n\}$. The discrete Choquet integral of function f: $X \to R$ with respect to μ is defined by:

$$C_\mu(f) = \sum_{i=1}^{n} f_{(i)}[\mu(A_{(i)}) - \mu(A_{(i+1)})]$$

where (i) indicates a permutation on X such that $f_{(1)} \le f_{(2)} \le \ldots f_{(n)}$ Also $A_{(i)} = \{I, \ldots, n\}$, $A_{(n+1)} = \phi$.

Since the fuzzy integral model does not need to assume independency of one criteria from another, it can be used in non-linear situations. The fuzzy integral of f with respect to μ gives the overall evaluation of an alternative. When the criteria are independent, the fuzzy measure is additive and the discrete Choquet integral coincide with the weighted arithmetic mean method. That is:

$$C_\mu(f) = \sum_{i=1}^{n} f_i \mu(\{i\})$$

In the following, we apply the Choquet integral operator to an example about the enterprise financial risk evaluation problem.

Example 1: The Commission on Risk Assessment and Risk Management of China Securities Regulatory Commission must prioritize the financial risk of the listed companies in Chinese electronic industry. The committee wants to prioritize these enterprises' financial risk from highest to lowest. In assessing the potential financial risk of each enterprise, a set of three factors are considered:

$$X = \{x_1, x_2, x_3\}$$

where, x_1 = working capital/total assets, reflecting the liquidity of the assets and company's solvency; x_2 = EBIT/total assets, reflecting the profitability of corporate assets; x_3 = sales/total assets, reflecting the company's operating capacity.

Assume there are seven companies to be evaluated and the raw data are in Table 1. The importance to each subset of P(X) are shown in Table 2. In Table 2, (0,0,0), (1,0,0), (0,1,0), (1,1,0), (0,0,1), (1,0,1), (0,1,1) and (1,1,1) represent empty set, $\{x_1\}$, $\{x_2\}$, $\{x_1, x_2\}$, $\{x_3\}$, $\{x_1, x_3\}$, $\{x_2, x_3\}$ and $\{x_1, x_2, x_3\}$, respectively. For the first firm, the raw scores are 0.061457, 0.035985 and 0.727159. First, rank the criteria values from the smallest to the largest, i.e., 0.035985, 0.061457 and 0.727159. Then, according definition 2, f_1 = 0.035985, f_2 = 0.061457 and f_3 = 0.727159. Finally, the overall financial performance evaluated by Choquet integral is computed by:

$$C_\mu(f_1, f_2, f_3) = f_1\times\mu(\{(1), (2), (3)\}) + (f_3 - f_1)\times\mu(\{(1), (3)\}) + (f_2 - f_3)\times\mu(\{(3)\}) = 0.035985\times\mu(\{x_1, x_2, x_3\}) + 0.061457 - 0.035985)*\mu(\{x_1, x_3\}) + (0.727059 - 0.061457)\times\mu(\{x_3\}) = 0.30605$$

Table 1: Raw data used to demonstrate financial risk evaluation by Choquet integral

Firm code	X_1	X_2	X_3
68	0.061457	0.035985	0.727159
600207	0.166224	0.038841	0.724676
600203	-0.250180	0.040350	1.004996
600083	0.078489	0.046250	0.054588
500355	0.440266	-0.120300	0.288413
58	0.123781	0.041910	0.201352
600330	0.171936	0.019871	0.744879

Table 2: A fuzzy measure used to demonstrate financial risk evaluation by Choquet integral

X_1	X_2	X_3	Fuzzy measure
0	0	0	0.000000
1	0	0	0.384346
0	1	0	0.321919
0	0	1	0.377289
1	1	0	0.683941
1	0	1	0.742042
0	1	1	0.670669
1	1	1	1.000000

By the same philosophy, the overall financial performance values of the second, third, fourth, fifth, sixth and seventh firms evaluated by Choquet integral are 0.34406, 0.30862, 0.061623, 0.24135, 0.13193 and 0.34887, respectively.

Entropy measure and granulation-based measure: We can see from Definition 2 that the Chqouet integral is based on fuzzy measures, so a suitable fuzzy measure is of importance for Choquet integral evaluation. We note that entropy measure proposed by Kojadinovic (2004) and granulation-based measure proposed in our study are qualified to be fuzzy measures.

