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Application of Grey Theory and Wavelet Neural Network in Slope Displacements Prediction

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Abstract: Slope displacement is the external performance of slope stability, predicting slope displacement according to the original displacement monitoring data and predicting slope stability, which can effectively prevent the occurrence of landslide disaster. Slope displacement can be regard as the sum of trend item and random item, grey system G (1, 1) model can simulate the trend of the slope displacement time series and the wavelet neural network can predict the random variations of slope displacement. Based on the grey-wavelet neural network algorithm, prediction analysis of displacement was carried, the displacement prediction of Changchun west railway station foundation pit verify the accuracy and practicability of this method.

Key words: Slope engineering, displacement monitor, prediction analysis, grey system, wavelet neural network

INTRODUCTION

Slope soil shear strength damage under the action of the external environment (rain, freeze-thaw cycles), resulting in slope stability decreased, at the same time the slope surface displacement changed obviously. Lots of studies show that it was able to calculate slope stability by slope surface displacement with some numerical algorithms (Liu *et al.*, 2007; Rechea *et al.*, 2008; Baroth and Malecot, 2010). Therefore, if the trend of slope surface displacement can be obtained, particularly accurately predict short slope displacement, it was able to provide effective reference for prevention of landslide disasters.

There were two research ideas of slope displacement forecasting at present. The first, the grey theory was used to establish slope displacement change model according to the monitoring data and then predict the displacement at some point in future. Zhao Jingbo, Li Li using grey system theory and methods to predict rock slope deformation and proposed to divided time series with the stage of controls factors changes, which improved the forecast accuracy (Zhao *et al.*, 2005). Chen Xiaobin, Zhang Jiasheng used GM (1, 1) model to predict bolt tension, ground subsidence and ground horizontal displacements, the results show that GM (1, 1) model was suitable for short-term forecasting and long-term forecasts ineffective (Chen *et al.*, 2006). Because of grey model need to generate number columns with grey exponentially by accumulation, so as to fit functional

relationship satisfy a certain precise. But, for most non-negative time series, its cumulative result can't meet or satisfy the exponential function accurately, so it is often difficult to achieve the target accuracy by use grey prediction model to predict. The second, as neural network algorithm with powerful nonlinear mapping ability, it was used in engineering practice more and more, there were also many examples of its applications in slope displacement forecast (Zhang *et al.*, 1999; Song *et al.*, 2003). But, the original weights and bias of neural network was random, which lead to calculation results of neural network had a certain instability. When the monitoring data was mutated, the neural network was difficult to obtain accurate predictions.

In a word, there were some defects in separate predict calculations as grey model and neural network algorithm. Many scholars carried studies in using grey neural network algorithm to predict and analysis. Gu Song used grey wavelet neural network to predict mine gas emission (Gu *et al.*, 2007) and Qu Jing used grey wavelet neural network to predict and study vessel traffic flow (Qu, 2010). All these studies show that the prediction accuracy of grey wavelet neural network was higher than grey BP neural network, as using these methods in short-term time series predict and analysis. Therefore, this paper introduced the grey theory and wavelet neural network algorithm to predict the slope displacement. The slope displacement was divided into a trend item and random item. Grey model is used to simulate the trend item and wavelet neural network was used to simulate the random item. At last the method

was used in engineering example to predict the displacement of deep foundation in Changchun west station.

DECOMPOSITION OF SLOPE DISPLACEMENT TIME SERIES

Typically, the slope displacement time series can divide into two kinds of ingredients and can use the model to represent as following:

$$y_t = u_t + v_t \quad (1)$$

where, u_t is the deterministic trend item of displacement, v_t is the uncertain random item, subjected to influences by seasonal (rain, freeze-thaw), period (groundwater) and random incidents (human activity and earthquakes), etc., so, it can be considered as random trend item.

Trend item of slope displacement can be describes by grey system. The random item was still a complex non-linear sequence, so, it can be describes by neural network model. At the same time, in order to overcome the shortcomings of traditional neural network model, wavelet neural network model was used to simulate this random item.

GREY SYSTEM MODELING OF SLOPE DISPLACEMENT TIME SERIES TREND ITEM

In order to obtain the trend item of displacement time series, at first, the scatter plot of slope displacement time series should be draw and observed the possible forms of displacement trends from the scatter plot. Since the displacement trend curve had convex trend, this curve was an exponential growth law, it was appropriate to describe the curve with grey system GM (1,1). It should be noted that, the grey system model was only used to obtain the trend item and no special requirements for its random item. So, there were no accuracy requirements in grey system model and only required the random item was no longer monotone increasing sequence.

