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# Joint Decision of Pricing and Advertising Investment for Online Group-Buying Under Intersecting Demand Regimes

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**Abstract:** Online group-buying is a new kind of online sales mechanism which is dominant posted pricing under intersecting demand regimes. This manucript base on the assumption of the intersecting demand regimes to build a model to analyze the relationship between advertising investment and pricing and discuss the optimal joint decision-making of advertising investment and price. We derive the seller's optimal strategy under three decision-making paths and compare the seller's revenue with those that obtain under the more conventional posted-price mechanism. Our results show that when the demand for advertisement is unrelated the demand for prices, the seller's revenue from online group-buying mechanism is strictly outperform the revenue from posted pricing.

**Key words:** Online group-buying, pricing, advertising investment, intersecting demand regimes, joint decision

## INTRODUCTION

As a new mechanism of network selling, online group-buying is developed and popular rapidly in recent years. The China e-Business Research Cente (2013) that the transaction scale of China online group-buying market grew to 21.6 billions with an annual rate of ten times from 2009-2011. In 2012, the growth was slow down but still reached 34.88 billion. In this process, A large number of online group-buying seller spend a lot of advertising costs to attract consumers to participate in group buying for market share. According to advertising monitoring data of Meihua Information, it show that online advertising investment of group-buying seller was over 1 billion in 2011 but decrease of 40% in next year. Advertising investment volatility illustrates the blindness of online group-buying seller. Therefore, how to make joint decision of pricing and advertising investment to maximize income is one of most concern practical issues for online group-buying seller currently.

In the existing literature, most studies focus on the principle mechanism of online group-buying and the behavior of buyers and sellers in the mechanism. Studies (2006) have shown that the feature of online group-buying is that products' prices are determined by the accumulative purchase quantity. This mechanism requires the seller to set buy price sequence (descending order) and the accumulative purchase quantity of each price conditions (increasing order) in a period of time. When purchases reach or exceed one or more price qualifications, the transaction deals according to the lowest price in the qualifications. Chen *et al.* (2003, 2010) regard online group-buying as an auction mechanism and put forward a method for sellers determining the online group-buying price sequence and price qualifications through analyzing online group-buying mechanism and bidder behavior. According to his study, when bidder's arrival rate was uncertain, the seller's revenue from online group buying may be exceeds his revenue from posted pricing. Anand and Aron (2003) also proved the revenue to the seller from online group buying strictly dominates the revenue from posted pricing under intersecting demand regimes and propose a method for online group-buying pricing using optimization theory. Although there are differences in the application of principles and methods, these studies have similar conclusions that domination of online group-buying requires two conditions: one is intersecting demand regimes, the other one is on-demand production and products with economies of scale. This conclusion highlights why online group-buying mechanism can applicant successfully. However, there is no literature to explore the role of advertising in online group buying and advertising investment impact on pricing.

Many literatures have discussed the relationship between the advertising and pricing based on posted price. Ferguson (1982) have point out that advertising was correlated with the pricing. With Conjoint Analysis of price elasticity of demand and advertising elasticity of demand, the optimal strategy of pricing and advertising investment can be solved by optimal methods (Ren *et al.*, 2001). In addition to the advertising optimization

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decision-making research of single enterprise, the mainstream research focused on relationship of advertising and pricing under more complicated environment were discussed in recent years. Such as Yun, 2006 study the advertisement and price decision analysis of inside and entry enterprises and discuss how to handle advertising and price policy conjugate to maximize their profit. Liao *et al* (2005) further relaxes the rational hypothesis and discusses the joint decisions of advertising and pricing with limited rational hypothesis. In recent years, more literature focused on the discussion the advertising and pricing decisions in the supply chain by competition-and-cooperation game analysis (Zhong *et al*., 2004; Yin and Xu, 2011; Huang *et al*., 2011).

At present, advertising is an important means to sellers. To study the relationship between advertising and pricing in online group buying, this manuscript aims to discuss the following issues: Under online group-buying mechanism, what happens to the demand for advertising and how it affects pricing? How to conduct the joint optimal decision of pricing and advertising investment for seller? When the demand changes for advertising, whether online group-buying still outperform the posted-price mechanism?

