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Noise Reduction of Rectangular Cavity with Sandwich Polycarbonate Window Pane for Automobile

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Abstract: In the automobile industry, the use of Polycarbonate (PC) window pane is well developed. However, the sound insulation performance of automobile's Polycarbonate composite pane is poor than traditional glass pane because of its high rate of stiffness to low weight. There is need to design polycarbonate window pane with superior acoustical insulating property for automobile consumer comfort. In this study, the innovative super thin sandwich structure (less than 10 mm) was developed using viscoelastic core layers and the outer layer of the panel was fabricated from Polycarbonate composite panel. This paper introduces the vibro-acoustic models to predict the response of the pane and to evaluate the effect of damping treatment on both structural vibration and noise transmission in the fluid domain. One of the particularities of the proposed model lies in the only storage of the panel impedance matrix reducing the numerical efforts of previous studies. The numerical results reveals that noise attenuation can be realized by appropriate selection of structural parameters such as the damping of structural and cavity, the thickness of the pane and density of viscoelastic core material.

Keywords: Noise reduction, polycarbonate window pane, fluid-structure interaction, sandwich structure, automobile

INTRODUCTION

It is a tendency that Polycarbonate (PC) materials replace the glass as the automobile window pane due to its high rigidity, lightweight, high freedom to processing. Control of interior noise levels in automobile is an important objective for providing passenger greater comfort. The transmission phenomenon is a typical fluid-structure interaction problem. The coupled structural vibration and acoustic field form a so-called vibro-acoustic system.

In the past, a comprehensive modal-based theoretical framework were presented by Lyon (1963), Dowell and Voss (1963), Pretlove (1965). Since then, Guy and Bhattacharya (1973), Narayanan and Shanbhag (1982) Pan *et al.* (1990) has been directed at improving analysis model of sound transmission through panels into cavities. They provide solutions for coupled responses in terms of the modal characteristics of the uncoupled structural and acoustic systems (Kim and Brennan, 1999). Considers the same problem but uses the impedance mobility approach with a simple conceptual structural-acoustic coupled

system. Dhandole and Modak (2010) used the Finite Element Method (FEM) predicting the interior noise in the low frequency range. Deckers *et al.* (2011) analyzes of same problem using a Wave Based Method (WBM) in the mid-frequency range. Nowadays, the combined usage of Boundary Element Methods (BEM) and FEM is becoming more and more popular for vibro-acoustic analysis. (Jeyaraj, 2010) adopts FEM for the structural dynamic simulation and BEM for the acoustic field assessment.

Based on the facts that (Narayanan and Shanbhag, 1982), the sandwich panel is more effective for noise attenuation than the single plate and a damping material can facilitate the design, a plausible method is to design optimal structure meeting both acoustical and structural demands. This paper presented a simple description on the vibro-acoustic modeling of the simply supported sandwich PC pane with close enclosures. In section 4, numerical analysis is performed to investigate the effect of proposed method on noise attenuations. Numerical results are elaborated to demonstrate the validity of the design, leading to some useful conclusions drawn in the final section.

ACOUSTIC FORMULATION

The model under investigation is a rectangular cavity filled with air consists of five rigid walls and a flexible vibrating sandwich pane on the top surface, as shown in Fig. 1. The sandwich pane is also simply-supported on all edges and an external sound field P_{ext} excites the top pane, which is a plane wave, in turn, radiates sound power into the cavity.

The acoustic theory starts from the well-known Helmholtz equation, which describes the acoustic pressure inside the cavity:

$$(\nabla^2 + k^2)P_c = 0 \tag{1}$$

The boundary conditions to be satisfied are:

$$\frac{\partial P_c}{\partial n} = -\rho \frac{\partial^2 w}{\partial t^2} \text{ on AF, } w = 0 \text{ on AR} \tag{2}$$

where ρ and k are the fluid density and the wavenumber of sound, respectively and w is the displacement of the flexible pane in the normal direction n , A_F and A_R , indicate the flexible and rigid surfaces of the cavity and t represents time.