First we introduce the fuzzy measures based on entropy method. In information theory, entropy is a measure of the uncertainty associated with a random variable (Shannon, 1948). The basic idea is that an item with large entropy in its ratings is more important in a user's interest than an item with small entropy. Based on this idea, an entropy-based method is in the following (Yu *et al*., 2001): For a random variable A, let p^A be the probability of A, the Shannon entropy, a measure of uncertainty and denoted by H(X), is defined as:

$$h(A) = -\sum p^A \log_2 p^A$$

where, $p^A > 0$. With the similar formula, let B be a discrete random vector which contains at least two discrete random variables, then generalize this idea to a random vector and call p^B be the joint probability and h(B) the joint entropy. By using the idea of joint entropy to calculate the entropy of the subsets of criteria of X, define the fuzzy measure μ_1 as the following:

$$\mu_1(S) = \frac{h(s)}{H(X)}$$

for all S⊆X (Kojadinovic, 2004) By using the idea of entropy, we need to decide the number of level to be used to classify the raw data into the level of the score for each criterion. For example, let the number of level to be used be 3 and S contain three random variables x_1, x_2, x_3. In addition, assume the raw data are in Table 1.

The raw data in Table 1 can be classified into Table 3 by histogram equalization of ‘’hist.m” program of Matlab 7.0 for each random variable. To generate the complete information of fuzzy measure μ_1, rst to compute h(X). A joint pattern (1, 2, 2) means that X1 = 1, X2 = 2 and X3 = 2 and (3, 1, 1) means that X1 = 3, X2 = 1 and X3 = 1 and so on. There are 2 of pattern (1, 2, 2), 1 of pattern (2, 2, 2), 1 of pattern (3, 1, 1) and 2 of pattern (2, 2, 1). Thus, p^S (X1 = 1, X2 = 2, X3 = 2), p^S (X1 = 2, X2 = 2, X3 = 1) = 2/6 = 0.3333, p^S (X1 = 2, X2 = 2, X3 = 2) = p^S (X1 = 3, X2 = 1, X3 = 1) = 1/6 = 0.16667. Therefore:

$$h(X) = -2\times 0.3333\times \log_2(0.3333)$$

$$-2\times 0.16667\times \log_2(0.1667) = 1.918294$$

Next, h(S) is computed when S = X1, X2, X3, {X1, X2}, {X1, X3} and {X2, X3}. In this case , there are 2 pattern “1”, 3 pattern “2” and 1 pattern “3” in X_1. From Table 3, p^{x_1} (x_1 = 1) = 2/6 = 0.3333, p^{x_1} (x_1 = 2) = 3/6 = 0.5, p^{x_1}(x_1 = 3) = 1/6 = 0.16667 and h(x_1) = $-0.3333\times\log_2(0.3333)-0.5\times\log_2(0.5)-0.16667\times\log_2(0.16667)$ = 1.459147.

Similar to the calculation of h(x1), h(x2) = 0.65002, h(x3) = 1, h(x1, x2) = 1.45915, h(x1, x3) = 1.918294, h(x2, x3) = 1.45915, By:

$$\mu_1(S) = \frac{h(s)}{H(X)}$$

for all S⊆X, the fuzzy measure μ_1 is completely defined by the following Table 4. Although our example is to compute the fuzzy measure of a random vector with two discrete random variables, the entropy method is also easy to compute the fuzzy measure of a random vector with more than two discrete random variables. However, the entropy-based weighting scheme might take the risk to estimate the probability for each criterion. If the sample size is small, it often makes a larger error to estimate the population probability. Under such circumstances, we propose a granulation method to improve the prediction.

Knowledge granularity is used to describe partition of domain U, so we can use equivalence class to describe granularity. Knowledge of different granularity

Table 3: Level of the score for each criteria classified from the raw data in Table 1 when the No. of level is three

Firm vode	X_1	X_2	X_3
68	1	2	2
600207	2	2	2
600203	1	2	2
600089	2	2	1
600355	3	1	1
58	2	2	1

Table 4: A fuzzy measure constructed by the entropy method

X_1	X_2	X_3	Fuzzy measure
0	0	0	0.000000
1	0	0	0.760651
0	1	0	0.338854
0	0	1	0.521298
1	1	0	0.760651
1	0	1	1.000000
0	1	1	0.760651
1	1	1	1.000000

corresponds to a different equivalence relation, also corresponds to a different division or classification. The more detailed the knowledge, the smaller the corresponding granularity division. After adding attribute X, the number of equivalence classes in domain U is increased, granularity smaller and knowledge discernibility degree raised. This concept is in agreement with our intuitive understanding that it is the elements making the greatest changes of domain partition that matters more.

