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Born Inversion Technique for Ultrasonic Scattering Measurements

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Abstract: In this article, three dimensional Born inverse scattering method is modified to convenient form (Born inversion technique) to reconstruct the shape of a three-dimensional flaw in a cylindrical specimen. In this technique, a measurement plane is restricted to the plane perpendicular to the axis of the cylindrical specimen. Thus the cross-sectional image of the flaw can be obtained. By moving the measurement plane along the axis of the cylindrical specimen, the cross-sectional image is obtained for each measurement plane. The three-dimensional flaw image is reconstructed by piling up the obtained cross-sectional images. Cylindrical specimen with an eccentric circular cylindrical cavity model is prepared. The performance of Born inversion technique to reconstruct the three-dimensional flaw is proved by using the experimentally measured waveforms. At the same time, the numerical results are obtained from all directions by finite element method. From the numerical analysis and experimental research for the eccentric circular cylindrical defect, it can be said that the modified method is an effective mean of shape reconstruction and it can reconstruct an eccentric defect that has a characteristic size of 4 mm. It is also proved that Born inversion technique works well for the volume type flaw.

Key words: Numerical analysis, born inversion technique, shape reconstruction, scattering

INTRODUCTION

Ultrasonic NDE (nondestructive evaluation) uses high-frequency acoustic/elastic waves to evaluate components without affecting their integrity or performance. This technique is commonly used in industry to inspect safety-critical parts for flaws during in-service use. In recent years, many studies about the detection and characterization of flaws in inhomogeneous material such as concrete have been developed (Chaix *et al*., 2003; Aggles *et al*., 2011). In addition the damage state of the material has been evaluated with ultrasonics (Shah and Ribakov, 2008). It is well known that the ultrasonic waves in inhomogeneous medium are scattered and diffracted by inclusions, therefore the material exhibits the attenuation (Baganas, 2005). The attenuation coefficient varies with frequency and depends on the size and volume fraction of inclusions. And the attenuation correlates with the dispersion of phase velocity. When the frequency dependency of phase velocity is taken into consideration, the accurate size and shape of the flaws in inhomogeneous medium can be evaluated.

In recent years, many studies about the shape reconstruction of the flaws in the inhomogeneous cylindrical rod structures such as concrete have been developed (Zheng *et al*., 2007a, b, 2008a, b). In this study, based on the frequency change, the phase velocity due to the inhomogeneous material is introduced into three-dimensional Born approximation formula and then the flaw shapes are reconstructed in the inhomogeneous material by three-dimensional formulas. That's the particular contribution. A shape reconstruction method of the macroscopic defect in inhomogeneous material containing inclusions is investigated and the three-dimensional flaw in a cylindrical structure is reconstructed by using the wave data measured on the side of the cylinder. Born inversion technique to reconstruct the shape of scatterer has been investigated by the numerical simulation and then by the experimental measurement in this article. The volume type of integral representation is formulated for the scattered field. Then Born approximation is introduced for the far-field expressions of scattering amplitudes in the integral representation. The shape reconstruction of the flaw is performed by the inverse Fourier transform of the

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approximated scattering amplitudes. Then the linearized inverse scattering method combined with the dispersion of phase velocity for inhomogeneous material is summarized. The experimental measurement is performed to collect the scattering data from flaw. The processed data from the measurement are fed into the inversion method and the performance of the shape reconstruction is confirmed. In the numerical simulation, the software named ANSYS 8.1 is used to calculate the far-field scattering amplitudes and reconstruct the shape of the flaws.

BORN INVERSION TECHNIQUE

In this study, an inversion method is applied to reconstruct the shape of defect. The method is based on the elastodynamic inverse Born approximation. The details of the inversion has been given by Kitahara and Hirose (1998) for the three dimensional elastodynamics. Here we adopt the Longitudinal- Longitudinal (L-L) pulse echo method for the ultrasonic measurement as shown in Fig. 1. The longitudinal wave from the defect D^c is received at the same transducer. In Fig. 1, the propagation direction of the incident wave $u^0(x)$ is defined to be $-\hat{y}$. If we choose the measurement point y at the far-field from flaws, the scattered longitudinal wave component $A_m(k_L, \hat{y})$ can be separated from the scattered transverse wave component. The scattered wave field $u_m^{sc}(y)$ at far-field measurement point can be separated into longitudinal and transverse waves as follows (Nakahata and Kitahara, 2000):

$$u_m^{sc}(y) = D(k_L|y|)A_m(\hat{y}) + D(k_T|y|)B_m(\hat{y}) \tag{1}$$

where, $A_m(\hat{y})$ and $B_m(\hat{y})$ are scattering amplitudes for longitudinal and transverse waves, respectively and $D(k_L|y|) = e^{ik_L|y|}/(4\pi|y|)$. The longitudinal scattering amplitude is used in the following inversion:

$$A_m(\hat{y}) = \frac{\kappa^2}{\mu}\hat{y}_i\hat{y}_m\int_D q_i(x)e^{-ik_L\hat{y}\cdot x}dV \tag{2}$$