## BASIC SETTINGS

Online group-buying pricing mechanism is a kind of quantity discounts mechanism and mainly characterized as: the seller set a sequence of prices and conditions; the sequence of prices constituted by a series of descending prices, called group-buying sequence prices, such as a sequence prices constituted by n group-buying prices $(p_1, p_2,\ldots, p_n)$ and $p_1>p_2>\ldots>p_n$; the conditions is made up of a sequence of increasing quantities $(q_1, q_2,\ldots, q_n)$ and $q_1<q_2<\ldots<q_n$ which each price corresponding to; the seller also should announce a closing date for the sale; Each customer can choose a price to order before the closing date; Eventually, the bargain price is made by lowest price which conditions are met and all winning customers which order price is higher than the transaction price get the product at the same price.

**Assumptions and parameters:** According to the conclusion from the existing literature (2003, 2010), Online group-buying can not outperform posted prices in the absence of demand uncertainty. so this manuscript set following assumptions:

- **Monopoly market:** In the monopoly market, the seller sells some kind of a product. The market has not other enterprise and there are no substitute products
- **Intersecting demand regimes:** There are two kinds of uncertain demand in the market. The two demand regimes cross and are irrelevant and independent. One regime may result in a higher quantity demanded for one range of prices while the other regime may dominate over another range of prices, respectively. We define these two types of demand as high demand and low demand. Figure 1 illustrates the case of intersecting demand curves
- **Product cost:** To simplify the analysis, assuming product cost is 0
- **On-demand production:** The seller's production should have sufficient flexibility, in accordance with actual demands and production capacity is enough to meet the market demand
- **Advertising elasticity of demand:** To stimulate the demand, the seller ads and pay the cost. Advertising demand is independent and not associated with the price demand. That is, whether the actual demand for the low demand or high-demand, advertising demand response consistent

Based on the above assumptions, we can build pricing model as shown in Table 1.

**Basic price model of intersecting demand regimes:** Base on the assumption of intersecting demand regimes, there are two uncertainty demand including low demand and high demand. The generic linear demand curve is given by $Q_L = b-np$ and $Q_H = a-mp$. Further, the two curves intersect only when $na/m<b<a$. The likelihood of two demands is $\delta$ and $(1-\delta)$, respectively. So, the seller's revenue functions under online group-buying and posted pricing are given by:

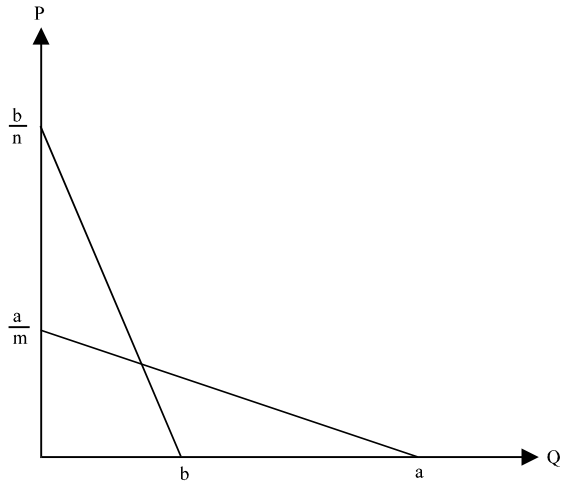


Fig. 1: Intersecting demand regimes

Table 1: Model parameters

| Parameter | Parameter meaning |
|---|---|
| b | Market capacity in low demand, b>0 |
| n | Price elasticity of demand in low demand, n>0 |
| a | Market capacity in high demand, a>0 |
| m | Price elasticity of demand in high demand, m>0 |
| $\gamma$ | Probability of low demand, probability of high demand is (1-$\gamma$), $\gamma \in (0, 1)$ |
| p | Price under posted pricing |
| $p_1$, $p_2$ | Online group-buying sequence prices, the $p_1$ is the price in low demand, $p_2$ is the price in high demand, $p_1>p_2>c$ |
| $q_1$, $q_2$ | Quantity condition corresponding to each price, $q_2=+\infty>q_1$ |
| $Q_L$, $Q_H$ | $Q_L$, $Q_H$ are the demand quantity in low demand and high demand, $Q_L>0$, $Q_H>0$ |
| $\theta$ | Advertising elasticity of demand |
| $c_a$ | Advertising cost |
| $\pi_g$ | Seller's expected revenue under online group-buying |
| $\pi_p$ | Seller's expected revenue under posted-price |
| D | Difference in revenues |

$$\pi_g = \gamma p_1(b - np_1) + (1-\gamma)p_2(a - mp_2)$$
$$\pi_p = \gamma p(b - np) + (1-\gamma)p(a - mp) \quad (1)$$

To maximize the seller's revenue within the feasible set of prices, we derive the seller's optimal strategy under online group-buying and posted pricing. Lemma 1 provides the optimal solution.

