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A Wholesale Price-Based Coordination Scheme to Optimize Supplier Profit in Newsvendor Problems

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Abstract: In this study, we present a single-period supplier-profit-oriented newsvendor supply chain problem in a stochastic demand framework, where market price will impact demand in multiplicative form. The Weibull distribution is chosen, since this inter-relationship will affect the scale parameter of the distribution. The model is game-theoretic. The supplier determines the wholesale price, whereas the retailer will respond with a market price and order quantity that maximize his own profit. The bargaining process ends when the supplier finds a wholesale price, which maximizes his sales profit minus product quality cost. The quality cost includes inspection cost and penalty cost of selling defective products to customers. Influence diagram is used to model a Bayesian inspection sampling plan that reduces product quality risk. Our model integrates market risk and product quality risk. We also analyze how the model parameters impact the optimal supply chain decisions and supply chain performance. The results show the effectiveness of the quantity discount strategy and product quality on the supply chain benefits in the proposed model.

Key words: Newsvendor problems, Quantity discounts, influence diagrams, weibull distributions

INTRODUCTION

This study studies a supply chain coordination issue of the newsvendor problem, taking into account market risk and product quality risk. The market elements usually include the following: (1) Market price, (2) Retailer order quantity, (3) Demand distribution, where expected value and/or variance may be affected by market price and (4) Wholesale price. The quality risk concerns the penalty cost when malfunctioning products are sold to customers. It is reasonable to assume that the supplier will bear this penalty.

The single-period newsvendor problem is one of the classical problems in inventory management (Silver *et al.*, 1998). This problem, with its intrinsically appealing optimal solution, has become an extensive research topic since 1950. The newsvendor problem is often observed in real life situations and used to assist management in decision making, especially in the fashion and sporting goods industries, both at manufacturing and retail levels (Gallego and Moon, 1993; Khouja, 1999). The problem can also help in order booking evaluation and managing capacity in service industries such as airlines and hospitality (Weatherford and Pfeifer, 1994).

Khouja (1999) summarized the contributions of published articles on the newsvendor problem and its

variants from 1988 to the end of 20th century. This study classified the extensions of the newsvendor problems into 11 categories, including (1) objectives, (2) supplier (wholesale) pricing policies and discount structures and (3) random yields. Qin *et al.* (2011) provided a review on studies analyzing the newsvendor problem in the past decades. They examined some specific extensions for the problem, including (1) how the market price, marketing effort and stocking quantity impact customer demand, (2) how supplier prices can serve as a coordination mechanism within the newsvendor supply chain framework and (3) how the retailer's risk profile moderates the newsvendor order quantity decision. Qin *et al.* (2011) in turn refer the reader to a review by Cachon (2003) on the newsvendor problem in supply chain settings with a primary focus on contractual mechanisms.

This research focuses on the study of newsvendor supply chain, which consists of one supplier, one retailer and customers. The study assumes that any excessive units at the end of the period are discarded or sold at a discount (salvage value). On the other hand, if the order quantity is less than the realized demand, there is a lost sales opportunity and shortage cost will occur. The objective of the retailer's problem is to find a market price and an order quantity that maximize his expected profit in a single period probabilistic demand framework, given a

certain wholesale price. In contrast, the supplier will determine the wholesale price which maximizes his own total profit minus the expected product quality risk. To the best of our knowledge, few studies on the newsvendor problems have considered product quality risk, which is highly related to the products' random yields. In our study, Bayesian analysis is applied to reduce product quality risk, since this method can utilize sampling information for computing the optimal quality loss. Additionally, the wholesale price in this study is a decision variable, differing from previous studies which define wholesale price as a given parameter (Mills, 1959; Karlin and Carr, 1962).

The remainder of this study is organized as follows: Section 2 describes the model; section 3 introduces the Bayesian sampling inspection plan; section 4 presents an illustrative example; section 5 provides analyses of model parameters; section 6 concludes the research and proposes future studies.