Born inversion: In the Born approximation, the total displacement field u in the equivalent source q_i of (Jain, 1982) is replaced by the incident wave field u^0. The equivalent source q_i is given in Eq. 4 and δC_{ijkl} and $\delta\rho$ are replaced by $\delta C_{ijkl} = -C_{ijkl}$, $\delta\rho = -\rho$ for the present case of strong scatterer. Originally, the Born approximation is applied to the weak in the low frequency range. The applicability to the strong scatterer has been discussed by Achenbach *et al.* (1982) in detail.

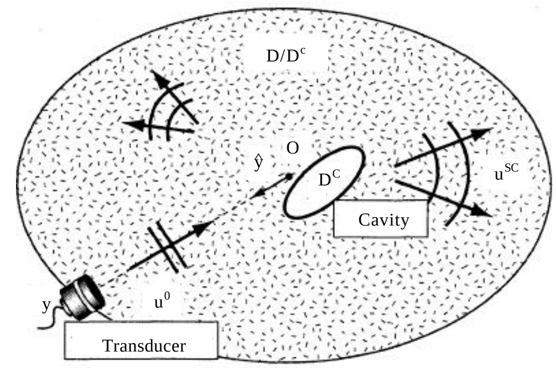

Fig. 1: Defect D^C and wave fields in host matrix $D\backslash D^C$

Introduction of the Born approximation to Eq. 2 leads to the following expression for the longitudinal scattering amplitude:

$$A_m(k_L,\hat{y}) \approx 2u^0k_L^2\hat{y}_m\int_D \Gamma(x)e^{-2ik_L\hat{y}\cdot x}dV \tag{3}$$

where the fact that the scattering amplitude A_m is the function of the wave number k_L has been notified in the above expression. The last integral in Eq. 3 is the Fourier transform $\tilde{\Gamma}(k)|_{k=2k_L\hat{y}}$ of the characteristic function Γ in the K-space with $K = 2k_L\hat{y}$. When the scattering amplitude is measured in the full frequency range on the measurement surface by the pulse-echo method, the Fourier transform of the characteristic function $\tilde{\Gamma}(k)$ in K-space can be determined from Eq. 3. Therefore, the characteristic function $\Gamma(x)$ is obtained by the inverse Fourier transform:

$$\Gamma(x) = \frac{1}{(2\pi)^3}\int\int_0^\infty \frac{\hat{y}_m}{2u^0k_L^2}A_m(k_L,\hat{y})e^{2ik_L\hat{y}\cdot x}8k_L^2dk_Ld\Omega(\hat{y}) \tag{4}$$

where, $\Omega(\hat{y})$ is the unit sphere in three dimensions. The shape of defects can be reconstructed from the characteristic function $\Gamma(x)$.

Three dimensional shape reconstruction from the side of cylinder: If we can obtain the scattering amplitude $A_m(k_L,\hat{y})$ from all directions around origin O, we can calculate the distribution of $\Gamma(x)$ and reconstruct the three-dimensional shape of flaw from Eq. 4. But in the field, it is difficult to measure from all directions.

In this study, the object to be inspected is cylindrical structure. And in the measurement, the size of flaw is larger than the diameter of the incident beam. We modify the three dimensional inverse scattering method to convenient form for cylindrical structure and try to reconstruct the three-dimensional eccentric flaw shape. The process of three dimensional shape reconstruction is