The sound pressure $P_c(r, \omega)$ at location r in the cavity can be described by the sound-pressure modal amplitude matrix $[P_A]$ and the cavity-mode shape matrix $[\Phi_N]$ as follows:

$$P_c(r, \omega) = [\Phi_N]^T [P_N] \tag{3}$$

The modal amplitude matrix is represented by:

$$[P_N] = [Z_A][V_M] \tag{4}$$

where, $[Z_A]$ is the internal radiation impedance matrix of the panel and can be represented by:

$$[Z_A] = \rho c \begin{bmatrix} B_{1,1}/\zeta_N^A & \dots & B_{1,M}/\zeta_N^A \\ \vdots & & \vdots \\ B_{N,1}/\zeta_N^A & \dots & B_{N,M}/\zeta_N^A \end{bmatrix} \tag{5}$$

where, $B_{i,j}$ is the modal coupling coefficient between the i th panel mode and j th cavity mode defined as:

$$B_{i,j} = \frac{1}{A_f} \left(\int_{A_f} \psi_i(\sigma) \phi_j(\sigma) d\sigma \right) \tag{6}$$

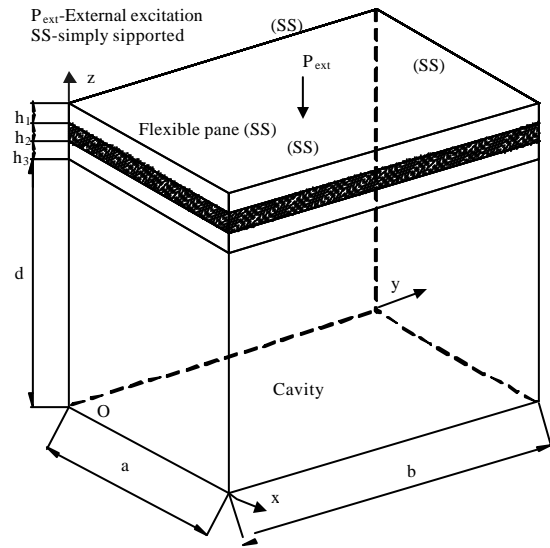


Fig. 1: Schematic diagram of a rectangular cavity with a vibrating sandwich pane

It should be mentioned that in the following discussions, the mode shape functions $\Psi_i(\sigma)$ and $\Phi_j(\sigma)$ for the uncoupled panel mode i and cavity mode j are elements of $[\Psi_M]$ and $[\Phi_N]$, respectively.

In Eq. 5, is defined as:

$$\zeta_N^A = j(M_N^A / \rho k A_F)(k_{\omega N}^2 - j\eta_{\omega N} k_{\omega N} k - k^2) \tag{7}$$

where, the modal masses defined as:

$$M_N^A = \rho \int_V \phi_N^2(r) dv \tag{8}$$

$$k_{\omega N} = \omega_{\omega N} / c \tag{9}$$

For a rigid-walled rectangular enclosure, the acoustic mode shape function is given by Bullmore *et al.* (1987):

$$\phi_j(r) = \cos(l\pi x / a) \cos(m\pi x / b) \cos(n\pi x / d) \tag{10}$$

where, is the integer indices of the j th cavity mode.

The corresponding acoustic natural frequency is given by:

$$\omega_{\omega N}^2 = c^2 [(l\pi/a)^2 + (m\pi/b)^2 + (n\pi/d)^2] \tag{11}$$

RESPONSE OF SANDWICH PANEL

The equations of motion of a uniform three layer sandwich plate with a damping layer were derived by Mead (1972) under the following assumptions:

- There is no significant direct strain in the core perpendicular to the plane of the plate. Both face-plates undergo identical transverse displacements
- There is no significant shear strain in either face-plate in planes perpendicular to the plane of the plate
- Direct stresses in the soft core are much smaller than the direct stresses in the face-plates and so may be ignored