PROBLEM DESCRIPTION

Notations:

- τ : Market price; $\tau \in T$
- ω : Wholesale price per unit; $\omega \in \Omega$
- q : Order quantity by the retailer
- $q^*(\omega)$: Optimal order quantity for ω
- X : Base stochastic demand, $X \sim \text{Weibull}(\lambda, m)$
- λ : Scale parameter
- m : Shape parameter
- $X(\tau)$: Stochastic demand for τ $X(\tau) = X \mu(\tau)$
- $\mu(\tau)$: Demand influence multiplier of τ :

$$\mu(\tau) = a \cdot \tau^{-b}$$

- c : Production cost per unit
- B : Shortage cost per unit
- s : Salvage value per unit
- $SP|\eta$: Supplier profit based on parameter η
- $RP|\eta$: Retailer profit based on parameter η
- $QC(q)$: Quality cost for q product units
- $\tau^*(\omega)$: Optimal market price with respect to ω
- ω^* : Optimal wholesale price for supplier

Model description: The newsvendor supply chain consists of one supplier, one retailer and customers. Figure 1 depicts the business operation and decision process of the model. The supplier seeks an optimal ω that will maximize the value of the sales profit minus the quality cost. To achieve this goal, each time the supplier will offer the retailer a wholesale price $\omega \in \Omega$; in response,

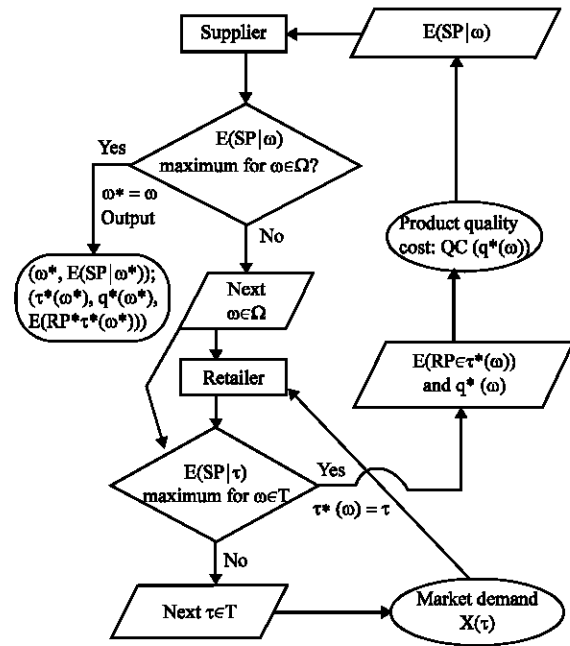


Fig. 1: Flow chart of the supply chain decision process

the retailer will strive to find the optimal market price $\tau^*(\omega)$ and order quantity $q^*(\omega)$ that maximizes his expected profit $E(RP|\tau^*(\omega))$.

The model assumes that the demand is sensitive to market price based on the following relation:

$$X(\tau) = \mu(\tau)X$$

where, $X \sim \text{Weibull}(\lambda, m)$. Thus, if market price is τ , $X(\tau)$ will be Weibull $(\mu(\tau)\lambda, m)$. The scale parameter will be changed from λ to $\mu(\tau)\lambda$ and the mean and variance will be multiplied by $\mu(\tau)$ and $\mu(\tau)^2$, respectively. Their cumulative distributions are as follows:

$$F(x) = \Pr(X \leq x) = 1 - e^{-(x/\lambda)^m} \quad (1)$$

$$\begin{aligned} \Pr(X(\tau) \leq x) &= \Pr(\mu(\tau)X \leq x) = \Pr\left(X \leq \frac{x}{\mu(\tau)}\right) \\ &= 1 - e^{-(x/\lambda\mu(\tau))^m} \end{aligned} \quad (2)$$

The retailer's expected profit for a market parameter setting (τ, ω, q) can be expressed as follows:

$$\begin{aligned} E(RP|\tau, \omega, q) &= \tau E(\text{Min}(q, X(\tau))) - \omega q \\ &\quad + s E(q - X(\tau))^+ - B E(q - X(\tau))^- \end{aligned} \quad (3)$$

where, $(q - X(\tau))^+ = \text{Max}(q - X(\tau), 0)$ and $(q - X(\tau))^- = \text{Max}(X(\tau) - q, 0)$.