**Lemma 1:** The optimal (revenue-maximizing) prices and seller seller's revenue under online group buying are given by:

$$\pi_g(p_1^*, p_2^*) = \frac{\gamma b^2 m + (1-\gamma)a^2 n}{4mn} \text{ where } \begin{cases} p_1^* = \dfrac{b}{2n} & \text{if } q_1 \le b - \dfrac{na}{2m} \\ p_2^* = \dfrac{a}{2m} & \text{otherwise} \end{cases} \quad (2)$$

And optimal strategy under posted price are given by:

$$\pi_f(p^*) = \frac{(\gamma b + (1-\gamma)a)^2}{4(\gamma n + (1-\gamma)m)} \text{ where } \quad p^* = \frac{\gamma b + (1-\gamma)a}{2(\gamma n + (1-\gamma)m)} \quad (3)$$

Based on lemma 1, we know the difference in revenues between online group-buying and posted pricing as follows:

$$D = \pi_g(p_1^*, p_2^*) - \pi_f(p^*) = \frac{(1-\gamma)\gamma(mb - na)^2}{4nm(\gamma n + (1-\gamma)m)} > 0 \forall \frac{na}{m} < b < a \quad (4)$$

Thus it can be seen that the seller's revenues from online group-buying outweigh the revenues from posted pricing in the case of intersecting demand regimes.

## PRICING AND ADVERTISING INVESTMENT

When the seller put into advertising costs, Advertising investment will affect the demand. Advertising effect can be determined by the reaction function. According to Shugan (1985), we build the advertising response function as:

$$Q(c_a) = \theta c_a \text{^} (1/2) \quad (5)$$

So, with advertising investment condition, the seller's revenue functions under online group-buying and posted pricing are given by:

$$\pi_g(p_1, p_2, c_a) = \gamma p_1(b - np_1 + \theta\sqrt{c_a}) + (1-\gamma)p_2(a - mp_2 + \theta\sqrt{c_a}) - c_a \quad (6)$$

$$\pi_f(p, c_a) = \gamma p(b - np + \theta\sqrt{c_a}) + (1-\gamma)p(a - mp + \theta\sqrt{c_a}) - c_a \quad (7)$$

For joint decision of pricing and advertising, the seller can adopt three kinds of decision path (2001, 2006). Firstly, the seller can set price before advertising; secondly, the seller determine the advertising costs and then solve pricing issue; finally, the seller combined with different demand response to make decisions.

Here, discusses deferent seller's optimal strategy through three kinds of decision path and whether online group-buying still dominant.

**Case of pricing firstly:** Pricing firstly means when the seller make advertising costs decision, prices have been set. Now we assume that the seller sale good under online group-buying and the prices are set to (p10, p20). So the seller's problem is how to invest advertisement cost to maximize revenue. It is actually a relatively simple optimization decision problem and can be solved by first-order conditions. The seller's revenue function is as follows:

$$\pi_g^* = \frac{4\gamma p_{10}(b - np_{10}) + 4(1-\gamma)p_{20}(a - mp_{20}) + \theta^2(\gamma p_{10} + (1-\gamma)p_{20})^2}{4}$$
$$\text{where } \quad c_a^* = \frac{\theta^2(\gamma p_{10} + (1-\gamma)p_{20})^2}{4} \quad (8)$$

Accordingly, the seller set price to p0 under posted-pricing and revenue is:

$$\pi_f^* = \frac{\theta^2 {p_0}^2 + 4p_0(\gamma b + (1-\gamma)a - p_0(\gamma n + (1-\gamma)m))}{4} \text{ where } \quad c_a^* = \frac{\theta^2 {p_0}^2}{4} \quad (9)$$

We now can derive sequence of prices and conditions under online group-buying from Lemma 1 and advertising response function as proposition 1.