Let I = (U, X) be a knowledge system, U is the domain, X = {x_1, x_2...x_n} be the set of attribute. For any subsets S⊆P(X), with the changes of r in X, on the one hand the number of equivalence classes is changed, on the other hand subordination equivalence class of the object in domain U is changed. Thus, the importance of subsets S⊆P(X) can be defined as follows:

$$G_1(S) = (1-\frac{|U/ind(X-S)|}{|U/ind(X)|})*$$

$$(\frac{||Pos_X(U/ind(X))|-|Pos_{X-S}(U/ind(X))||}{|U|})$$

where, U/ind(x) is an equivalence class for an example in domain U with respect to set x, Pos_C (U/ind(X)) is the set of all elements of U that can be uniquely classified into blocks of the partition U/ind(X), by means of U/ind(C) and |X| represent cardinal number of set X.

Moreover, it is very natural to define $G_1(\phi)$ to be zero, $G_1(X)$ to be 1, where, ϕ is an empty set. It is easy to check that G1 has property of monotonicity. That is, X⊆Y implies $G_1(X)\subseteq G_1(Y)$ for X, Y∈P(X). In addition, $G_1(\phi) = 0$. And $G_1(X) = 1$, By the definition of fuzzy measure, G_1 is a fuzzy measure.

Table 5: A fuzzy measure constructed by the granulation-based method

X_1	X_2	X_3	Fuzzy measure
0	0	0	0.0000
1	0	0	0.1250
0	1	0	0.0000
0	0	1	0.1250
1	1	0	0.5000
1	0	1	0.4167
0	1	1	0.1250
1	1	1	1.0000

Example 2: Let the number of level to be 3 and S contains only three random variables X1, X2 and X3. By using the raw data from Table 1, the raw data can be classified by histogram equalization of "hist.m" program of Matlab 7.0 for each random variable, as shown in Table 3. First compute the complete information of fuzzy measure G_1 for X_1:

In this case, U = {1, 2, 3, 4, 5, 6}, X = {X_1, X_2, X_3}. Compute equivalence class in domain U:

U/ind (X) = {{1, 3}, {2}, {4, 6}, {5}}; U/ind(X-X_1) = {{1, 2, 3}, {4, 6}, {5}}. Let P = U/ind(X), Q = U/ind (X-X_1), According the definition of granulation-based fuzzy measure, |U| = 6, |P| = 4, |Q| = 3, $|Pos_X(P)|$ = 6, $|P_{X-X1}(P)|$ = 3, thus $G_1(x_1) = (1-3/4)\times(|6-3|/6) = 0.125$. Similar to the calculation of fuzzy measure G_1 for X_1, we can compute the fuzzy measure G_1 for {X_2}, {X_3}, {X_1, X_2}, {X_1, X_3}, {X_2, X_3}, the fuzzy measure G_1 is completely defined by the following Table 5.

Although our example is to compute the fuzzy measure of a random vector with two discrete random variables, the granulation-based method is also easy to compute the fuzzy measure of a random vector with more than two discrete random variables.

In this study, three methods, including classical weighted sum method, the Choquet integral with the entropy method and our proposed Choquet integral with the granulation-based method, are applied in a case to evaluate the enterprise financial risk.

EVALUATION PROCEDURE USING THE DISCRETE CHOQUET INTEGRAL

Draw lessons from Calvo *et al.* (2001) a four step procedure of applying the Choquet integral is as follows:

Step 1: Use discretization method to classify the raw data into the level of the score for each criteria in our study. The range of level to be used in the discretization can be decided by Scott' rule and Sturge's formula (Scott, 1992). Assume m is the number of the level of scores and m = 3, 4, 5, 6, 7, 8, 9 are the range in our study. Then, transform the value of the raw data into the level of the scores for each criteria when m = 3, 4, 5, 6, 7, 8, 9

Step 2: For each m make the following calculations: (1) Use expert experience to get the weight for each criterion; (2) By using the results from Step 1, compute fuzzy measures based on entropy and joint entropy for each subset of all criteria. Then, the importance for each subset is resolved; (3) Use the results from Step 2, compute fuzzy measures based on the granulation for each subset of all courses. Thus, the importance for each subset is available

Step 3: Calculate the weighted sum among all criteria from the raw data. Use the Choquet integral with the entropy method and the granulation-based method to compute overall financial risk values discussed in Step 2 for each m = 3, 4, 5, 6, 7, 8, 9