To find the optimal values of (q, τ) that maximize $E(RP|\tau, \omega, q)$ for a given ω , we first differentiate the Eq. 3 with respect to q and then check the first and second order conditions. The result indicates that there exists a global optimal solution $q^*(\tau, \omega)$ (Eq. 4) which satisfies Eq. 3. By assuming the market price τ is a multiple of a certain number (e.g. dollars or half dollars), the optimal market price $\tau^*(\omega)$ can easily be computed using Eq. 5:

$$q^*(\tau, \omega) = F^{-1}\left(\frac{\tau - \omega + B}{\tau - s + B}\right) \alpha \cdot \tau^{-b} \quad (4)$$

$$\tau^*(\omega) = \operatorname{argmax}_{\tau \in T} (E(RP|\tau, \omega, q^*(\tau, \omega))) \quad (5)$$

For simplicity, we use notation $q^*(\omega)$ for $q^*(\tau^*(\omega), \omega)$. The optimal profit of the supplier for ω is given in (6):

$$E(SP|\omega) = (\omega - c) q^*(\omega) - QC(q^*(\omega)) \quad (6)$$

The supplier will select the wholesale price ω^* that maximize his total profit minus quality cost; that is:

$$\omega^* = \operatorname{argmax}_{\omega \in \Omega} \{E(SP|\omega)\} \quad (7)$$

REDUCING PRODUCT QUALITY RISK

In the proposed newsvendor model, the supplier will bear the product quality risk. In order to reduce expected total quality cost, a Bayesian rectifying inspection sampling model is applied to inspect product quality before product shipment. The model consists of a two-stage decision: (1) the first stage D_1 is to determine the optimal sample size for a lot of order quantity q , where the focus of optimality is on the expected total quality cost; (2) the second stage D_2 is to decide whether the remaining products of the lot should be subject to zero inspection or full inspection using the posterior distribution, which has been updated by the sampling information from the first stage. In the model, the cost considered involves inspection cost and product failure cost, the latter of which includes both the reproduction cost and transportation cost. The following introduces the notations used in the inspection model.

Notations:

- P : Probability that a unit is non-defective
- n : Sample size
- X_i : Value is 1 if the i -th product is defective and 0 if otherwise; $(X_1, \dots, X_q) | P = p \sim \text{i.i.d. Bernoulli}(p)$
- Y_n : Sum of defective units in n samples; $Y_n | P = p \sim \text{Binomial}(n, p)$; y is outcome

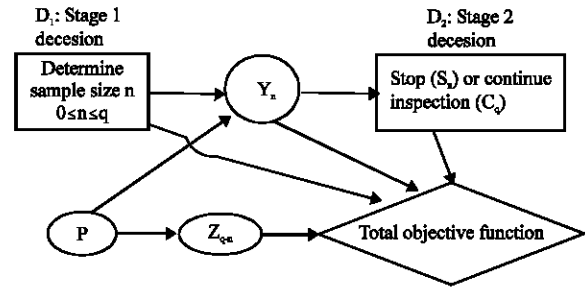


Fig. 2: Influence diagram for inspection model

- k_1 : Inspection cost per unit
- k_2 : Product failure cost per unit
- S_n : “Stop inspection” decision at the second stage
- C_q : “Full inspection” decision at the second stage
- $M(y)$: Extra inspections to obtain y non-defective units
- Z_{q-n} : Number of defective units in the remainder of the lot
- $M(Z_{q-n})$: Extra inspections to obtain Z_{q-n} non-defective units