**Proposition 1:** In the case of price firstly:

- Prices (p10-20) under online group-buying and the price under poster-price are given by:

$$p_{10} = \frac{b}{2n} \text{ if } q_{10} \le b - \frac{na}{2m}, p_{20} = \frac{a}{2m} q_{20} = \infty \; p_0 = \frac{\gamma b + (1-\gamma)a}{2(\gamma n + (1-\gamma)m)}$$

- Difference in revenues between online group buying and posted pricing is given by:

$$D = \pi_g^* - \pi_f^* = \frac{1}{16m^2n^2(\gamma n + (1-\gamma)m)^2}[(1-\gamma)\gamma(mb - na)^2$$
$$4mn(\gamma n + (1-\gamma)m) + \theta^2(1-\gamma)\gamma(mb - na)(m - n)(\gamma mb((1-\gamma)$$
$$m + (1+\gamma)n) + (1-\gamma)na(\gamma n + (2-\gamma)m))] \Rightarrow D = \pi_g^* - \pi_f^* > 0 \text{ if } \frac{na}{m} < b < a$$

- Thus, the revenue from group buying strictly dominates the revenue from posted pricing

**Case of advertising investment firstly:** In the case of advertising investment firstly, the seller offers advertising cost $c_{a0}$ and then looking for the best price under different pricing mechanisms. Correspondingly, because the revenue function of online group-buying is binary, he can solve the optimal price by first-order conditions. Proposition 2 derives the sellers revenues under both posted-prices and group-buying.

**Proposition 2:** In the case of advertising investment firstly, the difference in revenues between online group buying and posted pricing is given by:

$$D = \pi_g^* - \pi_f^* = \frac{\gamma(1-\gamma)\left(bm - an + \sqrt{c_{a0}}\theta(m-n)\right)^2}{4mn(\gamma n + (1-\gamma)m)} > 0 \text{ if } \frac{na}{m} < b < a$$

Likewise, the revenue from group buying strictly dominates the revenue from posted pricing.

**Case of decision-making simultaneously:** In addition to the above two cases, the seller may adjust advertising investment and price set at the same time to seek the optimal revenues. In order to explore the seller's adjustments issues of prices and advertising investment under online group-buying, we assume that the seller set the prices ($p_{10}$, $p_{20}$) and advertising cost $c_{a0}$ in the initial; then seller will pick the adjustments range of prices ($\Delta p_1$, $\Delta p_2$) and advertising cost $\Delta c_a$. when the seller adjust, changes $\Delta\pi_g$ in revenues is expressed by:

$$\Delta\pi_g = \gamma((p_{10}+\Delta p_1)(b - n(p_{10}+\Delta p_1) + \theta\sqrt{c_{a0} + \Delta c_a})$$
$$-p_{10}(b - np_{10} + \theta\sqrt{c_{a0}})) + (1-\gamma)((p_{20}+\Delta p_2)(a - m(p_{20}+\Delta p_2)) \quad (10)$$
$$+\theta\sqrt{c_{a0} + \Delta c_a}) - p_{20}(a - mp_{20} + \theta\sqrt{c_{a0}})) - (c_{a0} + \Delta c_a))$$

This revenues function shows the relationship between prices and advertising cost. This led us to optimal result under online group-buying about joint decision of pricing and advertising investment.

**Lemma 2:** In the Case of decision-making simultaneously:

- Relationship between prices and advertising cost are given by:

$$\begin{cases} \Delta p_1^* = \dfrac{b - 2np_{10} + \theta\sqrt{\Delta c_a + c_{a0}}}{2n} = \dfrac{b + \theta\sqrt{\Delta c_a + c_{a0}}}{2n} - p_{10} \\ \Delta p_2^* = \dfrac{a - 2mp_{20} + \theta\sqrt{\Delta c_a + c_{a0}}}{2m} = \dfrac{a + \theta\sqrt{\Delta c_a + c_{a0}}}{2m} - p_{20} \\ \Delta c_a^* = \dfrac{\theta^2(\gamma(\Delta p_1 + p_{10}) + (1-\gamma)(\Delta p_2 + p_{20}))^2}{4} - c_{a0} \end{cases}$$

