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A New Mathematical Model for Pipe Friction Coefficient Inversion in Oilfield Water Injection System

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Abstract: In simulation and operation optimization of oil field water-injection system, pipe friction factors are important parameters. Because pipelines have been corroded and they have been built for a long time, the pipe friction factors have been changed and we need to correct friction factors. Now the optimization model of friction factors inversion is mainly established by using multi-operating mode dates. Because dates of multi-operating mode are not easy to get, it is necessary to correct friction factors under single operating mode. This paper analyzes multiple solutions property of pipe friction coefficient inversion, presents two theorems about multiple solutions property of inversion. This paper especially establishes a new mathematical model of pipe friction coefficient inversion under single operating mode. The calculating results in example show the effectiveness of the method.

Key words: Water injection pipe, friction factor, inversion, mathematical model

INTRODUCTION

In simulation and operation optimization of oil field water-injection system, raw pipe friction factors are applied in mathematical model now (Liggett and Chen, 1994; Shayya and Sablani, 1998). Oil field water-injection network is the high-pressure system and the pipelines' diameter are relatively small and they have been corroded seriously and they have been built for a long time, so the pipe friction factors have been changed. So we need to correct friction factors. There were many results about pipe friction coefficient inversion (Wang, 2010; Sablani *et al*., 2003; Kapelan, 2002; Schaetzen, 2000), mainly the optimization model of friction coefficient inversion are established by using multi-operating mode dates. Both traditional optimization algorithms and intelligent optimization algorithms can not guarantee the global optimal solution of optimization problem is obtained, so the calculation result of this model is not accurate. In oil field production, dates of multi-operating mode are not easy to get, so calculating results of the multi-operating mode optimization model are not accurate.

This study analyzes multiple solutions property of pipe friction coefficient inversion, establishes a new mathematical model of pipe friction coefficient inversion under single operating mode.

MULTIPLE SOLUTIONS PROPERTY OF PIPE FRICTION COEFFICIENT INVERSION

In simulation calculation and operation optimization in oilfield water injection pipe network, nodes equations need be solved to get each node pressure. The continuous Eq. 1 and pressure reducing Eq. 2 are used to get node pressure.

$$\sum q_{ij} + Q_i = 0 \tag{1}$$

$$h_{ij} = H_i - H_j = s_{ij} \mid q_{ij} \mid^{n-1} q_{ij} \tag{2}$$

H_i and H_j denote nodes pressure on both ends of pipeline; s_{ij}denotes coefficient; n = 1.852,

$$s_{ij} = \frac{10.667 l_{ij}}{C_{ij}^{1.852} d_{ij}^{4.87}} \tag{3}$$

l_{ij} denotes the length of pipeline,its unit is meter, d_{ij} denotes pipeline diameter,its unit is meter, C_{ij} denotes pipe friction coefficient.

When all nodes' flow quantity, pipelines' length , pipelines' diameter and a reference point pressure are known, if all pipelines' friction coefficients are known

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each node pressure will be obtained by solving a equation system and the pressure of each node is the unique.

As follows we will debate how to obtain the pipeline friction coefficients if each note pressure is known and each pipeline friction coefficient is unique.

Theorem 1 Each node flow quantity Q_i, pipeline length l_{ij} and pipeline diameter d_{ij} in pipe network are known. Assuming that the number of pipes is m and the number of nodes is n, without loss of generality, assuming that the n-th node pressure is a reference point pressure, this node pressure is known. Removing the n-th equation in continuous system of linear Eq. 4:

$$\sum q_{ij} + Q_i = 0,\ i = 1, 2, \cdots, n \tag{4}$$

new system of linear Eq. 5:

$$\sum q_{ij} + Q_i = 0,\ i = 1, 2, \cdots, n-1 \tag{5}$$

is obtained. The new system of equations has infinitely many solutions.

Proof: q_{ij} that satisfies 1<j is as unknown quantity in the continuous system of linear Eq. 5. Because of $q_{ij} = -q_{ji}$ the number of the unknown quantity is m and the number of equations is n-1. Because of n-1<m,the new system of linear Eq. 5 has infinitely many solutions according to the theory of linear equations system.

Arbitrarily taking a set of pipe friction coefficient values $\{C_1, C_2, .., C_m\}$, the n-th node pressure is as a reference point pressure, this pressure value can be taken arbitrarily and calculation pressure values $\{H_1, H_2, \ldots, H_n\}$ of all nodes can be obtained by solving the node equations. All the pipeline flow quantity (q_{ij}) can be got by the calculation pressure values and the pressure reducing equations and pipe flow quantity must satisfy the continuity Eq.:

$$\sum q_{ij} + Q_i = 0 \tag{6}$$

so the new system of linear Eq. 5 has solutions, has infinitely many solutions.