GM(1,1) model: The differential equation of G (1,1) as following:

$$\frac{dx}{dt} + ax = b \quad (2)$$

Where a , b were coefficients, depended on different types and displacement change stages of slope, they can be calculated by grey theory.

Grey solution of a , b : (Original slope displacement monitoring data (the same time interval, Δt):

$$x^0(i), i = 1, 2, \dots, n$$

Accumulated original data and generated AGO, as following:

$$x^1(i) = \sum_{k=1}^i x^0(k), i=1, 2, \dots, n \quad (3)$$

$$B = \begin{bmatrix} \frac{1}{2}[x^1(1) + x^1(2)] & 1 \\ \frac{1}{2}[x^1(2) + x^1(3)] & 1 \\ \vdots & \vdots \\ \frac{1}{2}[x^1(n-1) + x^1(n)] & 1 \end{bmatrix} \quad (4)$$

$$Y = [x^0(2), x^0(3), \dots, x^0(n)]^T \quad (5)$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = [B^T B]^{-1} B^T Y \quad (6)$$

The GM (1,1) time response can be obtained as following:

$$x = (x_1 - \frac{b}{a})e^{-a(t-1)} + \frac{b}{a} \quad (7)$$

WAVELET NEURAL NETWORK ALGORITHM SIMULATED SLOPE DISPLACEMENT RANDOM ITEM

Wavelet theory: Wavelet theory developed based on the problem of Fourier transform. Fourier transform was a signal analysis method that was the most widely used in the field of signal processing. But it had a serious problem, which abandoned time information as transforming, the transform results can't determine signal generation time. That is Fourier transform had no ability to distinguish in time domain. The wavelet was a waveform with finite length and the average was 0. Its characteristic included:

- Time domain had compact set or approximate compact set
- The DC component was 0

Wavelet function was controlled by a mother wavelet get through pan and size telescopic, wavelet analysis decomposed the signal into superimposed of a series wavelet function.

The wavelet transform means put a fundamental of wavelet function $\psi(t)$ shift τ and then make the inner product with the signal $x(t)$ which was analyzed under different sizes α :

$$f_x(\alpha, \tau) = \frac{1}{\sqrt{\alpha}} \int_{-\infty}^{\infty} x(t) \psi\left(\frac{t-\tau}{\alpha}\right) dt \quad \alpha > 0 \quad (8)$$

The time domain expression was as following:

$$f_x(a, \tau) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(\omega) \psi(a\omega) e^{j\omega\tau} d\omega \quad a > 0 \quad (9)$$

where, τ and α were the wavelet transform parameters, τ was equivalent to a parallel shift lens relative to the target, α was equivalent to advancing towards or away from the lens.

Equation 8 and 9 show that wavelet theory can be used to analysis the local characteristic of the signal through transform analysis of wavelet function and it had the selectivity ability of signal direction in the case of two dimensional. Therefore, the method as a mathematical theory and analysis method had the better effect to extract the mutation feature of time series.

Wavelet neural network: Wavelet neural network was based on the BP neural network topology and the wavelet basic function was taken as the transfer function of the hidden layer, signal forward propagation and error back propagation. Wavelet neural network topology was shown in Fig. 1.

In Fig. 1, $X_1, X_2, \dots, X_k$ were the input parameters of wavelet neural network, $Y_1, Y_2, \dots, Y_m$ were the prediction output of wavelet neural network, ω_{ij} and ω_{jk} were wavelet neural network weights.

When the input signal sequence was $x_i (i = 1, 2, \dots, k)$, the hidden layer output can calculate as:

$$h(j) = h_j \left(\frac{\sum_{i=1}^k \omega_{ij} x_i - b_j}{a_j} \right) \quad j=1, 2, \dots, l \quad (10)$$

where, $h(j)$ was the output value for node j in the hidden layer, ω_{ij} was the connection weights between input layer and hidden layer, b_j was the shift factor of wavelet function h_j , a_j was the stretching factor of wavelet function h_j , h_j was wavelet function.

Wavelet neural network output layer can be calculated as:

$$y(k) = \sum_{i=1}^l \omega_{jk} h(i) \quad k=1, 2, \dots, m \quad (11)$$

where, ω_{jk} was the connection weights between hidden layer and output layer, $h(i)$ was the output value for node i in the hidden layer, l was the number of nodes in the hidden layer, m was the number of nodes in the output layer.

