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Research of an Othmer-stevens Chemotax Is Model with Reproduction Term

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Abstract: In this study, we study the asymptotical behavior of solutions for an Othmer-Stevens model with reproduction term. Making use of a function transformation and comparative method, we prove that the existence of global, blow-up or quenching solutions of the problem on different conditions and more interesting results are obtained. Under proper conditions, the species blow up while attractant quenches in finite time. The results of the paper not only verifies real biological phenomenon but also provides a theoretical groundwork for numerical problems of the chemotaxis model.

Key words: Chemotaxis, global existence, blow-up, quenching solution

INTRODUCTION

The ability to migrate in response to external signals is shared by many cell populations. The directed movement of cells and organisms in response to chemical gradients is called chemotaxis. The first chemotaxis equation was introduced by Keller and Segel (1970) to describe the aggregation of slime mold amoebae due to an attractive chemical substance. Othmer and Stevens consider another kind of chemotaxis phenomenons. In this case the diffusion of the attractant,such as slime, is ignorable and the diffusion coefficient is zero. Othmer and Stevens (1997) developed this kind model by random walking method and called it Othmer-Stevens model. Sleeman and Levine (1997) developed a particular system modeling angiogenesis. They found a explicit solution which could be global or blow up in finite time but they only considered this model in one dimension. Yang *et al*., (2001), Yang Yin considered Othmer-Stevens system in high-dimensional and concluded that the solution was either global or blow up in finite t.

In all works mentioned above, only this kind of chemotaxis model without any reproduction is discussed. Chen and Liu (2011), we considered a simplified Othmer-Stevens model with reaction term. In this study, we consider the following chemotaxis model:

$$\begin{cases} \dfrac{\partial u}{\partial t} = D\nabla\cdot\left(u\nabla\left(\ln\dfrac{u}{\alpha+\beta w}\right)\right)+u\left(a\text{-}b\dfrac{u}{\alpha+\beta w}\right), & x\in\Omega,\quad t>0 \\ \dfrac{\partial w}{\partial t} = \mu u-\delta w, & x\in\Omega,\quad t>0 \\ u\nabla(\ln\dfrac{u}{\alpha+\beta w})\cdot\bar{n}=0, & x\in\partial\Omega,\quad t>0 \\ u(x,0)=u_0(x),\qquad w(x,0)=w_0(x), & x\in\Omega \end{cases} \tag{1}$$

where, u represents the density or population of a biological species which could be a cell, a germ, or an insect while w represents an attractive resource of the species. a, b, μ and δ are parameters, α, β are positive constants. D is the diffusion coefficients of the cel u, $\delta>0$ is the rate of chemoattractant degradation.

For the readers' convenience, the following definition is first presented.

Definition 1: Suppose that (u(x, t), w(x, t)) is a positive solution to Eq. 1 defined in the interval [0, T), if $T\to\infty$, (u(x, t), w(x, t)) is said to be a global solution; otherwise, a non-global solution.

Definition 2: A non-global solution (u(x, t), w(x, t)) is said to be blow-up or quenching, respectively, in finite time (or eventually) if there is a finite T>0(or T = +∞) such that:

$$\lim_{t\to T}\sup_{x\in\Omega} u(x,t)=+\infty \quad \text{or} \quad \lim_{t\to T}\sup_{x\in\Omega} u(x,t)=0$$

FUNCTION TRANSFORMATION

Let the function transformation:

$$p=\frac{u}{\alpha+\beta w}$$

then the system (1) was transformed into the following equations:

$$\begin{cases} \dfrac{\partial p}{\partial t} = \Delta p+\nabla p\cdot\nabla\ln(\alpha+\beta w)+(\delta\dfrac{\beta w}{\alpha+\beta w}+a)p-(\mu\beta+b)p^2, & (x,t)\in Q_T \\ w(p)=\int_0^t \alpha\mu p(x,\tau)e^{\int_\tau^t(\mu\beta p(x,r)-\delta)dr}d\tau+w_0e^{\int_0^t(\mu\beta p(x,r)-\delta)dr}, & (x,t)\in Q_T \\ \dfrac{\partial p}{\partial n}=0, & (x,t)\in\Gamma_T \\ p(x,0)=p_0(x),w(x,0)=w_0(x), & x\in\overline{\Omega} \end{cases} \tag{2}$$