Theorem 2 Each node flow quantity Q_i, pipeline length l_{ij} and pipeline diameter d_{ij} in pipe network are known. Assuming that the number of pipes is m and the number of nodes is n. Assuming that all nodes pressure values $\{H_1, H_2, \ldots, H_n\}$ are known, the n-th node pressure is a reference point pressure, there exist infinitely many kinds of pipe friction coefficients $C = \{C_{ij}\}$. By using any friction coefficients $C = \{C_{ij}\}$ calculation pressure values are consistent with the measured pressure values.

Proof: The $\{q_{ij}\}$ that satisfy the equations set (6) have infinitely many kinds according to theorem 1, so infinitely many kinds of $\{S_{ij}\}$ can be obtained by using the known pressure $\{H_1, H_2\ldots, H_n\}$and pressure reducing Eq. 7:

$$H_i - H_j = s_{ij} \, | q_{ij} |^{n-1} \, q_{ij} \tag{7}$$

Again by using:

$$s_{ij} = \frac{10.667 l_{ij}}{C_{ij}^{1.852} d_{ij}^{4.87}} \tag{8}$$

infinitely many groups $\{C_{ij}\}$ can be obtained. Certainly for any group of friction coefficients $C = \{C_{ij}\}, \{q_{ij}\}$ that are obtained by using pressure reducing equation (7) must satisfy the new continuous Eq. 6.

Using any group friction coefficients $C = \{C_{ij}\}$ and the n-th node pressure as a reference point pressure , all nodes pressure are be obtained by solving node equation. The calculation pressure also satisfy the pressure reducing Eq. 7and $\{q_{ij}\}$ that is obtained with the pressure reducing equation must also satisfy the new continuous Eq. 6.

Because calculation pressure of the n-th node is equal to the known pressure and they both satisfy node equations, calculation pressure values are consistent with the known pressure values according to the uniqueness solution of nodes equation. So there exist infinitely many groups of pipeline friction coefficients $C = \{C_{ij}\}$, The nodes pressure values calculated by using any group are consistent with the known measured pressure values.
The proof process of theory 1 and theory 2 also show how to get friction coefficients by using the known nodes pressure values and the calculated pipe friction coefficients are not unique.

MATHEMATICAL MODEL OF PIPELINE FRICTION COEFFICIENT FOR SINGLE OPERATING MODEL

Assuming that the pipe network has n nodes, $H_1, H_2, \ldots, H_n$ denote respectively their pressure. The pipe network has m pipes,C_1, C_2, ..., C_m denote, respectively friction coefficients and assuming that $C = (C_1, C_2, \ldots, C_m)^T$.

For continuous system of linear Eq.:

$$\sum q_{ij} + Q_i = 0,\ i = 1, 2, \cdots, n \quad (9)$$

there exists at least a redundant equation that can be linearly expressed by other equations, the redundant equation should be removed. Without loss of generality the latest equation is removed and the new continuous system of linear equations is as follow:

$$\sum q_{ij} + Q_i = 0,\ i = 1, 2, \cdots, n-1 \quad (10)$$

I and j denote the nodes identifier in two ends of the k-th pipe according to number order i<j. Let $q_k = q_{ij}$, so $q_{ij} = -q_k$ and the continuous Eq. set (10) that regards q_1, q_2, ..., q_m as the unknown quantity has only m unknown terms.

To the convenience, the system of Eq. 10 is written as matrix vector form.

Assuming:

$$q = (q_1, q_2, \cdots, q_m)^T$$

A denotes coefficient matrix,obviously the order of A is (n-1)×m,the element values of A can be determined by designing computer programming as follows. The two node number of the j-the pipe are first(j) and second(j) and first(j)<second(j). VB program for generating coefficient matrix is as follows:

```
For i = 1 To n-1
For j = 1 To m
a(i,j) = 0
If first(j) = i Then
a(i,j) = 1
ElseIf second(j) = i Then
a(i,j) = -1
End If
Next j
Next i
```

So system of linear Eq. 10 can be written as matrix-vector form as follow

$$Aq = Q \quad (11)$$

Theorem 1 has already proved that this system of linear equations has infinitely many solutions. for any q,the pipe friction coefficient C can be obtained by using the known pressure H_1, H_2, ..., H_n.The proof process of theorem 2 shows that the calculation pressure values are consistent of the known ones.

C_0 denotes the experience friction coefficient, and $C_{min}<C<C_{max}$.