Step 4: Transform the results in Step 3 into 2 levels of the scores for each m = 3, 4, 5, 6, 7, 8, 9. Finally, calculate the accuracy for each method for each m = 3, 4, 5, 6, 7, 8, 9

EMPIRICAL CASE: GRANULATION-BASED MEASURE FOR RISK ASSESSMENT

Fifty-two sample cases are selected from Chinese electronic industry shown in Table 6. All data are from the electronic sector due to the fact that different industries may have different operating environments. The sample cases are composed of 52 corporations with 26 firms in each of the two groups. Group 1 includes 26 corporations selected from the companies with better financial statement. Among them, half are companies whose values of net asset profitability are more than 10%, the other half more than 5%. Group 2 includes 26 corporations from companies with worse financial statement, half of which are from ST companies, the other half's net asset profitability is less than 5%.

Previous research has demonstrated that financial ratios can be a good indicator to the assessment of enterprise financial risk. For a succinct statement of the application of the Choquet integral to the evaluation of enterprise financial risk, we use three variables (X_1 = Working Capital/Total Assets, X_2 = EBITDA/Total Assets, X_3 = Net Sales/ Total Assets) in Altman Z-Score model to evaluate the financial risk. The detailed information is depicted in Table 6.

According the evaluation procedure using discrete Choquet integral, the first step is to decide the range of level to be used to classify the raw data. The previous research efforts have provided two rules for us to decide the range of the number of level. One is Scott rule with the equation:

Table 6: Detailed information in the case study

Firm code	X_1	X_2	X_3	Firm code	X_1	X_2	X_3
68	0.051457	0.035985	0.727159	2188	0.366765	0.031915	0.505984
600207	0.166224	0.038841	0.724676	2017	0.388042	0.091451	1.272553
600203	-0.250180	0.040350	1.004996	2156	0.068515	0.048003	0.634462
600083	0.078489	0.046250	0.545880	2056	0.541534	0.063696	0.303833
600355	0.440266	-0.120300	0.288413	2222	0.518996	0.095627	0.223732
58	0.123781	0.041910	0.201352	823	0.137413	0.073647	0.847298
600330	1.171936	0.019871	0.744879	2079	0.261788	0.035270	0.910489
50	0.021584	-0.007810	0.273978	2049	0.239855	0.823570	0.549278
600980	0.277288	0.194320	0.538817	2119	0.088671	0.033241	0.702583
413	0.132315	0.015952	0.505003	2185	0.240151	0.068089	0.644188
200413	0.132315	0.015952	0.505003	2025	0.266640	0.064675	0.294060
600171	0.354561	0.002939	0.254258	2199	0.098600	0.078778	0.480887
2129	0.144006	0.094520	0.411260	2288	0.173982	0.125082	0.596484
600839	0.067996	0.016034	0.972327	2351	0.430467	0.233175	1.800480
2141	0.200117	0.022672	1.493962	2388	1.476512	0.148414	1.600140
727	-0.069010	0.050100	0.543254	2369	0.274325	0.139432	1.448836
600777	-0.141930	0.019994	0.299875	2289	0.349996	0.126555	1.443476
733	0.361341	0.022142	0.684031	2308	0.506087	0.313429	1.210835
2134	0.374384	0.025478	0.565123	600983	0.346937	0.112309	0.960980
30	-0.002460	0.023955	0.424029	300014	0.291728	0.179132	0.941114
200020	-0.002460	0.023955	0.424029	2236	0.690291	0.120162	0.730832
2076	0.385381	0.028685	0.798855	300053	0.324335	0.173370	0.704112
600360	0.057465	0.044521	0.401150	2104	0.495594	0.157098	0.662272
600363	0.068230	0.037995	0.717980	600563	0.408424	0.114687	0.604771
600353	0.431501	0.141191	0.535930	2179	0.296046	0.083701	0.591152
600584	-0.217900	0.047670	0.512080	2273	0.602466	0.155875	0.437818

$$m = \frac{R \cdot n^{1/3}}{3.49 \cdot \sigma}$$

where, R is full range, m is the number of the level, σ is the standard deviation of the distribution and n is the available sample of size (Scott, 1979). In practice, σ is replaced by the estimated standard deviation, s. In this study, the sample of size n is 52. From the raw data, R = 0.94047, 0.43373 and 1.74589 for each item and s = 0.20448, 0.06816 and 0.37705, respectively. By the above formula, m would be 4.9190, 6.8056 and 4.9522, respectively. Thus, m = 5, 6, or 7 are possible candidates. The other one is the Sturge's formula: $m = 1+3.3\times\log_{10}(n)$ (Scott, 1992). From the latter formula, m is 6.6628. Thus, m = 6 or 7 are possible candidates. In this study, set m = 3, 4, 5, 6, 7, 8, 9 for extending m values around the possible candidates by two levels. That is, m = 3, 4, 5, 6, 7, 8, 9.