Figure 2 shows the influence diagram (Shachter 1986) that represents the inspection sampling decision process with the objective of minimizing expected total quality cost. This model is a stochastic dynamic optimization problem. According to Clemen and Reilly, 2001), the total objective function can be derived as follows:

$$\operatorname{Min}_{0 \leq n \leq q} (n k_1 + \sum_{y=0}^n E(M(y)|n, y) \cdot k_1 \cdot \Pr(y|n) + \sum_{y=0}^n \operatorname{Min} (E(L(S_n, y)), E(L(C_q, y))) \cdot \Pr(y|n) \quad (8)$$

In Eq. 8, the first line represents the expected cost occurring in the first stage, where $n k_1$ is the sampling cost of size n , $E(M(y)|n, y)$ is the expected number of inspections to find y non-defective units given sampling information (n, y) and $\Pr(y|n)$ is the probability of y defective units in n samples. The second line shows the expected cost in the second stage. $E(L(S_n, y))$ is the expected cost given sampling outcome (n, y) when the second stage decision is “stop inspection”. Likewise, $E(L(C_q, y))$ is the expected cost for decision “complete inspection”. If P is Beta (α, β) , then pertinent calculations in (8) are as follows:

$$\begin{aligned} E(P|n, y) &= (\alpha + n - y) / (\alpha + \beta + n) \\ E(1/P|n, y) &= (\alpha + \beta + n - 1) / ((\alpha + n - y - 1)) \\ E(M(y)|n, y) &= y E(1/P|n, y) \end{aligned}$$

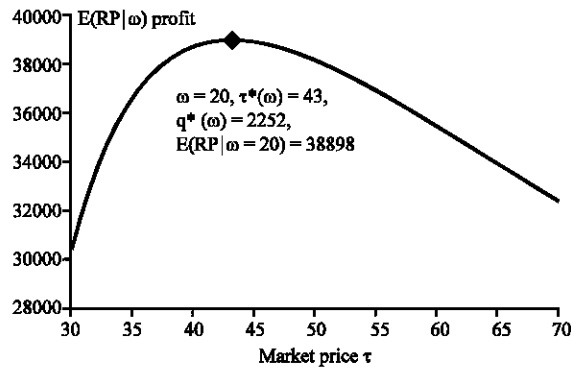


Fig. 3: Retailer profit curve for τ given $\omega = 20$

$$\Pr(y|n) = \binom{n}{y} [\Gamma(\alpha + \beta) / (\Gamma(\alpha) \Gamma(\beta))] \\ \frac{[\Gamma(\alpha + n - y) \Gamma(\beta + y)]}{\Gamma(\alpha + \beta + n)} \\ E(L(S_n, y)) = E(Z_{q-n}|n, y) k_2 + E(M(Z_{q-n})|n, y) k_1 \\ = (q-n) \cdot [(1-E(P|n, y)) k_2 + E((1-P)/P|n, y) k_1] \\ E(L(Cq, y)) = (q-n) k_1 + E(M(Z_{q-n})|n, y) k_1$$

where, Γ is a gamma function.

ILLUSTRATIVE EXAMPLE

The following example illustrates the proposed newsvendor model. Parameter values are set to as follows: $X \sim \text{Weibull}(\lambda = 3200, k = 5)$, demand influence multipliers are $(a = 2253.6, b = 2.15)$, which lead to the distribution of $X(\tau) \sim \text{Weibull}(3200, 2253.6, \tau^{-2.15}, 5)$, $c = 10$, $\omega \geq c+2$, $k_1 = 2$, $k_2 = 30$, $P \sim \text{Beta}(23, 2)$ with a mean of 0.92, $\tau \in (20, 60)$, $B = 8$ and $s = 4$.

