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Pedestrian Performance Measures of an M/G/C/C State Dependent Queuing Network in Emergency

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Abstract: Occupant safety is a major concern for a building in case of emergencies such as fire, bomb blast or earthquake. In such emergency situations, a quick and jam-free evacuation of pedestrian trapped in a network of corridors in buildings is most important. In this paper, we compared the throughputs for two different restrictions on the flow direction with the unrestricted flow for a facility. In this facility, occupants need to go through some source corridors which consists of multiple sources and from source corridors they can choose their nearest exiting corridors. The result indicates that in emergency situation a restricted flow direction of pedestrian increases the throughput of the network which eventually decreases the average evacuation time.

Key words: State dependent queuing networks, restricted flow, throughput, expected number of occupants, expected service time, expected evacuation time

INTRODUCTION

Congestion in a pedestrian facility occurs due to increased density of occupants which decays the service rate. Congestion in pedestrian traffic networks (Mitchell and Smith, 2001; Yuhaski and Smith, 1989), vehicular traffic networks (Jain and Smith, 1997), in material handling systems (Bedell and Smith, 2012; Smith and Kerbache, 2011) and many other systems where service rate decreases with increased density of occupants, can be appropriately modelled by using state dependent queuing networks.

Cruz *et al.* (2005a,b), Cruz and Smith (2007), Mitchell and Smith (2001), Smith (1991), Yuhaski and Smith (1989) and others had used state dependent queuing models to capture the congestion in pedestrian traffic flow with arbitrary topologies and for different types of multi-storied buildings. Kawsar *et al.* (2012) also used M/G/C/C state dependent queueing models to evaluate the performance measures of a complex facility and showed that in case of an emergency throughput can be increased by restricting the flow direction and controlling the arrival rate to each of its source corridors.

In this study, we considered the same facility as Kawsar *et al.* (2012) where occupants through the source corridors come from multiple sources and egress through the nearest corridor. The objective of this study is to estimate the different performance measures along with the average evacuation time for unrestricted and restricted flows during egress from the facility. These measures can be used to evaluate the optimal set-up for which the maximum throughput can be obtained.

MATERIALS AND METHODS

Analytical pedestrian flow model: Based on some empirical studies, Tregenza (1976) presents a number of relationships between the walking speed of a pedestrian and the crowd density which is re-created in Fig. 1. According to Fruin (1971) uni-directional flow models can be used to capture the bi-directional and multi-directional flows of occupants during an evacuation and the flow relationships are similar for stairwells and plane movements. Linear and exponential models for uni-directional walking speed has been developed by Yuhaski and Smith (1989) as follows:

- Linear:

$$V_n = \frac{A}{C}(C+1-n) \tag{1}$$

- Exponential:

$$V_n = A\exp[-(\frac{n-1}{\beta})^{\gamma}] \tag{2}$$

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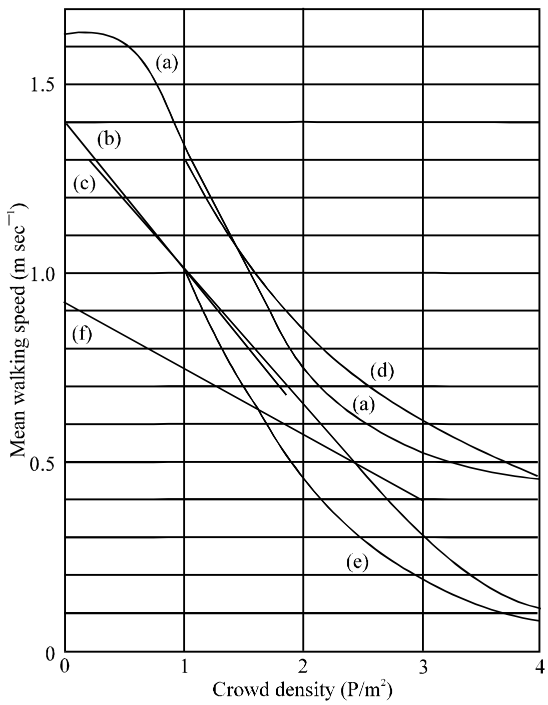

