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Modeling and Validation of a Dynamic Strongly Coupling Numerical Method for Giant Magnetostrictive Actuator

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Abstract: In order to analyze the non-linear dynamic characteristic in electromagnetic (EM) field and mechanical field exactly, it is key to establish a proper multi-physics coupling model for giant magnetostrictive actuator (GMA). Considering the eddy current effect, this paper presents a dynamic EM-mechanical strongly coupled model for GMA based on the energy variational principle. A prototype is built so that the axial displacements under different frequencies are measured. The results show that there is only a 5% relative error between the calculated results and the experimental ones, which indicates that the proposed EM-mechanical strongly coupling numerical model is valid.

Key words: GMA, magneto-mechanical strongly coupling, non-linear dynamic model

INTRODUCTION

There are great varieties of giant magnetostrictive material (GMM) with different geometric shapes, preparation technologies and application fields, all of which cause significant individual differences in sample products (Shimizu *et al.*, 2012; Saito and Nakagawa, 2008). Thus, the key of design is to accurately analyze and forecast the performance of EM, mechanical field and temperature field.

For GMM or GMA model, there is few work about numerical in which the problem of strongly coupling is concerned. In (Dean *et al.*, 2006) and (Watts *et al.*, 1997), a cantilever beam made by GMM is analyzed by J. Dean and Watts, based on finite element model, where the magnetic strain is assumed as an internal component of mechanical and thermal strain. However, it cannot truly represent the distribution of magnetostrictive strain when it is calculated by a constant coefficient. A 2D model of magnetostrictive film is proposed by (Benbouzid *et al.*, 1995) through weakly coupling method, in which magnetic field or stress field is solved separately. The deformation of finite element mesh can also cause solving error. The induced voltage is computed in time domain in (Yan *et al.*, 2010). However, the ignorance of non-diagonal elements in magneto-mechanical coupling term makes the solution precision decrease.

In this paper, based on the principle of minimum potential energy, a detailed dynamic numerical model of GMA which reflects the properties of strongly coupling of EM-mechanical field has been presented. In order to examine the validity of proposed model, axial displacement as a function of excitation current under different pressures has been computed and compared with the measured ones. Then its gain characteristic in a range of frequency from 1Hz to 1000Hz is used in comparative analysis, from which the accuracy of this model is verified.

DYNAMIC EM-MECHANICAL STRONGLY COUPLING MODEL FOR GMA

Coupling Relationship among Electric, Magnetic and Mechanical Field in GMA: GMA is a kind of device to convert electrical energy into mechanical energy with the participation of magnetic field. It has been applied into many fields such as micro-motor, location device and adaptive optical system, etc.

In general, the working principle and energy conversion relationship can be represented by Fig. 1 where Ω_1 and Ω_2 respectively indicate field domains of GMM rod and the rest of total GMA. Γ_1 and Γ_2 indicate field boundaries, i.e. the contours of the domains. When the excitation current passes through the coil, GMA starts to output mechanical force and displacement on the ejector pin following the frequency of EM field.

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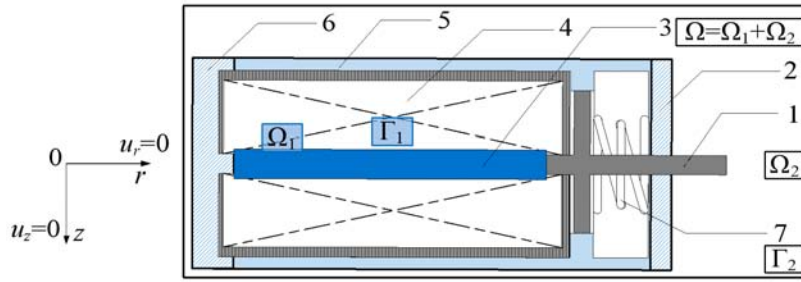


Fig. 1: Schematic diagram of a linear GMA

- 1: Output ejector pin, 2: Top cover, 3: GMM, 4: Excitation and bias coil, 5: Outer wall of sleeve,
6: Bottom cover, 7: Spring

Dynamic EM-mechanical Strongly-coupling Model: It is necessary to determine energy function of GMA when it is solved by FEM based on the principle of minimum potential energy. The total energy of the actuator consists of EM energy, potential energy of magnetic field boundary, elastic energy, work of external forces, mutual magneto- elastic energy, energy in the inductance coil, eddy current loss and hysteresis loss.