Figure 3 presents the retailer's expected profit as a function of market price τ for a given $\omega = 20$. In this example, the optimal market price is $\tau^*(\omega = 20) = 43$, which results in an order quantity $q^*(\omega = 20) = 2252$ and an expected profit $E(RP|\tau^*(\omega)) = 38898$. Fig. 4 displays the numerical results of optimal wholesale price $\omega^* = 23$, with corresponding order quantity $q^*(\omega^*)$, optimal supplier's profit $E(SP|\omega^*)$ and optimal retailer's profit $E(RP|\tau^*(\omega^*))$ with respect to ω . The maximum supplier's profit occurs at $\omega^* = 23$, to which the retailer responds with market price $\tau^*(\omega^*) = 49$ and order quantity $q(\omega^*) = 1684$. The maximum supplier's profit is $E(SP|\omega^*) = 18885$ and the corresponding supplier's profit is $E(RP|\tau^*(\omega^*)) = 33074$. This indicates that the supplier should offer a wholesale price 23 to earn the maximum profit under these model parameter settings.

Table 1: Effects of market price parameter b

	Case 2	Case 0	Case 1
$\tau^*(\omega^*)$	57	49	40
ω^*	25	23	20
$q^*(\omega^*)$	2249	1684	1494
$E(RP \tau^*(\omega^*))$	54833	33074	22231
$E(SP \omega^*)$	29734	18885	12243
$QC(q^*(\omega^*))$	4001	3007	2697

Table 2: Optimal values corresponding to P for three cases

	Case 3	Case 0	Case 4
$\tau^*(\omega^*)$	47	49	51
ω^*	22	23	24
$q^*(\omega^*)$	1848	1684	1541
$E(RP \tau^*(\omega^*))$	34823	33074	31483
$E(SP \omega^*)$	19611	18885	18405
$QC(q^*(\omega^*))$	2565	3007	3169

ANALYSES OF MODEL PARAMETERS

Several experiments were performed to analyze the impact of three model parameters on supplier and retailer benefit: (1) Demand influence multiplier b, (2) Quality factors P and k_1 and (3) Incremental-unit quantity discount.

Demand influence multiplier: In this study, the market price influences the demand in multiplicative form: $\mu(\tau) = \alpha \tau^{-b}$. We focus on the impact of parameter b, as it directly impacts demand exponentially through market price. Three cases are discussed, where Case 0 has the same parameter settings as the case in the illustrative example. Cases 1 and 2 have the same parameter settings as Case 0, with the exception of parameter b:

- Case 0 (base case): $\alpha = 2253.6, b = 2.15$
- Case 1 (price-sensitive): $\alpha = 2253.6, b = 2.3$
- Case 2 (less price-sensitive): $\alpha = 2253.6, b = 2.0$

Table 1 displays the effects of market price parameter b. Case 1 is price-sensitive. The market price will significantly influence the market demand. As a consequence, the optimal wholesale price, market price, order quantity, supplier and retailer's profits are lower than the base case and less price-sensitive case. On the other hand, for the less price-sensitive case (Case 2), all these values are larger than those of Case 0 and Case 1. In Case 2, both supplier and retailer will receive higher profits.

Quality factors: (P, k_1): First, we analyze the effects of product quality P on the newsvendor supply chain. Three cases of P are compared, including the base case:

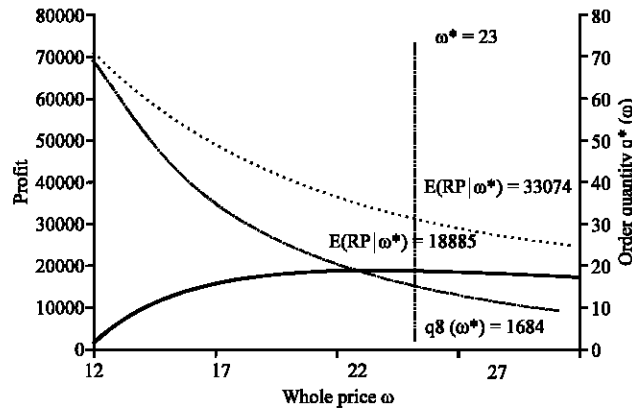


Fig. 4: Supplier and retailer profits w.r.t. market price τ

Table 3: Effects of inspection cost k_i for Case 0

	$\tau^*(\omega^*)$	ω^*	$q^*(\omega^*)$	$E(RP \tau^*(\omega^*))$	$E(SP \omega^*)$	$QC(q^*(\omega^*))$
k1 = 1	45	21	2036	36755	20325	2071
k1 = 2	49	23	1684	33074	18885	3007
k1 = 3	51	24	1541	31483	17975	3599