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For any given bounded positive smooth function w(x, t) on Q_T the problem:

$$\begin{cases} \frac{\partial p}{\partial t} = \Delta p + \nabla p \cdot \nabla \ln(\alpha + \beta w) + (\delta \frac{\beta w}{\alpha + \beta w} + a)p - (\mu\beta + b)p^2, & (x,t) \in Q_T \\ \frac{\partial p}{\partial n} = 0, & (x,t) \in \Gamma_T \\ p(x,0) = p_0(x) > 0, & x \in \overline{\Omega} \end{cases} \quad (3)$$

is an initial-boundary value problem of a semi-linear reaction-diffusion equation, to which the comparative method works.

By the well-known theory of parabolic equation, there is a unique local solution p(x, t) to (2.2), whose regularity depends on that of initial data, e.g., $p(x,t) \in C^{2+\varepsilon,\infty}(\Omega \times (0,T))$ if $p_0(x) \in C^{\varepsilon}(\Omega)$. For convenience, a solution in this study always means a function in $C^{2+\varepsilon,\infty}(\Omega \times (0,T))$ satisfying the equation and related initial-boundary condition. And then obviously we have the following lemma.

Lemma 1: $p(x,t) \in C^{2+\varepsilon,\infty}(\Omega \times (0,T))$ is a bounded solution to (2.2) if and only if $(u(x,t), w(x,t))$ is a bounded solution to(1.1), where, $p(x,t) = u(x,t)(\alpha + \beta w(x,t))$ and w(p) satisfying the second equation of system (2.1).

Let:

$$\overline{p}_0 = \sup_{x\in\Omega} p_0(x), \underline{p}_0 = \inf_{x\in\Omega} p_0(x)$$

and:

$$\overline{w}_0 = \sup_{x\in\Omega} w_0(x), \underline{w}_0 = \inf_{x\in\Omega} w_0(x)$$

It is easy to show that there are a super-solution $\overline{p}(x,t)$ and a sub-solution $\underline{p}(x,t)$ to (2.2) which read as follows:

$$\overline{p}(x,t) = \begin{cases} \frac{\overline{p}_0}{1 + \overline{p}_0(\mu\beta + b)t}, & \text{as} a + \delta = 0, \mu\beta + b \neq 0 \\ \overline{p}_0 e^{(a+\delta)t}, & \text{as} \mu\beta + b = 0 \\ \frac{\overline{p}_0 (a+\delta) e^{(a+\delta)t}}{(a+\delta) + \overline{p}_0(\mu\beta + b)(e^{(a+\delta)t} - 1)}, & \text{as} a + \delta \neq 0, \mu\beta + b \neq 0 \end{cases}$$

$$\underline{p}(x,t) = \begin{cases} \frac{\underline{p}_0}{1 + \underline{p}_0(\mu\beta + b)t}, & \text{as} a = 0, \mu\beta + b \neq 0 \\ \underline{p}_0 e^{(a+\delta)t}, & \text{as} \mu\beta + b = 0 \\ \frac{\underline{p}_0 a e^{at}}{a + \underline{p}_0(\mu\beta + b)(e^{at} - 1)}, & \text{as} a \neq 0, \mu\beta + b \neq 0 \end{cases}$$

By comparison principle, we see $\underline{p} \le p \le \overline{p}$. From the second Eq. 2, it can be seen that, if $\mu>0$, $w(\underline{p}) \le w(p) \le w(\overline{p})$; if $\mu<0$, $w(\underline{p}) \ge w(p) \ge w(\overline{p})$.