Assuming a harmonic vibration, under forced damped loading form $P = i\eta\mu\partial^2 w / \partial t^2$, After eliminating the in-plane displacements U, V, in terms of W, the equation of motion for the transverse displacement can be reduced to the form Narayanan and Shanbhag (1982):

$$\nabla^4 w - g'(1+Y)\nabla^2 w - \Omega^2 \nabla^2 w + g'\Omega^2 w = 0 \quad (12)$$

Where:

$$g' = g'(1 + j\beta) / a^2 = 2G(1 - \nu^2)(1 + j\beta) / Eh_1 h_2$$

$$\nabla^4 = \partial^4 / \partial x^4 + 3(\partial^4 / \partial x^2 \partial y^2) + 3(\partial^4 / \partial x^2 \partial y^4) + \partial^4 / \partial y^4$$

$$\nabla^4 = \partial^4 / \partial x^4 + 2(\partial^4 / \partial x^2 \partial y^2) + \partial^4 / \partial y^4$$

$$\Omega^2 = \omega^2(1 + i\eta)\mu / D$$

where, E is young's modulus of the face plate, ν is Poisson's ratio of the face PC pane, G is storage shear module of the damping core, β is loss factor of core, g' is shear parameter of the core, μ is mass per unit area of the entire sandwich, h_2 is thicknesses of constrained damping layer, $Y = 3(1 + h_2/h_1)^2$ is geometric parameter:

$$D = \frac{Eh_1^3}{6(1 - \nu^2)}$$

is total flexural rigidity of both face plate

It should be assumed that the pane edges for the simply supported case, the forced damped normal mode of panel can be obtained by:

$$\psi_1(\sigma) = \sin(u\pi x / a) \sin(v\pi y / b) \quad (13)$$

where, (u, v) is the integer indices of the Ith panel mode.

According to the method of modal analysis, the panel velocity at location σ on the panel can be expanded as:

$$W(\sigma) = [\Psi_M]^T [W_M] \quad (14)$$

where, $[W_M]$ is the modal velocity amplitude matrix.

Substituting Eq. 13 and 14 into Eq. 12 yields a characteristic polynomial equation in λ :

$$\lambda^3 + g'(1+Y)\lambda^2 - \Omega^2[\lambda^2 + g'] = 0 \quad (15)$$

Here $\lambda = (u\pi/a)^2 + (v\pi/b)^2$

For simply supported conditions, the complex natural frequency can be obtained by:

$$\omega_{pM}(1 + j\eta) = \frac{D}{\mu} \left[\frac{\lambda^3 + g'(1+Y)\lambda^2}{\lambda + g'} \right] \quad (16)$$

COUPLING RESPONSE OF STRUCTURAL BACOUSTIC

The fluid-structure interaction generates the coupling between the dynamic equations of fluid and panels. Consider the equation of forced motion of the Mth mode of vibration of panel:

$$M_M^p [\ddot{W} + \omega_{pM}^2(1 + j\eta_{pM})W] = P_M^{ext} \quad (17)$$

In this equation, M_M^p , P_M^{ext} , W are, respectively, the generalized mass, the generalized force and the generalized displacement of the panel mode defined by (u, v).

where, the generalized mass M_M^p is defined as:

$$M_M^p = \mu \int_{A_F} \psi_M^2(\sigma) d\sigma \quad (18)$$

If the incident pressures are harmonic functions of time then the external part of the generalized force is given by:

$$P_M^{ext} = - \int_{A_F} P_{ext} \psi_M(\sigma) d\sigma \quad (19)$$

The generalized force represented by matrix $[P_M^{ext}]$ and the modal amplitude matrices for the panel velocity $[V_M]$ is represented by:

$$[V_M] = [Z_p]^{-1} [P_M^{ext}] \quad (20)$$

In this equation, $[Z_p]$ is the panel input modal impedance matrix, which is defined by:

$$[Z_p] = [Z_{pp}] + [Z_{pa}] \quad (21)$$

where, $[Z_{pp}]$ is the panel modal input impedance without influence of the backed cavity (Pan *et al.*, 1990):

$$[Z_{pp}] = \rho c \begin{bmatrix} \zeta_1^p & & & \\ & \zeta_2^p & & \\ & & \ddots & \\ & & & \zeta_M^p \end{bmatrix} \quad (22)$$

with $\zeta_M^p = j(M_M^p / \rho k A_p) [k_{pm}^2 (1 + j\eta) - k^2]$:

$$[Z_{pa}] = \rho c \begin{bmatrix} B_{N,1} B_{N,1} / \zeta_N^A & \dots & B_{N,1} B_{N,M} / \zeta_N^A \\ \vdots & \ddots & \vdots \\ B_{N,1} B_{N,M} / \zeta_N^A & \dots & B_{N,M} B_{N,M} / \zeta_N^A \end{bmatrix} \quad (23)$$

In Eq. 23, $[Z_{pa}]$ is the contribution of the backed cavity to the panel input impedance.

The interaction of the interior sound field with the panel is represented by the velocity continuation at the internal surface.

Substituting Eq. 4 and 20 into Eq. 3 get:

$$P_c(t, \omega) = [\phi_N]^T [Z_A][Z_p]^{-1} [P_M^{ext}] \quad (24)$$

The mean-square sound pressure is related to the acoustical potential energy in the cavity as:

$$\langle PP^* \rangle = (1/V) \int_V P_c P_c^* dr \quad (25)$$

The noise reduction defined as:

$$NR = -10 \log \frac{\langle P_c P_c^* \rangle}{4 P_{ext}^* P_{ext}} \quad (26)$$

which is used to characterize the mean-square sound pressure response in the cavity relative to external sound pressure.

The panel average input impedance is defined as:

$$Z_{imp} = -10 \log \frac{\langle V_p V_p^* \rangle}{4 P_{ext}^* P_{ext}} \quad (27)$$

which is used to describe the mean-square panel-velocity response, ration to the average external sound pressure level.

NUMERICAL ANALYSIS

The validity of the all program was verified by Matlab software package. Numerical analyses are conducted

Table 1: Geometric and material properties of the PC pane-cavity system

Dimension of enclosure	a = 0.44, b = 0.94, d = 0.625
Density of air	$\rho_a = 1.21 \text{ kg m}^{-3}$
Speed of sound	c = 330 m sec ⁻³
Modulus elasticity of polycarbonate	E = 2.4 × 10 ⁹ N m ⁻²
Poisson's ratio	v = 0.38
Density of face plate	$\rho_p = 1200 \text{ kg m}^{-3}$
Density of core	$\rho_c = 1.20 \rho_p$
Thickness of face plate	$h_1 = h_2 = 0.002 \text{ m}$
Thickness of core	h2 = 0.002 m

Table 2: Natural frequencies and damping of the cavity

N	(l, m, n)	f_{nN} (Hz)
1	(0,0,0)	0
2	(0,0,1)	275
3	(0,2,0)	366
4	(0,2,1)	458
5	(0,0,2)	550
6	(0,2,2)	660
7	(2,0,0)	782
8	(0,0,3)	826
9	(2,0,1)	829
10	(2,2,0)	863

using the configuration shown in Fig. 1 and the modal damping factor of cavity is given by $\eta_{nN} = 4.4\pi/10\omega_{nN}$ with other parameters listed in Table 1. In case of vehicles, the interior noise generated by structural vibration that is caused by the external sources predominated at low frequencies. Hence, analytical frequency range has been restricted to 1000Hz in this study.

To study the performance of the structure and acoustic interaction problem, the natural frequencies analysis for the 2D pane and the 3D acoustic cavity are investigated first. The number of modes used for structural response and acoustic decomposition is the main factor affecting the accuracy of the solution. The modal coupling analysis is considerably simpler and efficient in computation and plate characteristic functions represent plate modes more accurately and yield more accurate estimation of noise transmission performance. The total twenty cavity and pane modes are used to satisfy the convergence of the accuracy.