The second step is to calculate the importance for each criteria by expert experience, in this research we use Altman (1968) for reference. And the results are summarized in Table 7. For the evaluation of the Choquet integral with the entropy-based and the granulation-based methods, calculate the importance for each subset generated by all financial ratios for m = 3, 4, 5, 6, 7, 8, 9. The numerical figures of fuzzy measures for each subset are computed by Matlab and provided in Table 8. We can see from Table 8 that, the importance of granulation-based method and entropy-based method for each subset of all criteria exist difference. The reasons may come from the error of estimating a population probability by a small sample of size 52.

Table 7: Weight for each course by the weighted arithmetic mean

Variable	X_1	X_2	X_3
Weight	1.2	3.3	1

Table 8: Entropy-based and granulation-based fuzzy measures with m = 3, 4, 5, 6, 7, 8, 9

Entropy-based				Granulation-based			
X_1	X_2	X_3	Fuzzy measure	X_1	X_2	X_3	Fuzzy measure
m = 3							
0	0	0	0.000000	0	0	0	0.000000
1	0	0	0.384346	1	0	0	0.451900
0	1	0	0.321919	0	0	1	0.480750
0	0	1	0.377289	0	0	1	0.432700
1	1	0	0.383941	1	1	0	0.812500
1	0	1	0.742042	1	0	1	0.812500
0	1	1	6706690	0	1	1	0.812500
0	1	1	0.67067	0	1	1	0.812500
1	1	1	1.00000	1	1	1	1.000000
m = 4							
m = 9							
0	0	0	0.000000	0	0	0	0.000000
1	0	0	0.535526	1	0	0	0.333330
0	1	0	0.434167	0	1	0	0.076920
0	0	1	0.492256	0	0	1	0.225636
1	1	0	85935400	1	1	0	0.753840
1	0	1	0.924361	1	0	1	0.774780
1	0	1	0.924364	1	0	1	0.774780
0	1	1	0.790787	0	1	1	0.800000
1	1	1	1.000000	1	1	1	0.000000

The third step is to compute the overall financial risk values of the 52 firms by the entropy-based and the

Table 9: Results of overall enterprise financial risk evaluated with value with entropy-based and granulation-based Choquet integral and their level of scores transformed with m = 3, 4, 5, 6, 7, 8, 9

Firm code	C1	C2	TC1	TC2	Code	C1	C2	TC1	TC2
m = 3									
68	0.344730	0.306050	1	1	2188	0.364220	0.33291	1	1
600207	0.383980	0.344060	2	1	0.002017	0.715160	0.64525	3	2
600203	0.403280	0.308620	2	1	2156	0.309550	0.27675	1	1
600083	0.063826	0.061623	1	1	2056	0.478900	0.44170	2	2
600355	0.280400	0.241350	1	1	2222	0.331400	0.30417	1	1
58	0.142000	0.131930	1	1	823	0.432620	0.38880	2	2
600330	0.391340	0.348870	2	1	2079	0.500001	0.44810	2	2
60	0.125280	0.109230	1	1	2049	0.344210	0.31597	1	1
600980	0.342100	0.309440	1	1	2119	0.343920	0.30599	1	1
413	0.271760	0.242910	1	1	2185	0.382820	0.34821	2	1
200413	0.271760	0.242910	1	1	2025	0.240640	0.22489	1	1
600171	0.252460	0.227980	1	1	2199	0.260300	0.23772	1	1
2129	0.250370	0.232070	1	1	2288	0.347630	0.32077	1	1
600839	0.449560	0.395790	2	2	2351	0.986280	0.89646	3	3
2141	0.826690	0.642500	3	2	2388	0.901190	0.81581	3	3
727	0.241160	0.196940	1	1	2369	0.757240	0.68266	3	3
600777	0.110740	0.072262	1	1	2289	0.781250	0.70492	3	3
733	0.437370	0.395590	2	2	2308	0.774910	0.72228	3	3
733	0.437370	0.395590	2	2	2308	0.774910	0.72228	3	3
2134	0.391500	0.356340	2	1	600983	0.568640	0.51808	2	2
20	0.192110	0.166200	1	1	300014	0.551610	0.50769	2	2
200020	0.192110	0.166200	1	1	2236	0.600930	0.55852	2	2
2076	0.497410	0.449370	2	2	300053	0.460360	0.42868	2	2
600360	0.203750	0.183800	1	1	2104	0.504250	0.47116	2	2
600363	0.343710	0.305570	1	1	600563	0.438310	0.40673	2	2
600353	0.403500	0.370220	2	1	2179	0.383920	0.35261	2	1
600584	0.198830	0.135430	1	1	2273	0.459360	0.42837	2	2
m = 4									
......									
m = 9									
......									