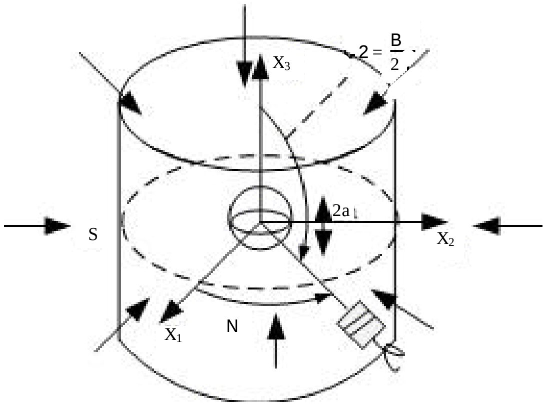

Fig. 2: Process of three-dimensional shape reconstruction

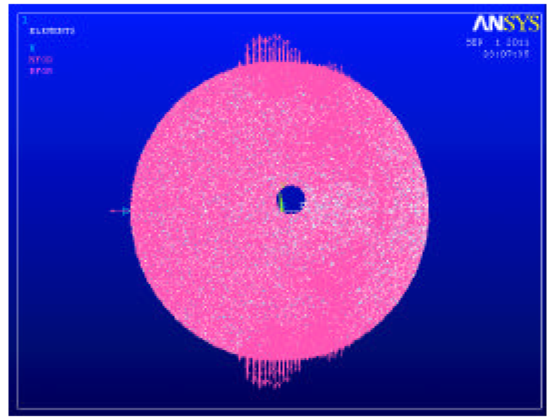

Fig. 3: Finite element's defect model

shown in Fig. 2. Here, the access point of transducer is limited to the surface of the cylinder. In this configuration, it is possible to measure from all directions in a measurement section S in the x_1-x_2 plane.

In this case, the scattering amplitude $A_m^L(k_L, \theta, \phi)$, in Eq. 4 can be represented as $A_m^L(k_L, \pi/2, \varphi)$. Then, we find:

$$\bar{\Gamma}(x) = \frac{1}{(2\pi)^3}\int_0^{2\pi}\int_0^{\infty}\frac{4}{u^0}\hat{y}_m A_m^L(k_L,\pi/2,\varphi)e^{2ik_L(x_1\cos\varphi+x_2\sin\varphi)}dk_L d\varphi \quad (5)$$

where, $\bar{\Gamma}(x)$ is the characteristic function reconstructed by scattering amplitude in x_1-x_2 plane. In this study, the frequency dependency of the phase velocity ($k_L = 2\pi f/c_L(f)$) is introduced into the integral variable k_L of Eq. 5 and we can get the expression for the characteristic function of defect in inhomogeneous material. Here, the phase velocity $C_L(f)$ is estimated from the ultrasonic attenuation by using the Kramers-Kronig relation (Brauner and Beltzer, 1985).

NUMERICAL RESEARCH

The performance of the inverse Born approximation is shown by numerical simulation. The defect model is shown in Fig. 3. It only represents one of the cross-sections along the x_3 axis. Here, the diameter of the cross-section is 100 mm, the eccentric circular cylindrical cavity of radius 4 mm exists in one mortar specimen. The backscattered longitudinal wave component is calculated by finite element method. In the calculation, the normalized wave number ak_L was considered in the range of 0.3~2.5 and Poisson's ratio is set to be 0.2 (Mortar). The calculated amplitudes are applied to the modified three dimensional shape reconstruction formula in Eq. 5.

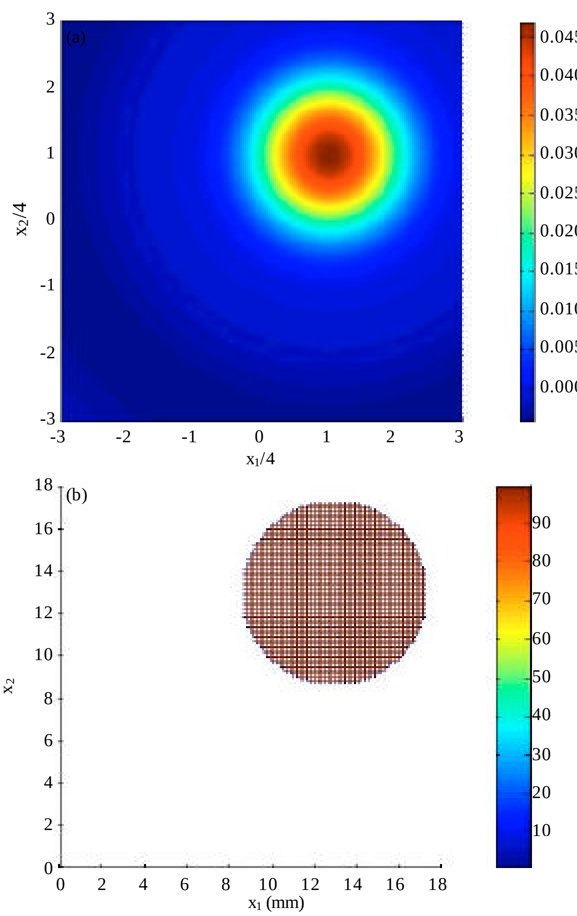

Fig. 4(a-b): $\bar{\Gamma}(x)$ reconstructed from all directions

The result of shape reconstruction is shown in Fig. 4 as the cross sectional view in the x_1-x_2 plane. The figure on the right side of the Fig. 4 is the result of the left figure