Table 3 gives the variation of the natural frequencies and modal loss factor with the core loss factor for all edges simply supported pane. In creasing the values of β increases the modal loss factors yet without appreciably changing the natural frequency values.

Table 4 gives the variation of the natural frequencies and modal loss factors with the core shear parameter g^* of pane. It is seen that increasing the values of the shear parameter g^* increases the natural frequency values but the loss factor is not regular form changing. The reason is that the core becomes high stiff with an increasing the value of g^* . There is no special rule to explain such a behavior, since the modal loss factor depends in a complex expression on the shear parameter, as can be seen from the equation (16) for simply supported edge.

Table 3: Natural frequencies for various the core loss factor β ($g^* = 10, Y = 12$)

M	(u,v)	$\beta = 0$		$\beta = 0.5$		$\beta = 1$	
		f_{nw}	η_{nw}	f_{nw}	η_{nw}	f_{nw}	η_{nw}
1	(1,1)	21	0	22	0.209	23	0.326
2	(1,3)	41	0	42	0.267	43	0.465
3	(1,5)	72	0	72	0.262	74	0.491
4	(3,1)	93	0	94	0.241	95	0.462
5	(3,3)	106	0	107	0.228	108	0.441
6	(1,7)	112	0	113	0.222	114	0.431
7	(3,5)	132	0	133	0.204	134	0.399
8	(1,9)	163	0	163	0.18	164	0.354
9	(3,7)	170	0	170	0.175	171	0.345
10	(5,1)	208	0	208	0.152	210	0.301

Table 4: Natural frequencies for various shear modulus g^* ($\beta = 0.3, Y = 12$)

M	(u,v)	$g^* = 1$		$g^* = 10$		$g^* = 100$	
		f_{nw}	η_{nw}	f_{nw}	η_{nw}	f_{nw}	η_{nw}
1	(1,1)	12	0.132	21	0.133	29	0.027
2	(1,3)	24	0.082	41	0.165	66	0.058
3	(1,5)	48	0.046	72	0.159	128	0.098
4	(3,1)	67	0.034	93	0.146	171	0.118
5	(3,3)	79	0.029	107	0.138	197	0.128
6	(1,7)	84	0.028	112	0.134	207	0.131
7	(3,5)	103	0.023	132	0.123	243	0.142
8	(1,9)	132	0.018	163	0.108	295	0.153
9	(3,7)	139	0.017	170	0.105	306	0.155
10	(5,1)	176	0.014	208	0.091	365	0.162

Table 5 gives the variation of the natural frequencies and modal loss factors with the geometric parameter Y of pane. It is seen that increasing the geometric parameter Y, increases the modal loss factors while keeping other parameters constant.

Figure 2 shows the effect on the noise reduction with frequency, for variation the core loss factor β . It is seen from Fig. 2 that there are dips in the noise reduction curves corresponding to the cavity resonant frequencies and the noise reduction curves tend to merge at these frequencies. The effect of the damping material is to smooth the noise reduction curves with higher noise reduction.

Figure 3 shows the effect on average panel input impedance, for variation the core loss factor β . It is seen from Fig. 3 that there are many peaks while $\beta = 0$, this is because the whole panel is high stiffness, the core loss factor has no attribution to the noise dissipation. It is seen from Fig. 3 that significant increase in the TL of the sandwich pane is obtained with increase in loss factor of core. But the effect of β on the TL is very minimal for lower value. The reason is that the loss factor β of high shear parameter is times of low shear parameters seen from Table 1.