- When ($\Delta p_1 \in [0, \Delta p_1^*]$, $\Delta p_2 \in [0, \Delta p_2^*]$) and $\Delta c_a \in [0, \Delta c_a^*]$, the seller' revenues increase by ($\Delta p_1$, $\Delta p_2$, $\Delta c_a$)
- When ($\Delta p_1 = \Delta p_1^*$, $\Delta p_2 = \Delta p_2^*$) and $\Delta c_a = \Delta c_a^*$, the seller maximize his revenues and the results are:

$$\pi_g^* = \frac{1}{4\left(4mn - \theta^2(\gamma m + (1-\gamma)n)\right)^2}\left\{(-2ab\theta^2(1-\gamma)\gamma\left(12mn + \theta^2(\gamma m + (1-\gamma)n)\right) + \right.$$
$$(1-\gamma)a^2\left(16mn^2 + ka^4\gamma(\gamma m + (1-\gamma)n) - 4\theta^2n(5(1-\gamma)n + 2\gamma m)\right) +$$
$$\left.\gamma b^2\left(16m^2n + \theta^4(1-\gamma)(\gamma m + (1-\gamma)n) - 4\theta^2m(2(1-\gamma)n + 5\gamma m)\right)\right\}$$

$$\text{where} \begin{cases} p_1^* = \dfrac{4bm + \theta^2(1-\gamma)(a-b)}{8mn - \theta^2(\gamma m + (1-\gamma)n)}, q_1 \le b - \dfrac{an(4n + \theta^2(2-3\gamma)) + \theta^2rb(2m+n)}{8mn - \theta^2(\gamma m + (1-\gamma)n)} \\ p_2^* = \dfrac{4an + \theta^2r(b-a)}{8mn - \theta^2(\gamma m + (1-\gamma)n)}, \text{otherwise} \\ c_a^* = \left[\dfrac{\theta(\gamma bm - (1-\gamma)an)}{4(4mn - \theta^2(\gamma m + (1-\gamma)n))}\right]^2 \end{cases}$$

**Lemma 2:** Optimal solution exists when seller make joint decision. Now the next issue that we need to solve is whether there are similar results under posted-pricing. With second order conditions, we get the following result:

$$\pi_g^* = \frac{(\gamma b + (1-\gamma)a)^2}{4(\gamma n + (1-\gamma)m) - \theta^2}$$
$$\text{where } p^* = \frac{2(\gamma b + (1-\gamma)a)}{4(\gamma n + (1-\gamma)m) - \theta^2}, c_a^* = \left[\frac{\theta(\gamma b + (1-\gamma)a)}{4(\gamma n + (1-\gamma)m) - \theta^2}\right]^2$$

The difference of seller's revenues can be stated by Proposition 3.

**Proposition 3:** In the case of decision-making simultaneously, the difference in revenues between online group buying and posted pricing is given by:

$$D = \pi_g^* - \pi_f^* = \frac{1}{4\left(4mn - \theta^2(\gamma m + (1-\gamma)n)\right)\left(4(\gamma n + (1-\gamma)m) - \theta^2\right)^2}$$
$$\{(1-\gamma)a^2\left(-16\theta^4n + \left(\theta^6 - 4\theta^4(m-6n) + 16\theta^2(2m-3n)n - 64mn^2\right)r\right) +$$
$$(1-\gamma)a^2\left(4\left(\theta^2 - 4n\right)^2(m-n)\gamma^2\right) - 2ab(1-\gamma)\gamma\left(\theta^6 - 64mn(\gamma n + (1-\gamma)m)\right) -$$
$$2ab(1-\gamma)\left(\gamma - 16\theta^2\left(2mn + m^2(1-\gamma) - \gamma n^2\right) + 4\theta^4(-n(-3+r) + m(2+\gamma))\right) +$$
$$+\gamma b^2\left(\theta^6(1-\gamma) - 16\theta^2m(1-\gamma)(m + 2m\gamma - 2n\gamma) + 64m^2(1-\gamma)(m(1-\gamma) - \gamma n)\right) +$$
$$\gamma b^2\left(4\theta^4(-n(1-\gamma)\gamma + m(-1 + \gamma(4+\gamma)))\right)\}$$