C1, C2 represent the granulation-based and entropy-based Choquet integral, TC1, TC2 represent the level of scores transformed with m = 3, 4, 5, 6, 7, 8, 9.

Table 10: Accuracy for each method with m = 3, 4, 5, 6, 7, 8, 9

Method	m = 3	m = 4	m = 5	m = 6	m = 7	m = 8	m = 9
Weighted arithemetic mean	0.76923						
Choquet integral based on entropy	0.80769	0.08769	0.08769	0.08769	0.08769	0.08769	0.08769
Choquet integral based on granulation	0.80769	0.84615	0.80769	0.80769	0.84615	0.84615	0.80769

granulation-based Choquet integral. The different numerical figures in the Choquet integral column depicted in Table 9 have different meanings. The higher the value of Choquet integral is, the smaller the enterprise financial risk is. Also we can see that the level of scores for each m = 3, 4, 5, 6, 7, 8, 9 between the entropy-based and the granulation-based Choquet integral have high similarity which prove the effectiveness of our proposed method.

Finally, the fourth step is to compare the prediction accuracy of different methods under different m. In order to compare the evaluation effectiveness of different methods, we first compute the potential enterprise financial risk by discriminant analysis model. The sample data were obtained from China Stock Market and Accounting Research Database. ST companies are considered as companies in financial distress and those never specially treated are regarded as healthy ones. According to the data between 2005 and 2008, 48 pairs of manufacturing companies listed in Shenzhen Stock Exchange and Shanghai Stock Exchange were selected for model construction. The selection of financial ratios in this study is based on the previous studies link to the topic of financial distress. The previous research efforts such as Altman (1968), Chen and Du (2009), Sun and Li (2008), Yeh *et al.* (2010) and Wu (2010) have provided useful guidance in choosing variables for predicting financial distress. Based on previous researches, originally six financial ratios are selected in this data set: Working capital/total assets, retained earnings/total assets, EBIT/total assets, sales/total assets, asset-liability ratio and equity ratio. Then the constructed model was used as the method to evaluate potential enterprise financial risk.

Though the comparison of evaluation results by weighted arithmetic mean, entropy-based and the granulation-based Choquet integral with the potential enterprise financial risk, the prediction accuracy of different methods under different m are depicted in Table 10, where higher value means better accuracy. Obviously, the Choquet integral with the Granulation-based method has the best accuracy among the four methods. The reasons may be that to estimate a population by the sample probability is worsen when m is greater than 4. It is worth to note that the Choquet integral method has better accuracy than the weighted arithmetic mean method since the weighted arithmetic mean method ignore the mutual interaction among enterprise financial risk factors.

CONCLUSION

Effectively evaluating corporate financial risk is an important and challenging issue for both listed companies and investors. But most of the traditional multi-attribute evaluation approaches use the weighted sum method as the aggregation operator with the assumption that there are no interactions among criteria. For the issue of evaluation of enterprise financial risk, an assumption of additivity may not be reasonable enough as the financial risk factors are not always independent of each other. However, the Choquet integral method is adequate to deal with interactions among criteria. So, in this study, we propose an evaluation and rating method that models interdependencies among enterprise financial risk factors based on discrete Choquet integral.

Moreover, we propose a granulation-based method to construct the fuzzy measures needed by the discrete Choquet integral. Compared with widely used weighted sum method, the granulation-based method has advantage of taking the interactions among criteria into consideration. The advantage of the proposed Choquet integral with the granulation-based method is that no population probability is to be estimated such that the error of estimating the population probability is reduced. Typically, in order to accurately estimate the population probabilities, the sample of size should be large enough.

Finally, an empirical study of applying the weighted sum method, Choquet integral with the entropy-based method and the proposed Choquet integral with the Granulation-based method is presented in this study to evaluate the overall enterprise financial risk in Chinese electronic industry. The results show that Choquet integral with the proposed granulation-based method is an appropriate approach to evaluate enterprise financial risk, especially when criteria are not mutually independent.

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