Figure 4 shows the effect on noise reduction, for variation the core shear modulus g^* . For very low frequencies, there are obviously more dips in the noise reduction curve while $g^* = 1$ corresponding to the

Table 5: Natural frequencies for various geometric parameter Y ($g^* = 10, \beta = 0.3$)

M	(u,v)	Y = 6.8		Y = 20.3		Y = 30.7	
		f_{nw}	η_{nw}	f_{nw}	η_{nw}	f_{nw}	η_{nw}
1	(1,1)	17	0.119	26	0.142	31	0.147
2	(1,3)	34	0.139	49	0.184	58	0.194
3	(1,5)	63	0.123	84	0.188	96	0.207
4	(3,1)	83	0.108	107	0.179	121	0.202
5	(3,3)	96	0.100	120	0.173	135	0.197
6	(1,7)	102	0.096	126	0.170	141	0.195
7	(3,5)	122	0.086	147	0.159	163	0.187
8	(1,9)	152	0.073	178	0.145	195	0.174
9	(3,7)	159	0.071	185	0.142	203	0.171
10	(5,1)	198	0.060	224	0.127	242	0.156

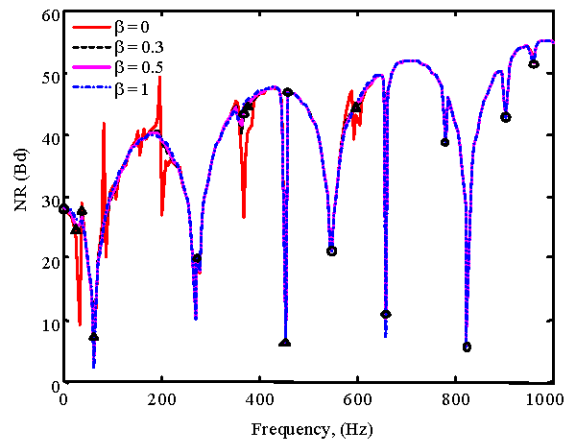


Fig. 2: Effect of core loss factor β on noise reduction \circ , cavity resonances; Δ , structural resonance

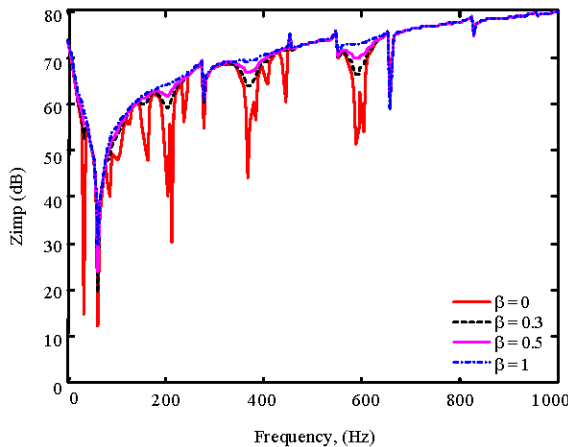


Fig. 3: Effect of core loss factor β on average panel input impedance

structural resonant frequencies. But the effect on the noise reduction is small at high frequencies. Hence it could be improved in noise reduction by choosing proper shear modulus of core.