Weight parameter correction algorithm of wavelet neural network was similar to BP neural network. The gradient correction method was carried to correct the network weights and wavelet function parameters, so, the prediction output of wavelet neural network was closer and closer to the desired output. The correction steps of wavelet neural network as follows:

- Calculate network prediction error:

$$e = \sum_{k=1}^m y_n(k) - y(k) \quad (12)$$

where, $y_n(k)$ was the desired output, $y(k)$ was the prediction of wavelet neural network

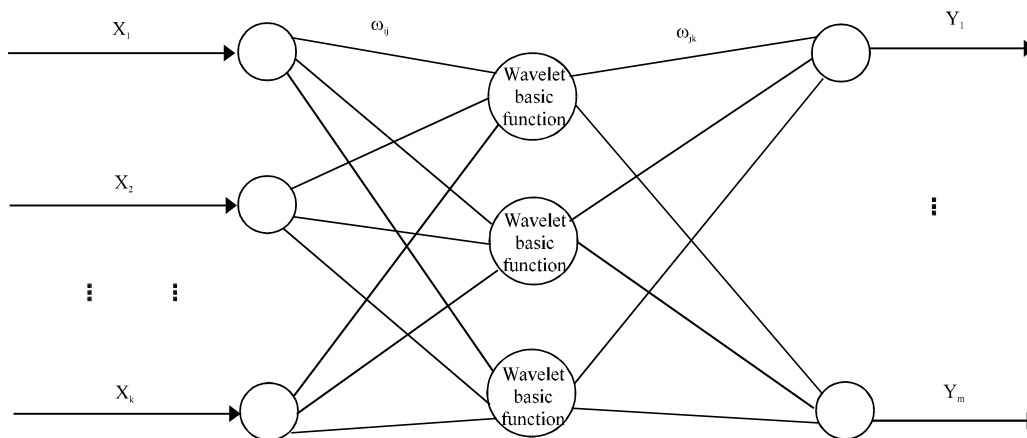


Fig. 1: Wavelet neural network topology

- Corrected wavelet neural network weights and wavelet coefficients according to the prediction error:

$$\omega_{n,k}^{(i+1)} = \omega_{n,k}^i + \Delta\omega_{n,k}^{(i+1)} \quad (13)$$

$$a_k^{(i+1)} = a_k^i + \Delta a_k^{(i+1)} \quad (14)$$

$$b_k^{(i+1)} = b_k^i + \Delta b_k^{(i+1)} \quad (15)$$

where, $\Delta\omega_{n,k}^{(i+1)}$, $\Delta a_k^{(i+1)}$ and $\Delta b_k^{(i+1)}$ were calculated by network prediction error:

$$\Delta\omega_{n,k}^{(i+1)} = -\eta \frac{\partial e}{\partial \omega_{n,k}^i} \quad (16)$$

$$\Delta a_k^{(i+1)} = -\eta \frac{\partial e}{\partial a_k^i} \quad (17)$$

$$(18)$$

where, η was the learning efficiency.

ENGINEERING APPLICATION

Engineering overview: This engineering was Changchun West Railway Station south square deep excavation. The internal transport of this project contained rail, bus, taxi and other cars, the total gross floor area reached 103550 m² (including subway 20833 m²). The deep excavation was divided into three layers, two floors and basement. This excavation had features as large area, deep, more hierarchical. The south square area was about 115,000 m², vertical excavation had two steps, the excavation depth of first stage was based on surface topography varies, the deepest was 8.4 m, the most shallow depth was 3.4 m, bottom elevation was 13.2 m. The second stage excavation on the basis of the first stage and excavation depth was about 7.9~8.2 m, bottom elevation was 21.1~24 m.

Slope displacement monitoring data: Wireless remote method was used to obtain slope horizontal displacement in real time, through automatic data monitoring and transmission. The monitoring cycle was 10 minutes, the monitoring results of slope horizontal shown as Fig. 2.

Slope displacement prediction: The original monitoring data accumulated with grey theory as Eq. 3, changes of displacement shown in Fig. 3.