By using known nodes pressure, pressure reducing equation and experience friction coefficient C_0, we can obtain pipe experience flow quantity q_0 and scope:

$$Q_{min}>q>q_{max} \quad (12)$$

So the mathematical model of this problem is as follows

$$\begin{aligned} &\min_q \| q - q_0 \|_2 \\ &\text{s.t. } Aq = Q \\ &\quad q_{min} < q < q_{max} \end{aligned} \quad (13)$$

Because system of linear equations:

$$Aq = Q$$

must have solution and the rank of A is less than the number of unknown terms,the solutions of:

$$Aq = Q$$

are consistent with the least square solutions,they have both infinitely many solutions and any solution can satisfy:

$$| Q - Aq \|_2^2 = 0$$

So the mathematical model is changed as follow Determining q to satisfy:

$$\begin{aligned} &\min_q \| q - q_0 \|_2 \\ &\text{s.t. } \min\|Q-Aq\|_2^2 \\ &\quad q_{min} < q < q_{max} \end{aligned} \quad (14)$$

Let:

$$q' = q-q_0,$$

$$Q' = Q-Aq_0,$$

$$q'_{min} = q_{min}-q_0,$$

$$q'_{max} = q_{max}-q_0,$$

then mathematical model is changed as follows.

Determining q' to satisfy:

Table 1: Inversion results of friction factors

Pipe ID	The real friction factor	The calculating friction factor	Error
1	85	85	0
2	85	84.6	0.4
3	95	94	1
4	85	84.5	0.5
5	85	86.4	1.4
6	95	94.3	0.7
7	95	95	0
8	85	84	1
9	85	85	0
10	95	96.2	1.2
11	85	85.3	0.3
12	105	105	0
13	115	114.3	0.7
14	95	95.5	0.5
15	105	104.2	0.8
16	105	105	0
17	115	115	0
18	105	105	0
19	105	103.5	1.5
20	115	114.2	0.8
21	115	114.5	0.8
22	105	105	0
23	115	115	0
24	115	114	1

$$\min \|q'\|_2 \quad \text{s.t.} \begin{cases} \min \|Q' - Aq'\|_2^2 \\ q'_{min} \le q' \le q'_{max} \end{cases} \tag{15}$$

So the calculation steps of pipe friction coefficient calibration under single operating mode are as follows:

- **Step 1:** Coefficient matrix A is determined by using the known datas
- **Step 2:** Experience flow q_0 in pipelines is determined by the known nodes pressure and experience friction coefficients
- **Step 3:** q_{min} and q_{max} are determined by using the known nodes pressure, pressure reducing equation, friction coefficient scope C_{min} and C_{max}
- **Step 4:** Solving the optimization problem (15) by using the least square method
- **Step 5:** Let $q = q' + q_0$
- **Step 6:** s_{ij} is solved by formula $s_{ij} = |H_i - H_j| / |q_{ij}|^{1.852}$
- **Step 7:** Friction coefficients C_{ij} of all pipe lines are determined by formula

$$s_{ij} = \frac{10.667 l_{ij}}{C_{ij}^{1.852} d_{ij}^{4.87}}$$

Example: There are 2 water injection stations, 16 nodes, 24 pipes, 9 rings in an ideal pipe network. The simple graph of ideal pipe network is shown in Fig. 1.

The mathematical model (15) is solved and the inversion results are shown in Table 1.

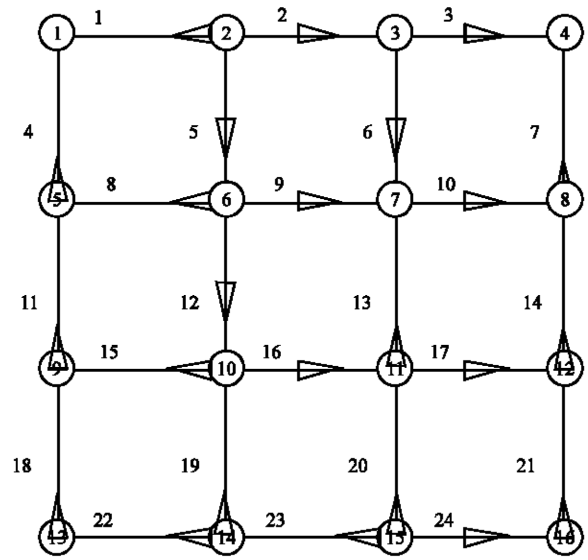

Fig. 1: Simple graph of ideal pipe network

According to these dates in Table 1, the result was satisfied, the average value of errors between the real friction factor and computing friction factor is 0.525.

CONCLUSIONS

This study analyzed multiple solutions property of pipe friction coefficient inversion, presented two theorems about multiple solutions property of inversion, established a new mathematical model of pipe friction coefficient inversion under single operating mode. About the uniqueness of solution of this mathematical model will be detailed in the other articles. In example the calculating results show the effectiveness of the method.

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