Fig. 1: Variation of mean walking speed with crowd density (a) Hankin and Wright (1958), (b) O'Flaherty and Parkinson (1972), (c) Older (1968), (d) Togawa (1955), (e) Togawa (1955) and (f) Foot (1973)

Where:

$$\gamma = \ln\left[\frac{\ln(V_a/A)}{\ln(V_b/A)}\right]/\ln\left(\frac{a-1}{b-1}\right),\ \beta = \frac{a-1}{[\ln(A/V_a)]^{1/\gamma}} = \frac{b-1}{[\ln(A/V_b)]^{1/\gamma}}$$

Where:

γ, β = Shape and scale parameters for the exponential model
V_n = Average walking speed for n occupants in a corridor
V_a = Average walking speed when crowd density is 2 ped m^{-2} = 0.64 m sec^{-1}
V_b = Average walking speed when crowd density is 4 ped m^{-2} = 0.25 m sec^{-1}
A = V_1 = Average walking speed of a lone occupant = 1.5 m sec^{-1}
n = Number of occupants in a corridor
a = 2 LW
b = 4 LW
C = 5 LW
L = Length of the corridor
W = Width of the corridor

Cheah (1990) provided exponential walking speed models for bi- and multi-directional corridor which are similar in the form to the uni-directional model. In this study, the analysis is carried out solely for uni-directional traffic flows through corridors using the exponential pedestrian speed model.

M/G/C/C state dependent queuing system: In M/G/C/C state dependent queueing model the arrival rate of pedestrians are assumed to be Markovian. The total number of pedestrians that can enter the system is equal to the capacity of the corridor which is also the number of servers, since the corridor behaves as a server to the occupants. The service rate is the rate at which pedestrians pass across the entire length of the corridor. This rate is dependent on the number of occupants (n) within the corridor and follows a general distribution G. Hence, the queuing model is state dependent. Cheah (1990) showed that M/M/C/C and M/G/C/C state dependent queues are stochastically equivalent and developed the limiting probabilities for the number of pedestrians in an M/G/C/C state dependent queuing model as follows:

$$P_n = \frac{[\lambda E(S)]^n}{n!f(n)f(n-1)...f(2)f(1)}P_0,\ n = 1, 2, 3, ..., C \quad (3)$$

where:

$$P_0^{-1} = 1 + \sum_{n=1}^{C}\left[\frac{\{\lambda E(S)\}^i}{i!f(i)f(i-1)...f(2)f(1)}\right]$$

In this model, E(S) is the expected service time of a lone occupant in a corridor of length L that is:

$$E(S) = \frac{L}{1.5}$$

P_n is the probability when there are n occupants in the corridor and P_0 is the probability of the corridor being empty. The service rate, f(n), is the ratio of the average walking speed of n pedestrians (V_n) in the corridor to that of a lone pedestrian (V_1) that is:

$$f(n) = \frac{V_n}{V_1}$$

When a pedestrian attempts to enter a corridor, but cannot because the corridor is currently at capacity then the blocking occurs and the probability of such blocking is equal to P_n where n equals C, the capacity of the corridor. The different performance measures can be computed as:

$$\theta = \lambda(1 - P_C),\ E(N) = \sum_{n=1}^{C} nP_n \text{ and } E(T) = \frac{E(N)}{\theta}$$

Where:

θ = Steady state throughput through corridor
E(N) = Expected number of occupants in the system
E(T) = Expected service time in seconds

Following the work of Yuhaski and Smith (1989), Kawsar *et al*. (2012) modeled corridor with multiple arrival sources and represented the facility as a network of series, split, merge or a combination of these topologies.