BEHAVIOR OF SOLUTIONS AS

$$a + \delta = 0, \ \mu\beta + b \neq 0$$

In this section, as $a + \delta = 0, \ \mu\beta + b \neq 0$, a sub-solution and super-solution to Eq. 3 read as follows:

$$\overline{p}(x,t) = \frac{\overline{p}_0}{1 + \overline{p}_0(\mu\beta + b)t}$$

$$\underline{p}(x,t) = \frac{\underline{p}_0 a e^{at}}{a + \underline{p}_0(\mu\beta + b)(e^{at} - 1)}$$

It can be seen easily that, if $a + \delta = 0, \mu\beta + b > 0$, we have:

$$\lim_{t\to+\infty} \overline{p}(x,t) = \lim_{t\to+\infty} \underline{p}(x,t) = 0$$

which implies that:

$$\lim_{t\to+\infty} p(x,t) = 0$$

If $\mu\beta + b < 0$:

$$\underline{p}_0 > \frac{a}{\mu\beta + b}$$

there are:

$$T_1 = \frac{1}{\overline{p}_0(\mu\beta + b)}$$

$$T_2 = \frac{1}{a} \ln[1 - \frac{a}{\underline{p}_0(\mu\beta + b)}]$$

such that:

$$\lim_{t\to T_1} \overline{p}(x,t) = \lim_{t\to T_2} \underline{p}(x,t) + \infty$$

which implies that there is $T \in [T_1, T_2]$ such that:

$$\lim_{t\to T} p(x,t) = +\infty$$

Let:

$$\lambda = \frac{\mu\beta}{\mu\beta + b}$$

from the second equation of system (2):

$$w\left(\bar{p}\right)=e^{-\delta t}\left[1+\bar{p}_0\left(\mu\beta+b\right)t\right]^{\lambda}\left\{\int_0^t \alpha\mu\bar{p}_0[1+\bar{p}_0\left(\mu\beta+b\right)\tau]^{-\lambda-1}e^{\delta\tau}d\tau+w_0\right\} \quad (4)$$

$$w\left(\underline{p}\right)=e^{-\delta t}\left[1+\frac{\underline{p}_0}{a}\left(\mu\beta+b\right)\left(e^{at}-1\right)\right]^{\lambda}\left\{\int_0^t \alpha\mu\underline{p}_0[1+\frac{\underline{p}_0}{a}\left(\mu\beta+b\right)\left(e^{a\tau}-1\right)]^{-\lambda-1}e^{\delta\tau}d\tau+w_0\right\} \quad (5)$$

Theorem 1: Suppose that $a+\delta=0,\ \mu\beta+b>0,$ then there is a unique global solution (u(x, t), w(x, t)) to the Eq. 1, where:

- if $\mu\geq 0$, then both u(x, t) and w(x, t) quench eventually
- if $\mu<0$ then u(x, t) keeps to be uniformly bounded and **quenches in finite time.

Proof: Since $a=-\delta<0, \mu\beta+b>0,$ then:

$$\lim_{t\to+\infty} p(x,t)=0$$

As $\mu>0$, $\lambda>0$ from Eq. 4, 5 and:

$$\lim_{t\to+\infty}\frac{\int_0^t \alpha\mu\bar{p}_0\left[1+\bar{p}_0\left(\mu\beta+b\right)\tau\right]^{-\lambda-1}e^{\delta\tau}d\tau}{\left[1+\bar{p}_0\left(\mu\beta+b\right)t\right]^{-\lambda}e^{\delta t}}=0$$

it can be seen that:

$$\lim_{t\to+\infty} w(\bar{p})=0,\ \lim_{t\to+\infty} w(\underline{p})=0$$

which implies by comparison that:

$$\lim_{t\to+\infty} w(x,t)=0$$

Since u = p(α+w) then:

$$\lim_{t\to+\infty} u(x,t)=0$$

As $\mu<0$, $\lambda<0$:

$$\lim_{t\to+\infty}\int_0^t \alpha\mu\bar{p}_0\left[1+\bar{p}_0\left(\mu\beta+b\right)\tau\right]^{-\lambda-1}e^{\delta\tau}d\tau=-\infty$$

which implies that there exists T_1 such that:

$$\lim_{t\to T_1} w(\bar{p})=0$$

At the same time, if $\lambda+1<0$, then:

$$1\leq[1+\frac{\underline{p}_0}{a}(\mu\beta+b)\left(e^{a\tau}-1\right)]^{-\lambda-1}\leq[1+\frac{\underline{p}_0}{\delta}(\mu\beta+b)]^{-\lambda-1}$$

If $\lambda+1>0$, then:

$$1\geq[1+\frac{\underline{p}_0}{a}(\mu\beta+b)\left(e^{a\tau}-1\right)]^{-\lambda-1}\geq[1+\frac{\underline{p}_0}{\delta}(\mu\beta+b)]^{-\lambda-1}$$

If $\lambda+1 = 0$, then:

$$\alpha\mu\underline{p}_0\int_0^t[1+\frac{\underline{p}_0}{a}(\mu\beta+b)\left(e^{a\tau}-1\right)]^{-\lambda-1}d\tau=\alpha\mu\underline{p}_0 t$$

From above analysis, for any $\lambda\in R$:

$$\lim_{t\to+\infty} w(\underline{p})=-\infty$$

which implies that there is T_2 such that:

$$\lim_{t\to T_2} w(\underline{p})=0$$

where $T_1<T_2$. Thus, there is $T\in[T_1, T_2]$ such that:

$$\lim_{t\to T} w(p)=0$$

and then:

$$\lim_{t\to T} u(x,t)=\lim_{t\to T} p(\alpha+w(p))=0$$

As $\mu = 0$, $w(x,t)=w_0(x)e^{-\delta t}$, then

$$\lim_{t\to+\infty} w(x,t)=0,\ \lim_{t\to+\infty} u(x,t)=0$$

Remark 1: In this chemotaxis model 1, $\delta>0$ means that the attractant can self-reproduce, e.g., slime produced by the species, or mineral in favor with the species. And then, the coefficient μ takes positive when attractant w, e.g., slime, is produced by the species u, or negative when, e.g., mineral, is consumed by u. Thus, as $\mu>0$, the more is u, the more is w; while $\mu<0$, the more u, the less w. If $\mu = 0$, it means w is a kind of mineral which beneficial and non-profit to the species. Species u is effected by attractant w but u does not effect and consume w. That is consistent with our result.

By the similar analysis in the proof of Theorem1, the following theorem can be concluded.

Theorem 2: Suppose that $a+\delta=0, \mu\beta+b<0$ and:

$$\underline{p}_0 > \frac{a}{\mu\beta+b}$$

then there is a unique solution (u(x, t), w(x, t)) to the Eq. 1 such that:

- if $\mu>0$, both u(x, t) and w(x, t) blow up in finite time T:

$$T \in [\frac{-1}{\bar{p}_0(\mu\beta+b)}, \frac{-1}{\underline{p}_0(\mu\beta+b)}]$$

- If $\mu<0$, u(x, t) blow up in finite time T and w(x, t) quenches in finite time T'<T
- If $\mu = 0$, $w(x, t) = w_0 e^{-\delta t}$ and then u(x, t) blow up in finite time T