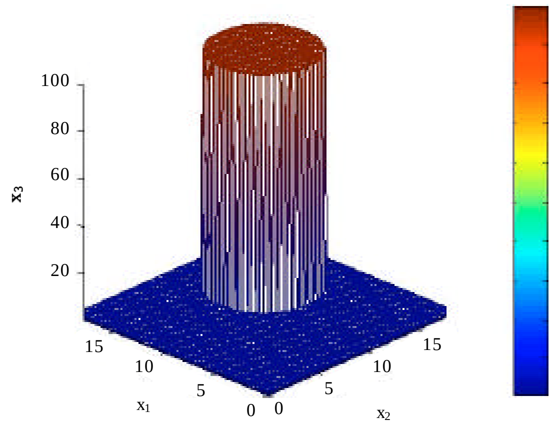

Fig. 5: Reconstructed shape of cylindrical hole

by introducing the threshold value (about 0.036). The condition of numerical simulation is perfect because the radius of the eccentric circular cylindrical cavity that has been reconstructed is 4.15 mm while the truth value is 4 mm, so the results of shape reconstruction from the other cross-sections should be the same. Total cross-sectional images (Fig. 4) are piled up, then three dimensional shape reconstruction is carried out by introducing the interpolation for the piled up cross-sectional images. The results of three-dimensional shape reconstruction are shown in Fig. 5. From the results, it can see that the Born model is supposed to reconstruct the flaw volume. The values of $\bar{\Gamma}(x)$ greater than each threshold value are displayed. The shapes of flaw models are well reconstructed. From Fig. 4 and 5, it is understood that the Born inversion reconstructs the volumetric part of defect.

EXPERIMENTAL RESEARCH

Description of mortar specimen: The inhomogeneous material specimen is the cement paste matrix with the fine aggregates of inclusions and this is called mortar. The mortar specimen includes an eccentric circular cylindrical hole with the diameter 8 mm as flaw model. The specimen with diameter 100 mm is prepared as shown in Fig. 6 and the aggregates are distributed randomly as shown in Fig. 6. The main component of aggregates is granite having mean diameter of grains of 300 microns. The volume fraction is defined as $\Delta V_{incluson}/V_{total}$ and the Fig. 6 is the specimen 5% volume fraction. The wave velocities of cement paste and granite are listed in Table 1. The cement paste property is obtained by compressive test experimentally. The granite property is obtained from the published data (Clark, 1966). The ultrasonic scattering

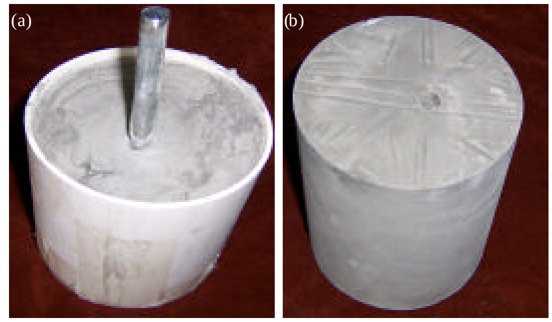

Fig. 6(a-b): Mortar specimen with an eccentric cylindrical hole and its mould

Fig. 7: Experimental setup

Table 1: Cement and granite properties

Material	L-wave velocity c_L (km sec^{-1})	T-wave velocity c_T (km sec^{-1})	Density (g cm^{-3})
Cement	3.89	2.22	2.1
Granite	5.12	3.03	2.6

from micro grains of cement is negligible, so the scattering from the granites determines the frequency dependence of wave attenuation and phase velocity in this mortar.

Experimental setup: The experimental setup is shown in Fig. 7. The experimental system consists of five parts: ultrasonic pulser-receiver (Model 5800), digital oscilloscope (Tektronix DPO3000), transducer (f = 0.5 MHz), PC and the specimen. The scattered wave from flaw model is measured by longitudinal- longitudinal pulse-echo method. A contact type transducer with diameter 12.7 mm moves on the surface of the specimen 10 degrees a step in x_1-x_2 plane and 20 mm a step in x_3 direction from x_3 = 0 to 100 mm. Glutinous honey syrup is used as a couplant and it brings good acoustic coupling and fixation of the transducer. The transducer is driven by 300 V square wave pulser-receiver and digital oscilloscope