Description of the facility: In this study, the same facility as Kawsar *et al*. (2012) is considered which is presented in Fig. 2. The numbers represent the corridors, the alphabets S, T, U, V, W, X, Y and Z represent the different seating arrangements and A', B', C' and D' are the exits to other corridors. The number of seats in S, T, U, V, W, X, Y and Z are 220, 220, 309, 259, 100, 55, 98 and 77, respectively. The main exit door situated at the end of corridor 3 is modelled as one large door. Smith (2009) showed that node splitting at double width exits allowing for a support beam has no effect on throughput. Each of corridors 10 and 11 is modelled as a single corridor with 20 and 16 arrival sources, respectively and each of corridors 6, 7, 8 and 9 as a single corridor with three arrival sources. In cases when there is a split, the throughput from the splitting corridor is divided according to the probabilities of the branches. The arrival rate to a merging corridor equals the sum of the throughputs of the previous corridors.

Table 1 presents the dimension, number of sources and average travelling distance of the source corridors. Considering the arrival rate of a source corridor as the sum of the arrival rates of all related sources and the average travelling distance, the different performance measures of these corridors can be calculated. The different dimensions of the exiting corridors are presented in Table 2.

Table 1: Dimensions, number of sources and average travelling distance of source corridors

Corridor	Length (m)	Width (m)	No. of sources	Average travelling distance
6	10.10	2.8	3	2.156
7	8.50	2.8	3	1.780
8	10.10	2.0	3	2.156
9	8.50	2.0	3	1.780
10	9.45	1.8	20	2.700
11	7.35	1.8	16	2.275

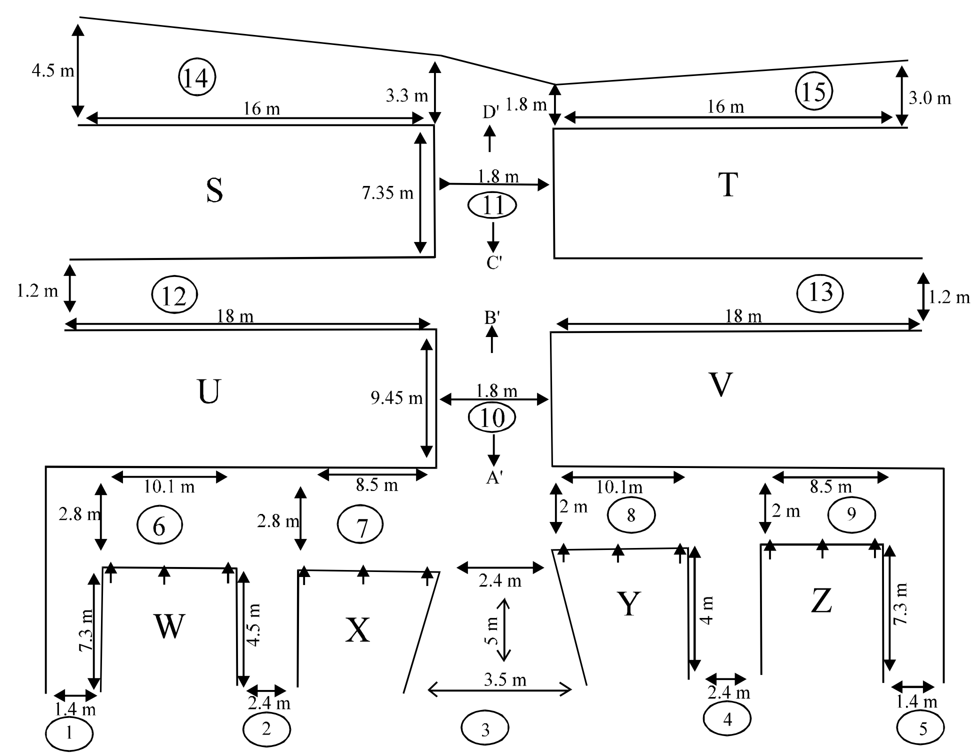

Fig. 2: Simplified representation of corridors of the facility

Table 2: Dimensions of the exiting corridors

Corridor	Length (m)	Width at entrance (m)	Width at exit (m)
1	7.3	1.4	1.4
2	4.5	2.4	2.4
3	5.0	2.4	3.5
4	4.0	2.4	2.4
5	7.3	1.4	1.4
12	18.0	1.2	1.2
13	18.0	1.2	1.2
14	16.0	3.3	4.5
15	16.0	1.8	3.0