BEHAVIOR OF SOLUTIONS AS

$\mu\beta+b = 0$

Suppose that $\mu\beta+b = 0$, the sub-solution and super-solution to system 3 as follows:

$$\bar{p}(x,t) = \bar{p}_0 e^{(a+\delta)t}$$

$$\underline{p}(x,t) = \underline{p}_0 e^{at}$$

From the second equation of Eq. 2:

$$w(\bar{p}) = e^{\frac{\mu\beta}{a+\delta}\bar{p}_0[e^{(a+\delta)t}-1]-\delta t}\left\{\alpha\mu\bar{p}_0\int_0^t e^{(a+2\delta)\tau}e^{\frac{\mu\beta}{a+\delta}\bar{p}_0[1-e^{(a+\delta)\tau}]}d\tau + w_0\right\}$$

$$w(\underline{p}) = e^{\frac{\mu\beta}{a}\underline{p}_0[e^{at}-1]-\delta t}\left\{\alpha\mu\underline{p}_0\int_0^t e^{(a+\delta)\tau}e^{\frac{\mu\beta}{a}\underline{p}_0[1-e^{a\tau}]}d\tau + w_0\right\}$$

By the similar analysis in the last section, the following theorems can be concluded.

Theorem 3: Suppose that $\mu\beta+b = 0$ and $a>0$, then there is a global solution to the problem 1 such that:

- If $\mu<0$, both u(x, t) and w(x, t) blow up eventually;
- If $\mu<0$, u(x, t) will blow up eventually while u(x, t) quenches in finite time
- If $\mu = 0$, w(x, t) quenches eventually and **blow up eventually

Theorem 4: Suppose that $a+\delta<0$ and $\mu\beta+b = 0$, then there is a global solution (u(x, t), w(x, t)) to the problem 1 such that:

- As $\mu\geq 0$, both u(x, t) and w(x, t) quench eventually
- As $\mu<0$, if:

$$2\delta + a < 0,\ \underline{w}_0 > \frac{\alpha\mu\bar{p}_0}{a+2\delta}e^{\frac{\mu\beta}{a+\delta}\bar{p}_0}$$

both u(x, t) and w(x, t) quench eventually;

- If:

$$2\delta + a \geq 0, \underline{w}_0 > \frac{\alpha\mu\underline{p}_0}{a+\delta}$$

or:

$$2\delta + a < 0,\ \bar{w}_0 \leq \frac{\alpha\mu\underline{p}_0}{a+\delta},$$

u(x, t) quench eventually and w(x, t) quenches in finite time

- If $\lambda>1$, both u(x, t) and w(x, t) blow up eventually

Theorem 5: Suppose that $a+\delta = 0$ and $\mu\beta+b = 0$, then there is a global solution (u(x, t), w(x, t)) to the problem 1 such that:

- As $\mu>0$, $\mu\beta\bar{p}_0 < \delta$, both u(x, t) and w(x, t) are bounded uniformly
- As $\mu>0$, u(x, t) keeps to be bounded uniformly, u(x, t) quenches in finite time
- As $\mu = 0$, w(x, t) quenches eventually and u(x, t) keeps to be bounded uniformly

BEHAVIOR OF SOLUTIONS AS

$a+\delta\neq 0$, $\mu\beta+b\neq 0$

As $a+\delta\neq 0$, $\mu\beta+b\neq 0$ the sub-solution and super-solution can be written as follows:

$$\bar{p}(x,t) = \frac{\bar{p}_0(a+\delta)e^{(a+\delta)t}}{(a+\delta)+\bar{p}_0(\mu\beta+b)(e^{(a+\delta)t}-1)},\quad \underline{p}(x,t) = \frac{\underline{p}_0 a e^{at}}{a+\underline{p}_0(\mu\beta+b)(e^{at}-1)},\quad as\ a\neq 0$$

$$\bar{p}(x,t) = \frac{\bar{p}_0\delta e^{\delta t}}{\delta+\bar{p}_0(\mu\beta+b)(e^{\delta t}-1)},\quad \underline{p}(x,t) = \frac{\underline{p}_0}{1+\underline{p}_0(\mu\beta+b)t},\quad as\ a=0$$

It can be seen easily that:

- If $a>0$, $\mu\beta+b>0$:

$$\lim_{t\to+\infty}\bar{p}(x,t) = \frac{a+\delta}{\mu\beta+b}$$

$$\lim_{t\to+\infty}\underline{p}(x,t) = \frac{a}{\mu\beta+b}$$

which implies that:

$$\frac{a}{\mu\beta+b} \le \lim_{t\to+\infty} p(x,t) \le \frac{a+\delta}{\mu\beta+b}.$$

- If $-\delta<a\le 0$, $\mu\beta+b>0$:

$$\lim_{t\to+\infty} \bar{p}(x,t) = \frac{a+\delta}{\mu\beta+b}$$

$$\lim_{t\to+\infty} \underline{p}(x,t) = 0$$

which implies that:

$$0 \le \lim_{t\to+\infty} p(x,t) \le \frac{a+\delta}{\mu\beta+b}$$

- If $a+\delta<0$, $\mu\beta+b>0$, then:

$$\lim_{t\to+\infty} \bar{p}(x,t) = 0$$

$$\lim_{t\to+\infty} \underline{p}(x,t) = 0$$

which implies that:

$$\lim_{t\to+\infty} p(x,t) \le 0$$

- If $a\ge 0$, $\mu\beta+b<0$, there exist $t_0<t_2$ such that:

$$\lim_{t\to t_1} \bar{p}(x,t) = +\infty$$

$$\lim_{t\to t_2} \underline{p}(x,t) = +\infty$$

which implies that there is a $T\in[t_1, t_2]$ such that:

$$\lim_{t\to T} p(x,t) = +\infty$$

- If $-\delta<a\le 0$, $\mu\beta+b<0$:

$$\underline{p}_0 > \frac{a}{\mu\beta+b}$$

then there exist $t_1<t_2$ such that:

$$\lim_{t\to t_1} \bar{p}(x,t) = +\infty$$

$$\lim_{t\to t_2} \underline{p}(x,t) = +\infty$$

which implies that there is $T\in[t_1, t_2]$ such that:

$$\lim_{t\to T} p(x,t) = +\infty$$

- If $a+\delta<0$, $\mu\beta+b<0$, as:

$$\bar{p}_0 < \frac{a+\delta}{\mu\beta+b}$$

$$\lim_{t\to+\infty} \bar{p}(x,t) = 0$$

$$\lim_{t\to+\infty} \underline{p}(x,t) = 0$$

which implies that:

$$\lim_{t\to+\infty} p(x,t) = 0$$

as:

$$\underline{p}_0 > \frac{a}{\mu\beta+b}$$

there exist $t_1<t_2$ such that:

$$\lim_{t\to t_1} \bar{p}(x,t) = +\infty$$

$$\lim_{t\to t_2} \underline{p}(x,t) = +\infty$$

which implies that there is $T\in[t_1, t_2]$ such that:

$$\lim_{t\to T} p(x,t) = +\infty$$

From the second equation of problem 2:

$$\begin{aligned} w(\bar{p}) &= \bar{\upsilon}(t)^{\lambda} e^{-\delta t}\left\{\alpha\mu\bar{p}_0\int_0^t \bar{\upsilon}(\tau)^{-\lambda-1} e^{(a+2\delta)\tau} d\tau + w_0\right\}, \quad \bar{\upsilon}(t) = 1 + \frac{\bar{p}_0(\mu\beta+b)}{a+\delta}(e^{(a+\delta)t}-1) \\ w(\underline{p}) &= \underline{\upsilon}(t)^{\lambda} e^{-\delta t}\left\{\alpha\mu\underline{p}_0\int_0^t \underline{\upsilon}(\tau)^{-\lambda-1} e^{(a+\delta)\tau} d\tau + w_0\right\}, \quad \underline{\upsilon}(t) = 1 + \frac{\underline{p}_0(\mu\beta+b)}{a}(e^{at}-1) \end{aligned} \tag{6}$$