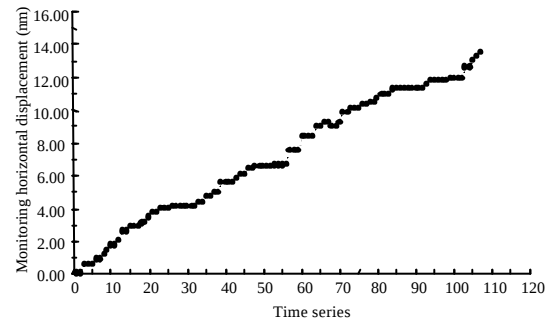


Fig. 2: Horizontal displacement

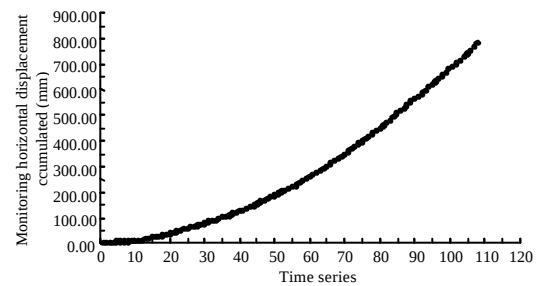


Fig. 3: Data samples after grey accumulated

As can be seen from Fig. 3, the monitoring data sample had better regularity, using Eq. 4~6 can obtain the G (1, 1) model of displacement trends:

$$x = 201.54 e^{0.0153(t-1)} - 201.48 \quad (19)$$

Slope displacement trend item at any point time can be calculated according to Eq. 19, slope displacement random item can be calculated according to Eq. 1 as the time point is obvious. Then, displacement random item of six time point before time point t was taken as wavelet neural network input, displacement of time point t was taken as wavelet neural network output, carried neural network training. In this Study, 102 time points displacement were as wavelet neural network samples, the wavelet neural network structure was 6-8-1, that is, six input nodes, eight hidden nodes and on output node. When the network iterative calculation number reached 1000, the wavelet neural network training error was 0.0719 and program operation time was 10 sec. Training error of wavelet neural network shown as Fig. 4.

Input random item of monitoring data with time number between 97 and 102, prediction value of monitoring data random item with time number of 103 can be obtained, the result was -1.90. Then put $t = 103$ into Eq. 13, displacement trend item with time number of 103 can be obtained, the result was 15.57. Summed the random item and trend item was displacement prediction

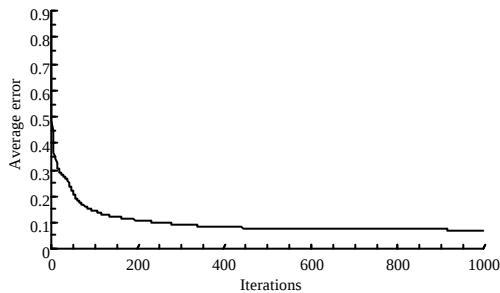


Fig. 4: Iterative error of wavelet neural

Table 1: Measured displacement and the model output

t	y_t	u_t	v_t	x_t	δ (%)
103	12.60	14.57	-1.90	12.67	0.58
104	12.60	14.80	-1.96	12.84	1.88
105	13.02	15.02	-1.99	13.03	0.07
106	13.31	15.26	-1.99	13.27	0.34
107	13.54	15.49	-1.85	13.64	0.77
108	13.60	15.73	-1.99	13.74	1.04

t: Time series, y_t : Measured data, u_t : Trend term prediction value, v_t : Random term prediction value, x_t : Prediction value by grey theory and wavelet neural network and δ : Relative error

with time number $t = 103$. In the same way, displacement prediction with time number between 104 and 108 can be obtained, the results shown as Table 1.

As can be seen from Table 1, the prediction accuracy was less than 2% as using grey theory and wavelet neural network to predict slope displacement. Connecting remote real time monitoring system, the slope displacement of next time in future can be obtained accurately by this algorithm, all this can effectively prevent the occurrence of landslides.

CONCLUSION

In this study, grey theory and wavelet neural network was used to predict and analysis slope displacement. Focusing on analysis the basic principle of wavelet neural network and carrying researches on steps of using the algorithm to predict slope displacement. G (1,1) model was used to simulate slope displacement trend term, which can obtain slope displacement trend term prediction value in any time. Wavelet neural network was used to predict slope

displacement random term and random term of six time node before prediction time was taken as network input, prediction random term as network output. Practical engineering application indicated that this method can accurately predict slope displacement according to the original monitoring data, which can provide references for effectively preventing the occurrence of landslides.

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