Table 4: Effects of quantity discount policy

	Case 0	Case 5	Case 6
$\tau^*(\omega^*)$	49	49	49
$\omega^*(\bar{\omega}^*)$	23	23 (22.88)	23 (22.88)
$q^*(\bar{\omega}^*)$	1684	-	-
$q^*(\bar{\omega})$	-	1743	1743
$E(RP q^*(\omega^*))$	33074	-	-
$E(RP q^*(\bar{\omega}^*))$	-	33237	33237
$E(SP q^*(\omega^*))$	18885	-	-
$E(SP q^*(\bar{\omega}^*))$	-	19558	19558
$QC(q^*(\omega^*))$	3007	-	-
$QC(q^*(\bar{\omega}^*))$	-	3101	3101

- Case 0: $P \sim \text{Beta}(23.0, 2.0)$, $E(P) = 0.92$
- Case 3: $P \sim \text{Beta}(23.0, 1.5)$, $E(P) = 0.94$
- Case 4: $P \sim \text{Beta}(22.5, 2.5)$, $E(P) = 0.90$

Table 2 presents the numerical results of the three cases. For Case 3 where product quality is the best, the order quantity is the highest, both supplier and retailer receive greatest profits and the quality cost is the smallest. The supplier is confident in his product quality and hence will offer a smaller wholesale price to promote the order quantity and market sales. The result is on the other way for Case 4 where product quality is the lowest.

Table 3 presents the effects of inspection cost k_i on the supply chain for base case. When inspection cost increases, the quality cost increases. In this case, the expected outgoing quality is likely lower than those of smaller inspection cost and thus will decrease the order quantity and market sales. In Table 3, we observe that high inspection cost will result in lower profits of supplier and retailer.

Incremental-unit quantity discount policy: Under this policy, the price quoted by the supplier is only applied to incremental units ordered by the retailer. That is, if the order quantity is $q_{i-1} < q < q_i$, the wholesale unit prices are ω_1 ($0, q_1$), ω_2 for (q_1, q_2) , ..., ω_i for $q - q_{i-1}$. In this case, the average cost per unit is as follows:

$$(\omega_1, q_1 + \omega_2 (q_2 - q_1) + \dots + \omega_i (q - q_{i-1})) / q \quad (9)$$

The expected value and variance of market demand $X(\tau)$ are as follows:

$$E(X(\tau)) = \mu(\tau) E(X) = a \tau^{-b} \lambda \Gamma(1 + \frac{1}{m}) \quad (10)$$

$$\text{Var}(X(\tau)) = (\mu(\tau))^2 \text{Var}(X) = \lambda^2 \Gamma(1 + \frac{2}{m}) - \Gamma^2(1 + \frac{1}{m}) \quad (11)$$

Let v and σ be the mean and standard deviation for the demand distribution of Case 0, where $v = 1565$, $\sigma = 358$. Table 4 exhibits the results of two step-function (incremental-unit) quantity discount policies: Case 5 and Case 6. The parameter settings of these two cases are the same as Case 0, with the exception of price discount:

- Case 0: $\omega_1 = \omega^* = 23$ for order quantity $q^*(\omega^*)$
- Case 5: Set $q_1 = v$, $q_2 = \max(0, q^*(\omega^*) - v)$; if $v < q^*(\omega^*) \leq v + 0.5\sigma$ and $q_3 = \max(0, q^*(\omega^*) - v - 0.5\sigma)$. In the study, $w_1 = 23$ for q_1 ; $\omega_2 = 0.95 \omega_1$ for q_2 ; $\omega_3 = 0.90 \omega_1$ for q_3
- Case 6: Similar to the rule of Case 5, with the exception that the gap 0.5σ is changed to 1.0σ

The average wholesale cost per unit $\bar{\omega}$ for the incremental-unit discount policy of Case 5 and 6 can be obtained by Eq. 9 using $q = q^*(\omega^*)$.