RESULTS AND DISCUSSION

The corridor capacity depends on its size and the throughput of a corridor depends on the arrival rate. Considering the whole facility as a network of corridors, the throughputs of the source and exiting corridors for different arrival rates are presented in Fig. 3 and 4, respectively. The patterns of these graphs are similar with that obtained by Mitchell and Smith (2001). However, the magnitudes of the curves vary as the throughput depends on the capacity of corridor as well. From Fig. 3, we observe that the highest throughput for corridors 6 and 7 occur for an arrival rate of more than 14 ped sec^{-1}. Similarly for corridors 8 and 9, this rate is more than 10 ped sec^{-1} and for corridors 10 and 11 it is more than 6 ped sec^{-1}. From Fig. 4 it is observed that the throughputs for some of the exiting corridors coincide, because of approximately same size of the corridors. These are corridors 1 and 5; corridors 2 and 4; corridors 12 and 13 and corridor 14 and 15.

To increase the throughput of the entire system we have put a new restriction in place along with the restriction used previously by Kawsar *et al.* (2012). We restrict occupants from corridor 10 to exit only through corridors 12 and 13 and occupants from corridor 11 to exit only through corridors 14 and 15. As Xiang (2007) observed that people looked for the nearest exit which is shorter, occupants from all other source corridors are assumed to choose their nearest corridor to exit. Under this restriction, the average travelling distance for the occupants in corridor 10 and 11 is changed to 4.95 and 4.095 m, respectively. Also the arrival rates for corridors 12, 13, 14 and 15 are changed. The new arrival rate for each of corridors 12 and 13 is now half of the throughput of corridor 10 and that for each of corridors 14 and 15 is half of the throughput of corridor 11. Also the new arrival rate for corridor 3 is half of the throughput of corridor 7 plus half of the throughput of corridor 8. The throughputs for the source and exiting corridors under the restrictions are presented in Fig. 5 and 6, respectively. It shows that the new restriction decreases the throughputs of both source corridors 10 and 11. This happens because the restriction increases average travelling distance and arrival rate. This coincides with the findings