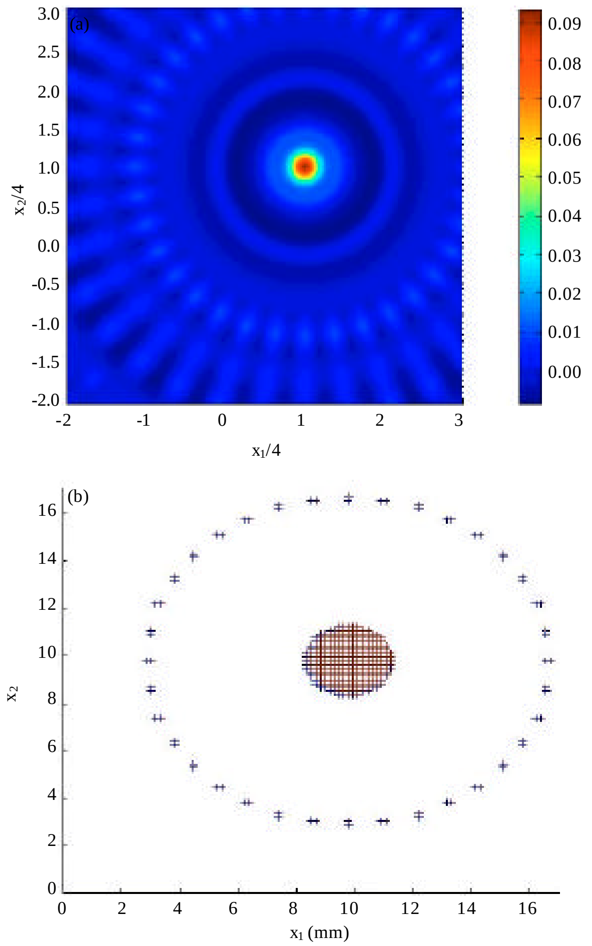

Fig. 8(a-b): $\bar{\Gamma}(x)$ reconstructed from mortar specimen with eccentric cylindrical hole (x_3 = 50 mm)

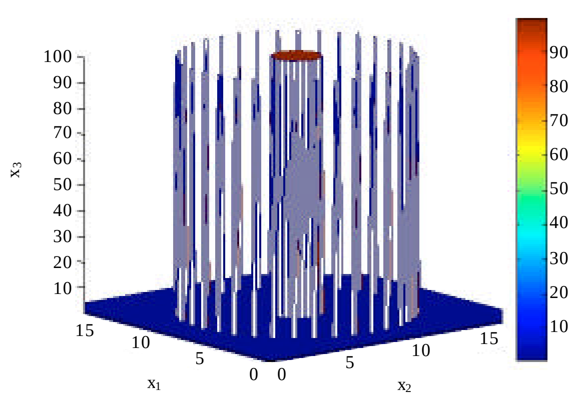

Fig. 9: Reconstructed shape of eccentric cylindrical hole

and PC are used for data processing and shape reconstruction. The center frequency of transducer is 0.5 MHz.

Shape reconstruction of defect: The processed wave (Zheng *et al*., 2008a) can be used as the longitudinal scattering amplitude $\hat{y}_m A_m^L(k_L, \pi/2, \varphi)$ in Eq. 5. Obtained cross-sectional image of $\bar{\Gamma}(x)$ for eccentric cylindrical hole is shown in Fig. 8 as an example. From the Fig. 8, we can see the radius of the eccentric circular cylindrical cavity that has been reconstructed deviates from the truth value a little more than the numerical result. That was caused by systematic error. Since we extract the scattering amplitude on the surface of the specimen 10^0 a step in x_1-x_2 plane, each cross-sectional image has 36 distinct markings around a cross-section.

Total cross-sectional images (Fig. 9) are piled up, then three dimensional shape reconstruction is carried out by introducing the interpolation for the piled up cross-sectional images. The result of three dimensional shape reconstruction is shown in Fig. 9. Here, the values of $\bar{\Gamma}(x)$ greater than each threshold value are displayed. The shape of flaw model is well reconstructed.

CONCLUSION

Three-dimensional linearized inverse scattering method is modified to convenient form to use measured waveforms in the x_1–x_2 plane and cross-sectional images are obtained with numerically and experimentally measured waveforms. Then, three-dimensional shape reconstruction of flaw model in mortar specimen is performed by piling up the cross-sectional images. From the results of the shape reconstruction of the defect, it can be said that the modified method is an effective method for shape reconstruction and it can reconstruct the eccentric defect that has a characteristic size down to 4 mm in inhomogeneous material. It is shown that the modified Born inversion works well for the volume type defect in inhomogeneous material.

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