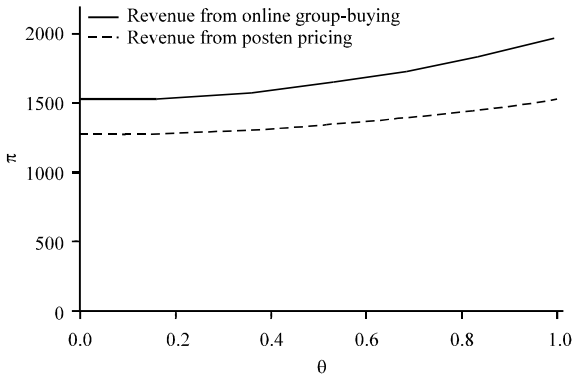


Fig. 2: Difference of revenue in the case of pricing firstly

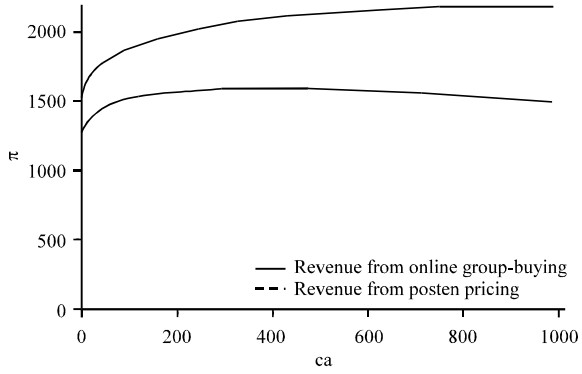


Fig. 3: Difference of revenue in the case of advertising investment firstly

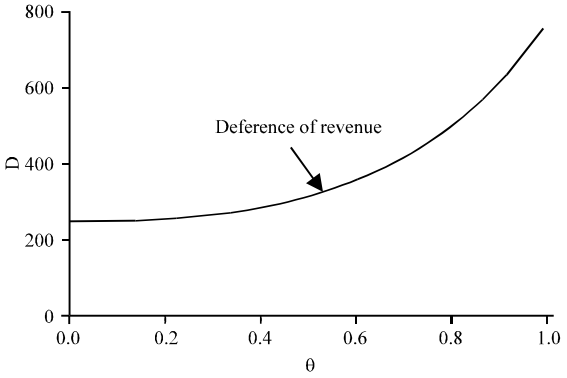


Fig. 4: Difference of revenue in the case of decision-making simultaneously

And the revenue from group buying outperforms the revenue from posted pricing.

It is difficult to visually compare the difference of revenues from proposition 3. We will prove the dominance of online group-buying by numerical simulation in next section.

## NUMERICAL SIMULATION

Here, we build a simulation example to illustrate the relationship between prices and advertising investment. We also verify the propositions by graphic results of this example.

Consider there are two demands uncertain and each can occur with equal likelihood ($\gamma$). The demand functions of price are expressed as $Q_L = 60-0.5p$ and $Q_H = 100-2p$. It is obvious that the demand functions meet the conditions of intersecting demand regimes ($na/m<b<a$, $m<n$).

Now we do numerical simulation calculation on the basis of the seller's decision path to verify the propositions. Figure 2 illustrate the relationship between the seller's revenue and advertising investment in the case of pricing firstly.

Simulation results in Fig. 2 clearly illustrate there is a positive correlation between revenue and advertising elasticity of demand and the revenue from online group-buying is greater. Given $\theta = 1$, Fig. 3 shows the same dominance of online group-buying in the case of advertising investment firstly.

Consider the case of decision-making simultaneously, the seller in this case must determine the multiple variables including the prices and advertising costs. To verify the proposition 3 visually and succinctly, we select the deference of revenues under deferent pricing mechanism and advertising elasticity of demand to explore the comparison of revenues. Figure 4 shows that for any level of $\theta$, the revenues difference is always greater than zero. This also demonstrates that the seller can obtain more revenue under online group buying.

## CONCLUSION AND EXTANSIONS

When the seller adopts the advertising to promote sales under online group-buying, he must consider the joint decision of pricing and advertising investment. This study explores the relationship between price and advertising on the basis of previous research. We briefly summarize our findings in each context below.

The basic model in section 2 defines the research assumption and parameters. Expanding on the existing model, we obtain optimal pricing strategy under online group-buying. And then setting three cases of decision path, we draw the following conclusions though mathematical derivation (section 3) and numerical simulation (section 4): Under online group-buying, prices raised when advertising costs increase. Then the seller

should conduct the joint optimal decision of pricing and advertising investment by optimization method. When the demand changes for advertising, the seller can adjust the prices to maximize his revenue and the revenue from online group-buying still outperform the posted-pricing.

Our discussion made no mention of the seller's decision when advertising demand is not independent or associated with the price demand. These issues should be addressed in future research. Another important area for future research is the joint decision of pricing and other promotions such as purchase rebate, customer referral, etc.

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