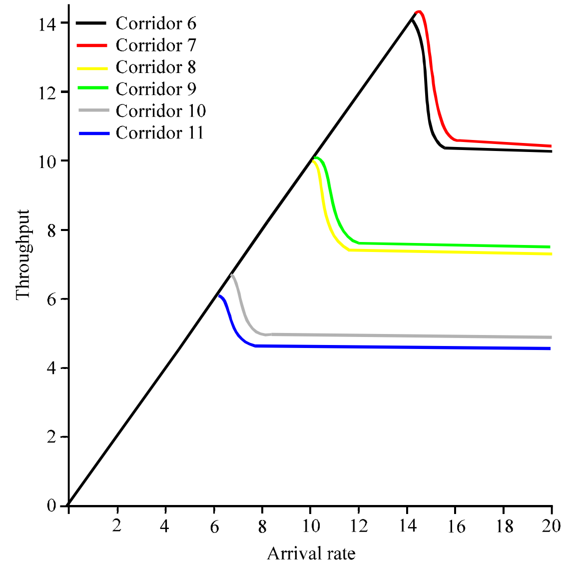

Fig. 3: Throughputs of source corridors for varying arrival rates

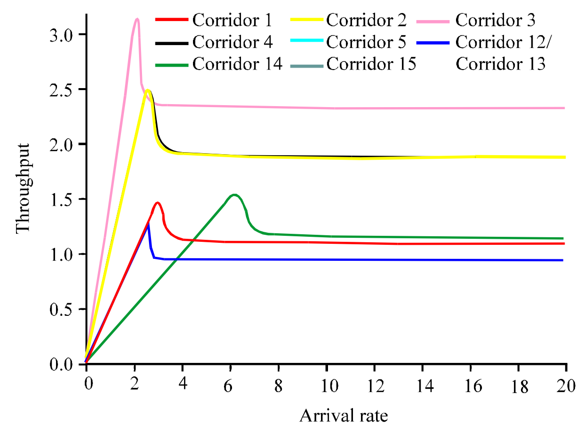

Fig. 4: Throughputs of exiting corridors for varying arrival rates

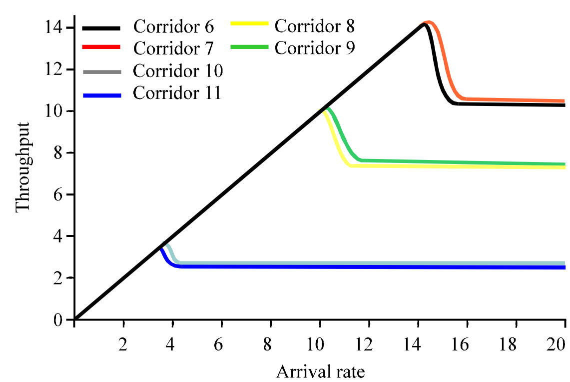

Fig. 5: Throughputs of source corridors for varying arrival rates under restriction

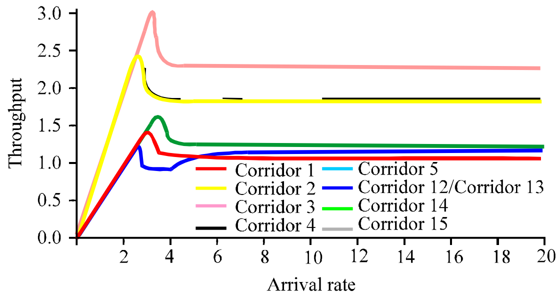

Fig. 6: Throughputs of exiting corridors for varying arrival rates under restriction

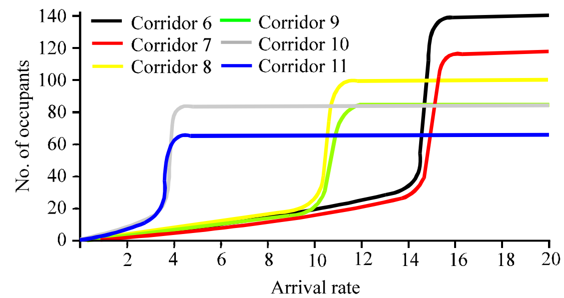

Fig. 8: Expected number of occupants of the source corridors under restricted flow

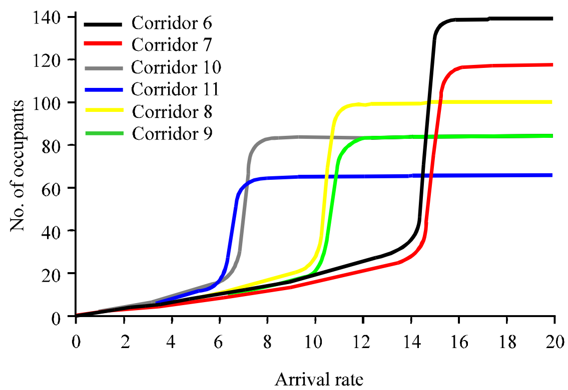

Fig. 7: Expected number of occupants of the source corridors for unrestricted flow