By neglecting distortions of the rest parts of GMA, the magnetostrictive rod is considered as computed domain (Ω_1) of mechanical problem whose surface is represented by Γ_1 . The mechanical field boundary condition is shown in Fig. 1 in which z -axis is chosen as direction of distortion; u_r and u_z are the displacement components r and z directions, respectively. According to Newton's second law, the dynamic equation of magnetostrictive rod is expressed as

$$\rho \frac{\partial^2 u}{\partial t^2} - \nabla \cdot \sigma = f_{\Omega_1} \quad (1)$$

where ρ and f_{Ω_1} indicate density and volume force of the rod. So the potential energy function of mechanical field I_1 , which is shown as (2), is derived by volume integration and Green's formula. The four parts in the right side of (2) indicate elastic energy, inertial energy and the work of force in Ω_1 and on Γ_1 , respectively.

$$I_1 = \int_{\Omega_1} \left(\int_0^{\epsilon} \sigma d\epsilon \right) d\Omega_1 + \int_{\Omega_1} \left(\int_0^u \rho \frac{\partial^2 u}{\partial t^2} du \right) d\Omega_1 - \int_{\Omega_1} \left(\int_0^u f_{\Omega_1} du \right) d\Omega_1 - \int_{\Gamma_1} \left(\int_0^u n \cdot \sigma du \right) d\Gamma_1 \quad (2)$$

The relationship between stress and strain tensor is written as (3) where α , D and K^H denote respectively Poisson's ratio, elastic module matrix and Young modulus under certain magnetic strength. For a 2D problem, $\sigma_{r,z}$ and $\epsilon_{r,z}$ indicate components of stress and strain along r and z axis.

$$\sigma = \begin{bmatrix} \sigma_r \\ \sigma_z \end{bmatrix} = \frac{K^H}{(1+\alpha)(1-2\alpha)} \begin{bmatrix} 1-\alpha & \alpha \\ \alpha & 1-\alpha \end{bmatrix} \cdot \begin{bmatrix} \epsilon_r \\ \epsilon_z \end{bmatrix} = [D] \cdot \nabla u \quad (3)$$

As for constraint equations in EM field, they can be written as (4) based on Maxwell equations when the rod is excited by a time-varying current and displacement current is ignored.

$$\begin{cases} \nabla \times H = J_s + J_e \\ \nabla \times E = -\frac{\partial B}{\partial t} \\ \nabla \cdot B = 0 \end{cases} \quad (4)$$

Here, J_s and J_e indicate current density of source current and eddy current. Because the divergence of B equals zero, a complex vector potential A is introduced, i.e. $B = \nabla \times A$, then (4) can be written as

$$\nabla \times \left(E + \frac{\partial A}{\partial t} \right) = 0 \quad (5)$$

Therefore, for a 2D axisymmetric problem, potential energy function I_2 of EM field is shown as

$$I_2 = \int_{\Omega} \left[\int_0^{A'_z} (\mu^{-1} \nabla \cdot \nabla A'_z) dA'_z \right] d\Omega + \int_{\Omega} \left[\int_0^{A'_z} \gamma \left(\frac{\partial A'_z}{\partial t} \right) dA'_z \right] d\Omega - \int_{\Omega} \left[\int_0^{A'_z} J_s dA'_z \right] d\Omega \quad (6)$$

where A'_z indicates component of A' on z -axis while Γ_1 satisfies first kind boundary condition. Items in the right side of (6) from left to right indicate the energy of magnetic field, EM induction and source current respectively.

In magnetostrictive phenomena, GMA is under simultaneous action of magnetic field and stress field. So, the

characteristic of EM-mechanical coupling between fields is described by first-class piezomagnetic equation as (Clark, 1980)

$$\begin{cases} \varepsilon = (K^H)^{-1} \sigma + d_{33}H \\ B = d_{33}\sigma + \mu^\sigma H \end{cases} \quad (7)$$

where μ^σ indicates permeability under certain pressure and d_{33} indicates piezomagnetic coefficient. Since overall coefficient matrix consists of A'_z and u , σ is expressed by ε and B as

$$\sigma = \frac{\mu^\sigma K^H}{\mu^\sigma - d_{33}^2 K^H} \varepsilon + \frac{d_{33} K^H}{\mu^\sigma - d_{33}^2 K^H} B \quad (8)$$

Thus, the potential energy function of EM-mechanical coupling ΔI can be derived by (8) as

$$\Delta I = \int_{\Omega_2} \left(\frac{d_{33} K^H}{\mu^\sigma - d_{33}^2 K^H} \nabla u \cdot B^T \right) d\Omega_2 \quad (9)$$