Theorem 6: Suppose that $a>0$, $\mu\beta+b>0$, then there is a unique global solution (u(x, t), u(x, t)) to (1) which satisfies that:

- As $\mu>0$, if $\lambda a>\delta$, then both and blow up eventually, if $\lambda(a+\delta)<\delta$, then both u(x, t) and w(x, t) are uniformly bounded
- As $\mu<0$, u(x, t) is uniformly bounded while u(x, t) quenches in finite time
- As $\mu = 0$, then u(x, t) is uniformly bounded and w(x, t) quenches eventually

Proof: Since a>0, μβ+b>0, then:

$$\frac{a}{\mu\beta+b} \le \lim_{t\to+\infty} p(x,t) \le \frac{a+\delta}{\mu\beta+b}$$

From Eq. 6, if μ>0, μa>δ:

$$\lim_{t\to+\infty} w(\underline{p}) = +\infty$$

If μ>0, λ(a+δ)<δ:

$$\lim_{t\to+\infty} w(\underline{p}) = \alpha\mu\underline{p}_0 \lim_{t\to+\infty} \underline{\upsilon}(t)^{\lambda} e^{-\delta t}\int_0^t \underline{\upsilon}(\tau)^{-\lambda-1} e^{(a+\delta)\tau} d\tau = \frac{a\mu\alpha}{(\mu\beta+b)(\delta-\lambda a)}$$

Similarly, when λ(a+δ)<δ:

$$\lim_{t\to+\infty} w(\overline{p}) = \frac{(a+\delta)\mu\alpha}{(\mu\beta+b)[\delta-\lambda(\delta+a)]}$$

From above analysis:

- As λa>δ:

$$\lim_{t\to+\infty} w(p) = +\infty$$

then:

$$\lim_{t\to+\infty} u(x,t) = +\infty$$

- As λ(a+δ)<δ, then u(x, r) is bounded and:

$$\lim_{t\to+\infty} w(p) = \widetilde{w}(x)$$

Where:

$$\frac{a\mu\alpha}{(\mu\beta+b)(\delta-\lambda a)} \le \widetilde{w}(x) \le \frac{(a+\delta)\mu\alpha}{(\mu\beta+b)[\delta-\lambda(\delta+a)]}$$

If μ<0, then λ<0, λ(a+δ)<δ:

$$\lim_{t\to+\infty} w(\underline{p}) < 0$$

$$\lim_{t\to+\infty} w(\overline{p}) < 0$$

Thus, there exist $t_1<t_2$ such that:

$$\lim_{t\to t_1} w(\overline{p}) = 0$$

$$\lim_{t\to t_2} w(\underline{p}) = 0$$

which implies that there is $T\in[t_1, t_2]$ such that:

$$\lim_{t\to T} w(p) = 0$$

$$\lim_{t\to T} u(x,t) = \alpha p(x,T)$$

If μ = 0, w(x, t) = $w_0e^{-\delta t}$, then u(x, t) is uniformly bounded and**quenches eventually.

By the similar analysis of proving Theorem 1 and Theorem 6, the following Theorems can be concluded.

Theorem 7: Suppose that $a<0, a+\delta>0, \mu\beta+b>0,$ then there is a unique global solution (u(x, t), w(x, t)) to (1) which satisfies that:

- As μ>0, λ(a+δ)<δ, then both u(x, t) and w(x, t) are uniformly bounded,
- As μ<0, u(x, t) is uniformly bounded while u(x, t) quenches in finite time
- As μ = 0, then u(x, t) is uniformly bounded and w(x, t) quenches eventually

Theorem 8: Suppose that $a=0, \mu\beta+b>0,$ then there is a unique global solution (u(x, t), w(x, t)) to (1) which satisfies that:

- As μ>0, 0<λ<1, then both u(x, t) and w(x, t) are uniformly bounded
- As μ<0, u(x, t) is uniformly bounded while u(x, t) quenches in finite time
- As μ = 0, then u(x, t) keeps to be bounded and w(x, t) quenches eventually