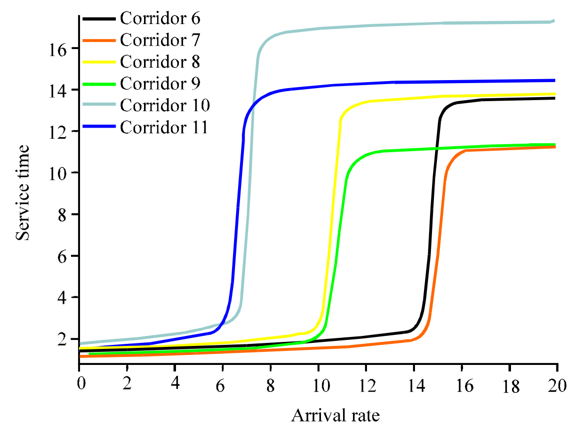

Fig. 9: Expected service time for the source corridors for unrestricted flow

of Lo and Fang (2000) and Cruz *et al*. (2005a,b). Lo and Fang (2000) showed travel distance decreases flow rate at the congested points and Cruz *et al*. (2005a,b) showed that throughput and expected number of customers are closely dependent upon the arrival rate. The decrease in the throughputs of the source corridors results a low occupant density in the exiting corridors and decreases the evacuation time. Liu *et al*. (2009) also found a similar result that occupant density around the exit influences the evacuation time.

The expected number of occupants and the expected service time for each of the source corridors against varying arrival rate for the unrestricted and restricted flows are presented in Fig. 7-10. The figure shows that for unrestricted flow, both corridors 10 and 11 reaches the capacity for an arrival rate slightly greater than 8 ped sec^{-1} and the expected service times are approximately 17 and 14 sec, respectively. Under the restriction, for an arrival rate of approximately 4.5 ped sec^{-1} both corridors reach the capacity and the expected service time increased to approximately 31 and 26 sec, respectively. This reduces congestion in exiting

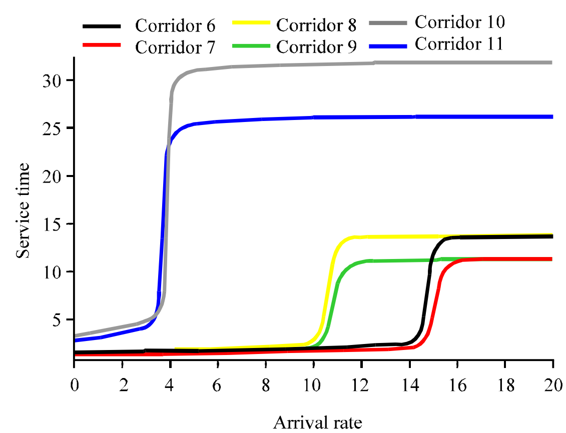

Fig. 10: Expected service time for the source corridors under restricted flow

corridors and results an increase in the throughputs of the exiting corridors. A similar result was obtained by Cruz (2009) for a finite single server general queuing network.

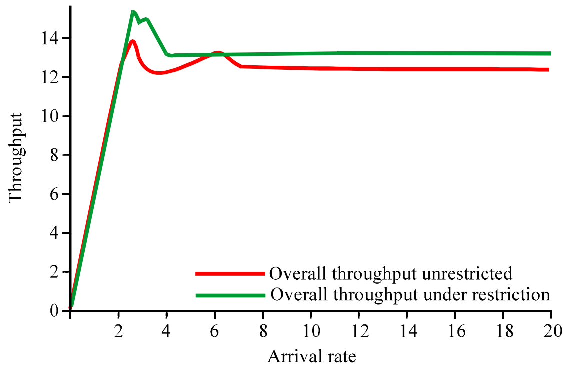

Fig. 11: Total throughput of the facility for unrestricted and restricted flows

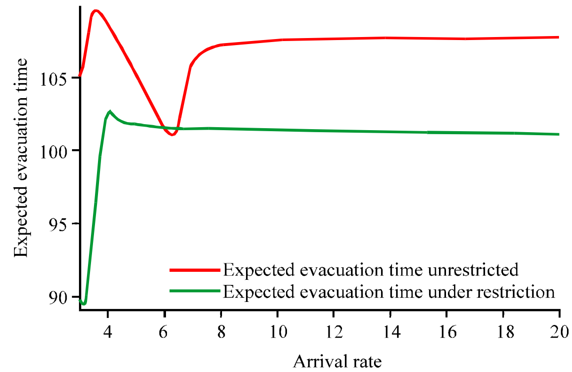

Fig. 12: Expected evacuation time from the facility for unrestricted and restricted flows