Here the item $\nabla u \cdot B^T$ stands for the direct coupling of the two fields, and variables of the fields are solved simultaneously. Note that when GMA is sinusoidally excited, it is convenient to represent the variables in a complex vector. This means that each and any of the EM and mechanical variables are represented by a phasor quantity j . Consequently, the overall potential energy function I can be expressed with angular frequency ω as

$$\begin{aligned} I = I_1 + I_2 + \Delta I = & \int_{\Omega_1} \left[\frac{\mu^\sigma K^H}{\mu^\sigma - d_{33}^2 K^H} [D] \cdot (\nabla u) \cdot (\nabla u)^T \right] d\Omega_1 \\ & - \int_{\Omega} (J_s \cdot A'_z) d\Omega + \int_{\Omega_1} \left(-\frac{1}{2} \rho \omega^2 \mu^\sigma \cdot (\mu^\sigma)^T \right) d\Omega_1 \\ & + \int_{\Omega} \left\{ \frac{1}{2} (\mu^\sigma)^{-1} \nabla \cdot \nabla [A'_z \cdot (A'_z)^T] \right\} d\Omega + \int_{\Omega} \left[\frac{1}{2} j \omega \gamma A'_z \cdot (A'_z)^T \right] d\Omega \\ & + \int_{\Omega_1} \left[\frac{d_{33} K^H}{\mu^\sigma - d_{33}^2 K^H} \nabla u \cdot B^T \right] d\Omega_1 \end{aligned} \quad (10)$$

It contains energy involved in EM and mechanical field of GMA. Then overall coefficient matrix is obtained by its partial derivatives of each discrete element.

Analysis of Simulation and Experiments: By experiment, a linear GMA prototype is built to examine the correctness of this strongly coupling model. The GMM rod used in this subject is $Tb_{0.3}Dy_{0.7}Fe_{1.92}$ with high energy density and electro-mechanical coupling coefficient. As for numerical solution, the solving domain of GMA is

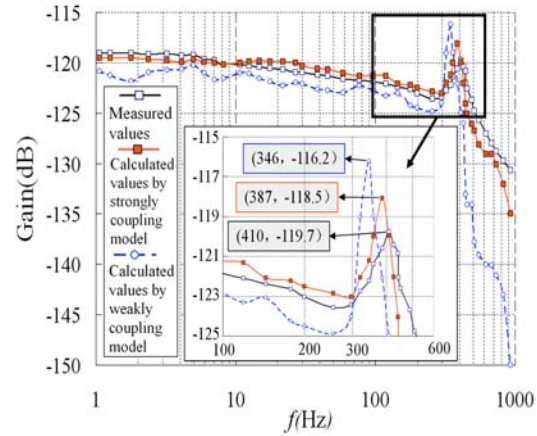


Fig. 2: Geometric structure diagram and discrete mesh for GMA. (a) Structure diagram of GMA (b) Discrete mesh for GMA

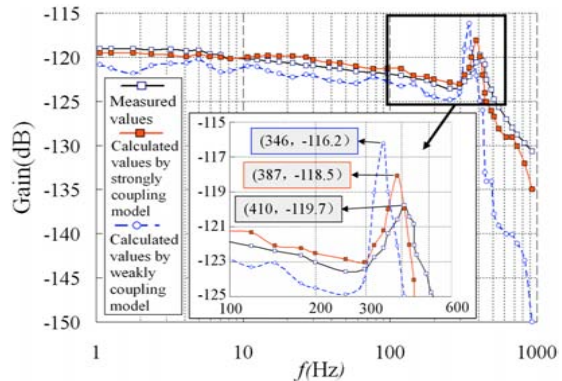


Fig. 3: The gain curves of axial displacements from 1Hz to 1000Hz while excitation keeps current 0.3A

converted to a 2D axisymmetric region as shown in Fig. 2(a) and is discretized by first-order triangular element as shown in Fig. 2(b) where the elements on GMM rod are densified to acquire higher precision. Based on previous analysis, the global stiffness matrix is assembled and solved by using large sparse matrix solution technique and Newton-Raphson iteration method. Here, iterative error is set to 1×10^{-4} . When GMA is excited by a varying field with frequency range from 1Hz to 1000Hz, the comparison of the gain curves of axial displacements achieved by three methods is shown in Fig. 3. It can be seen that although numerical results by weakly coupling and strongly coupling reflect the frequency characteristics, there is a greater error in results from the former, especially when frequency is larger than 100Hz. And resonant frequency of GMA is predicted by strongly

coupling method at 387Hz with a calculation error about 5.6% which is less than that by weakly coupling, i.e., 346 Hz, 15.6%.

CONCLUSION

In order to analyze the dynamic performance of GMA, a dynamic EM-mechanical strongly coupled model proposed in this paper. Based on principle of minimum potential energy, the overall potential energy function of GMA is derived by representing the relationship of EM and mechanical field with first-class piezomagnetic equation. For the aspect of experiment, calculated results of displacement on *z*-axis are compared with measured ones on a linear GMA, from which it have been found that they are qualitatively in agreement with an error about 5%. Characteristics of GMA excited by alternating current can be predicted clearly which demonstrate the accuracy and effectiveness of this model.

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