Theorem 9: Suppose that $a+\delta<0, \mu\beta+b>0,$ then there is a unique global solution (u(x, t), w(x, t)) to (1) which satisfies that:

- As μ≥0, both u(x, t) and w(x, t) quench eventually;
- As μ<0 and λ+1<0
- If $\overline{w}_0 < \min\{d_1, d_2\},$ u(x, t) quenches eventually, w(x, t) quenches in finite time;
- If $2\delta+a<0, \underline{w}_0 \ge \max\{d_5, d_6\}$ both u(x, t) and w(x, t) quench eventually
- As μ<0 andλ+1 ≥0
- If $\overline{w}_0 < \min\{d_5, d_6\},$ u(x, t) quenches eventually, w(x, t) quenches in finite time
- If $2\delta+a<0, \underline{w}_0 \ge \max\{d_1, d_2\}$ both u(x, t) and w(x, t) quench eventually

Here:

$$d_1 = \frac{\alpha\mu \underline{p}_0}{a+\delta}, d_2 = \frac{\alpha\mu \bar{p}_0}{a+2\delta}, d_3 = \frac{\mu\beta + b}{a}\underline{p}_0, d_4 = \frac{\mu\beta+b}{a+\delta}\bar{p}_0 \quad (5)$$

$$d_5 = d_1(1-d_3)^{-(\lambda+1)}, d_6 = d_2(1-d_4)^{-(\lambda+1)}$$

Theorem10: Suppose that $a+\delta > 0,\ \mu\beta + b < 0$.

If a≥0 or a<0:

$$\underline{p}_0 > \frac{a}{\mu\beta + b}$$

then there is a unique non-global solution (u(x, t), w(x, t)) to (1) which satisfies that:

- As μ≥0, then both u(x, t) and w(x, t) blow up in finite time
- As μ<0, w(x, t) quenches in finite time T whileu(x, t) blows up in finite time T', where T'>T

Theorem 11: Suppose that $a+\delta < 0, \mu\beta + b < 0,$ then there is a unique solution (u(x, t), u(x, t)) to (1) which satisfies that:

- Asμ>0
- If d_4<1, both u(x, t) and w(x, t) quench eventually
- If d_3<1, both u(x, t) and w(x, t) blow up in finite time
- As μ<0
- If 2δ+a<0, d_4<1, $\underline{w}_0 \ge \max\{d_5, d_6\}$, both u(x, t) and w(x, t) quench eventually
- If d_4<1, $\bar{w}_0 < \min\{d_1, d_2\}$, u(x, t) quenches eventually and ** quenches in finite time
- If d_3>1, u(x, t) blow up in finite time T and w(x, t) quenches in finite time T'<T
- As μ = 0, d_3>1, then u(x, t) blows up in finite time and w(x, t) quench in finite time

SUMMARY

In this study, we discuss the chemotaxis model (1). The existence of unique solution to this problem is given by sup-sub-solution method. The solution exists globally when the initial value:

$$\frac{u_0(x)}{\alpha + \beta w_0(x)} < \min\{\frac{a}{\mu\beta + b}, \frac{a+\delta}{\mu\beta + b}\}$$

While when the initial value:

$$\frac{u_0(x)}{\alpha + \beta w_0(x)} > \max\{\frac{a}{\mu\beta + b}, \frac{a+\delta}{\mu\beta + b}\}$$

any non-global solution (u(x, t), w(x, t)) satisfies that:

- If μ>0, both u(x, t) and w(x, t) blow up in finite time
- If μ<0, w(x, t) quenches in finite time, after that, u(x, t) blow up in finite time

The main results are shown by above theorems in this study. In fact, we wonder and are studying, what will happen when:

$$\underline{p}_0 < \frac{a}{\mu\beta + b} < \bar{p}_0$$

now.

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