The overall throughput of the system for the restricted along with the unrestricted flows is presented in Fig. 11. It is observed that for the unrestricted flow, the curve has two peaks. The throughput reaches its maximum value 13.86 ped sec^{-1} when the arrival rate is approximately 2.60 ped sec^{-1}. The second peak occurs at 13.26 ped sec^{-1} for arrival rate of 6.30 ped sec^{-1}. As the arrival rate increases from 6.30 ped sec^{-1}, the congestion begins to increase and the service rate of each occupants decreases. Thus, despite an increased arrival rate within the corridors, the decreased service rate of each occupant affects the overall throughput as arrival rate increases past 6.30 ped sec^{-1}. As the number of occupants in corridors approaches to corridor capacity, an increase in arrival rate does not have a significant effect on the overall throughput and the throughput appears to reach a limit as arrival rate increases.

For restricted flow, the throughput reaches the maximum value of 15.35 ped sec^{-1} when arrival rate is approximately 2.60 ped sec^{-1}. As the arrival rate increases from 2.60 ped sec^{-1} the throughput decreases and for an arrival rate of 3.90 ped sec^{-1} it reaches the limit which is slightly greater than 13 ped sec^{-1}. That is, if the arrival rate increases past 3.90 ped sec^{-1}, the throughput remains approximately same. Such findings agree with the findings of Mitchell and Smith (2001). The figure shows that the overall throughput increases for the restricted flows compared to the unrestricted flow.

Figure 12 shows the expected evacuation time for the different types of flows for varying arrival rates. It shows that the expected evacuation time is less for the restricted flow compared to that for unrestricted flow. Under restricted flow, the highest expected evacuation time is achieved for an arrival rate of approximately 4 ped sec^{-1} and it approaches to the limit as arrival rate increases past 4 ped sec^{-1}.

It can be observed that restrictions on the flow direction and arrival rates of the source corridors reduce the average evacuation time. If from all the seating arrangements occupants rush to exit, then there will be a high probability of congestion. Since majority of the occupants sit surrounding corridors 10 and 11 and they need to travel a long way for the exit, the arrival rate to these corridors need to be controlled in such a way that the highest throughputs from the exiting corridors can be maintained. Shende *et al.* (2011) has also achieved smooth flow of pedestrians using control-algorithm. In case of an emergency, occupants from seating arrangements S and T may be suggested to go through corridors 14 and 15 and from arrangements U and V they may be suggested to go through corridors 12 and 13. While it may seem that it will decrease throughputs from source corridors, there will be less congestion in exiting corridors and will reduce the average evacuation time. Xiang (2007) also showed the similar result for lecture theater type rooms that when there are congestions in the aisle area connecting with each row, the doorway was not fully occupied. In addition, Shende (2008) showed that for uncontrolled case, flow got interrupted within a few seconds of starting, however it did not happen in the controlled case and revealed the usefulness of the pedestrian control approach.

CONCLUSION

We have used state dependent M/G/C/C queuing model to capture the bottleneck effects of pedestrian flow within the corridors of a facility involving combinations of merges and splits. The different performance measures of all the corridors of this hall room during egress have been computed. These measures are used to compute the evacuation time of the hall room in case of an emergency and can be used to evaluate the optimal internal set up for which the maximum throughput can be obtained. We have calculated the different performance measures for restricted and unrestricted flows. The higher arrival rate to the source corridor is found as a cause of higher congestion in the exiting corridors. The congestion may be controlled by putting some restrictions on arrival to the source corridors and on travelling direction from source corridor to exiting corridors. Further extensions of this work will be to investigate the optimal internal set up for such facilities.

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