The optimal order quantity for Case 5 can be obtained by the following procedure:

- Calculate the optimal order quantity $q_8(\omega_1)$ (for ω_1 If $q^*(\omega_1) > v$, then calculate $q^*(\bar{\omega}_2)$ for:

$$\bar{\omega}_2 = \frac{\mu(\tau)\omega_2 + \sigma\omega_2}{\mu(\tau) + \sigma}$$

If $q^*(\bar{\omega}_2) \leq v + 0.5\sigma$ apply the binary search to find the optimal $q^*(\bar{\omega}^*)$ in the interval $(v, v + 0.5\sigma)$; otherwise, seek an upper bound q' such that the optimal order quantity $q^*(\bar{\omega}^*)$ is within the interval $(v + 0.5\sigma, q')$. Then, apply the binary search to find the optimal $q^*(\bar{\omega}^*)$ within $(v + 0.5\sigma, q')$.

The optimal order quantity $q^*(\bar{\omega}^*)$ for Case 6 can be calculated in the same manner. Clearly, the optimal $q^*(\bar{\omega}^*)$ will also shift right for both Case 5 and 6, when compared to the no-discount case (Case 0). Table 4 displays the numerical results of the incremental-unit quantity discount policy. In this example, the optimal order quantity of Case 0 compared to those of Case 5 and Case 6 is:

$$q^*(\omega^*) = 1684 > q^*_{\text{case 5}}(\bar{\omega}^*) = q^*_{\text{case 6}}(\bar{\omega}^*) = 1743$$

as a consequence, the expected profits of supplier and retailer are greater than those of Case 0. In this example, the optimal $q^*(\omega^*)$, $\tau^*(\omega^*)$, ω^* for no-discount are respectively 1684, 49, 23 and for the two incremental quantity discounts these three values are the same, 1743, 49 and 22.88, respectively. Furthermore, if the salvage value s is larger, or the shortage cost B is smaller, then it is likely that the optimal $q^*_{\text{case 5}}(\bar{\omega}^*) < q^*_{\text{case 6}}(\bar{\omega}^*)$ and $E(SR_{\text{case 5}}|\bar{\omega}^*) > E(SR_{\text{case 6}}|\bar{\omega}^*)$.

CONCLUSION

News vendor problems have been widely studied during the past decades. In this study, we study a market control power sharing problem which takes into account the product quality risk and market risk. The supplier owns the decision power of wholesale price and the retailer determines the market price and order quantity based on the wholesale price and his knowledge of the market price's effects on market demand. The supplier's optimal decision on the wholesale price is based on the sales profit deducting the product quality risk.

A Bayesian rectifying inspection sampling plan is developed to minimize the product quality risk for the supplier. In the proposed model, the supplier will offer a wholesale price to maximize his own benefit and the retailer will respond with optimal decisions on order quantity and market price with the given wholesale price. Several analyses of model parameters were discussed, including inspection cost, product yield, market price and quantity discount policy.

This research studies a newsvendor problem of a game-theoretic type, where the supplier possesses more market control power than the retailer. Incremental-unit quantity discount policy is applied to alleviate the profit conflict between the supplier and the retailer. However, there are many other schemes coordinating the supply chain operations equally well or even better than quantity discount policies. In particular, it is well known that centralized systems usually produce higher total supply chain than decentralized systems. In such situations, developing suitable revenue sharing schemes which take into account the risk attitudes of the supplier and retailer will be a good approach to generate a successful supply chain business operation. Other extensions of the study may include multiple suppliers or retailers.

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