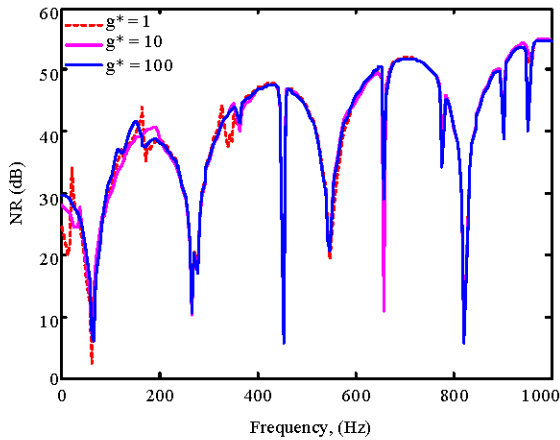


Fig. 4: Effect of core shear modulus g^* on noise reduction

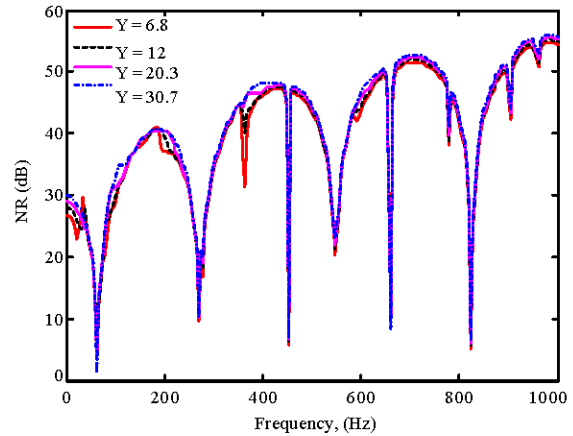


Fig. 6: Effect of geometric parameter Y on noise reduction

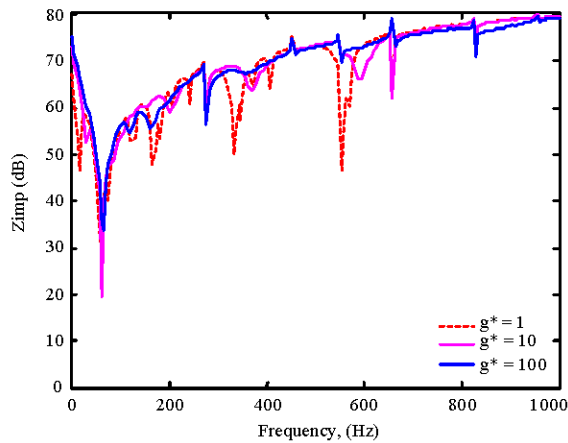


Fig. 5: Effect of core shear modulus g^* on average panel input impedance

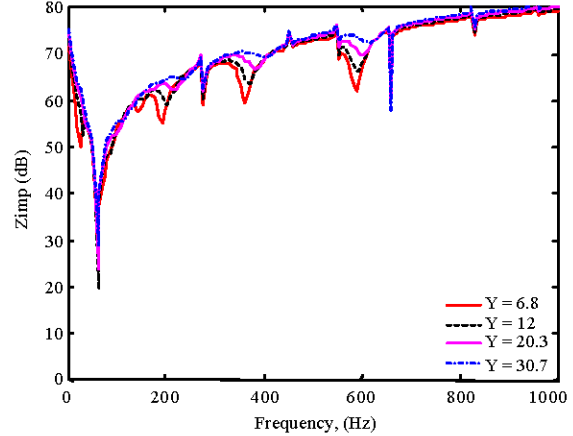


Fig. 7: Effect of core geometric parameter Y on average panel input impedance

Figure 5 shows the effect on average panel input impedance, for variation the core shear modulus g^* . It is seen from Fig. 5 that there is an optimum value of the core shear parameter Y for maximum damping effectiveness. The dips of average panel input impedance is lower, if the modal loss factor is higher according to Table. 4. It should be choose proper core shear parameter g^* to ensure improvement the noise reduction at the pane resonant frequencies.

Figure 6 shows the effect on noise reduction for variation the geometric parameter Y . It is seen from Fig. 6 that the pane noise reduction is improved with increasing the geometric parameter Y in low frequencies. Hence, it should be chose proper geometric parameter ensure better noise transmission performance at the structural resonance. There are dips in the noise reduction curves also corresponding to the cavity resonant frequencies.

Figure 7 shows the effect on average panel input impedance, for variation the geometric parameter Y . It is seen from Fig.7, if noise reduction curves exhibits dips at structural resonance frequencies, there are also peaks shown in the average panel input impedance curve.

CONCLUSION

The work presented in this note demonstrates the possibility of using sandwich PC pane to control noise in an acoustic enclosure for automobile application. The core of sandwich PC pane can be made from a more lightweight and less expensive material, while polycarbonate face sheet bearing load capacity. Simulation results reveal that, a high noise absorption capability over a wide frequency can be obtained using the proposed design, which provides an alternative way

for noise control of the discussed configuration using passive technique. However, it is noted that the dimension of the cavity is another factor affecting the effort.

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