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Deformation Behavior of Functionally Graded Poisson's Ratio Cellular Structure

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Abstract: Based on the computational design methodology and innovative structural material concept, the functionally graded Poisson's ratio cellular material is developed in this paper for investigated the deformation behaviour, which can be used for protection application. The optimal shape design and Poisson's ratio distribution have been introduced for further improve the protection materials.

Key words: Functionally graded, poisson's ratio, cellular structure

INTRODUCTION

Cellular materials, which are attractive for multifunctional applications, have been used as advanced structural components in automotive, aerospace, naval and biomedical industries. The structure of cellular materials ranges from the near-perfect order of the bee's honeycomb to the disordered, two and three dimensional networks. The mechanical properties of cellular solids have been traditionally modeled using periodic unit cell representative volumes. However, cellular structures have shown a specific directionality in terms of average orientation and shape geometry of its cells, which provide a global anisotropic mechanical behaviour.

For the application and manufacturing processes, the study and evaluation of cell configurations with potential unusual deformation mechanisms are required (Ajdari *et al*., 2009; Ajdari *et al*., 2011; Taylor *et al*., 2012; Babaee *et al*., 2012). Functionally graded cellular structures are novel class of materials, where variations in cell size, shape and wall thickness results in a functional variation in the relative density and organization of the cellular structure (Kumar and McDowell, 2009; Velea and Lache, 2011; Schraad, 2007). Natural materials as bamboo, banana peel and elk antler are functionally graded cellular structures. Researches have shown that this kind of structures exhibit excellent energy absorption ability (Covaciu *et al*., 2011; Pal *et al*., 2010; Ju and Summers, 2011).

The behavior of the material under deformation is governed by one of the fundamental mechanical properties of materials-Poisson's ratio. The Poisson's ratio of a material is defined as the ratio of the lateral contractile strain to the longitudinal tensile strain for a material undergoing tension in the longitudinal direction, which has historically been studied as one of elastic constants for isotropic materials. And based on the thermodynamic considerations of strain energy, the Poisson's ratio of isotropic materials can not only take negative values, but can have a range of negative values much larger than of positive ones, which has been accepted consequence of classical elasticity theory for over 200 years (Greaves *et al*., 2011). Material with negative Poisson's ratio means that it can undergo lateral expansion when stretched longitudinally.

This study describes a functionally graded Poisson's ratio material and develops a combined computational design methodology and structural material concept by optimal shape design and Poisson's ratio material distribution to further improve the deformation characteristics of the advanced structures. The cellular structures with both positive and negative values of Poisson's ratio are considered, in which assembles are translated into 2-Dimensional (2D) and 3-Dimensional (3D) finite element mesh, where the ribs are modeled using beam elements.

FUNCTIONALLY GRADED POSITIVE POISSON'S RATIO MATERIAL

Configuration: Figure 1 illustrates the conventional two dimensional hexagonal cellular structure with positive Poisson's ratio. Three design variables are introduced to describe the unit cell, where $(0, \alpha, \pi)$ denotes angle of the V shape formed by two adjacent inclined ribs with the same length I and h is the horizontal rib of the cell. And the basic configurations of the PPR materials have significant effects on effective material properties of the PPR structures. And the inclined and horizontal ribs can be made of different materials, which can satisfy the functional requirements of the structure and lead to a function-oriented design for kinds of applications.

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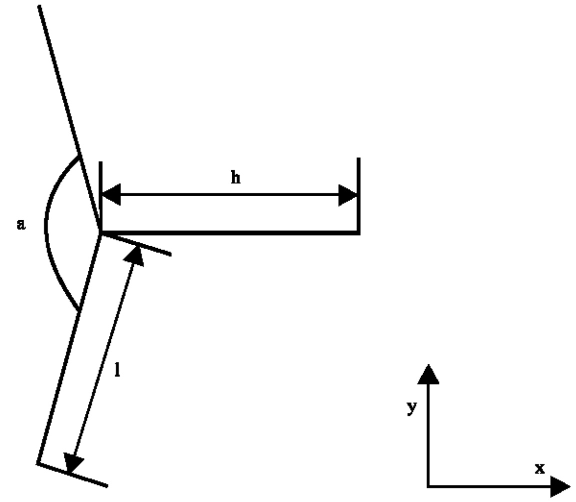


Fig. 1: Hexagonal configurations

In the following consideration, assuming a uniform thickness t for all ribs of cell, the effective Poisson's ratio, Young's modulus of cellular material can be defined as:

$$\nu_{yx} = \frac{\sin^2\frac{\alpha}{2}}{(h/l+\cos\frac{\alpha}{2})\cos\frac{\alpha}{2}} \tag{1}$$

$$E_x = E(\frac{t}{l})^3 \frac{\sin\frac{\alpha}{2}}{(h/l+\cos\frac{\alpha}{2})\cos^2\frac{\alpha}{2}} \tag{2}$$

$$E_y = E(\frac{t}{l})^3 \frac{(h/l+\cos\frac{\alpha}{2})}{\sin^3\frac{\alpha}{2}} \tag{3}$$

In the calculations, the raw material of structure is considered to be aluminum and material properties are given as Young's modulus E = 69 Gpa, Poisson's ratio v = 0.33 and density $\rho = 2700$ kg m^{-3}. Figure 2 shows how the effective material properties vary with respect to the design variables h^{-1} and $(0, \alpha, \pi)$. It can be seen that both effective Poison's ratio and Young's modulus E_x increased with α and Young's modulus E_y decreased with α.

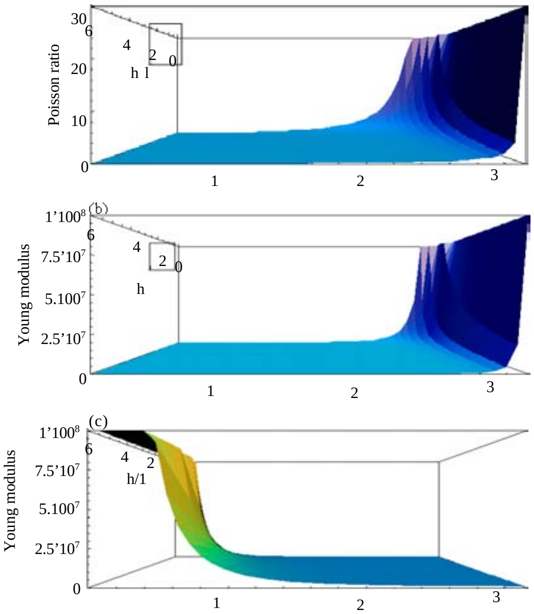


Fig. 2(a-c): Material properties of hexagonal cell with variation of h/l and α

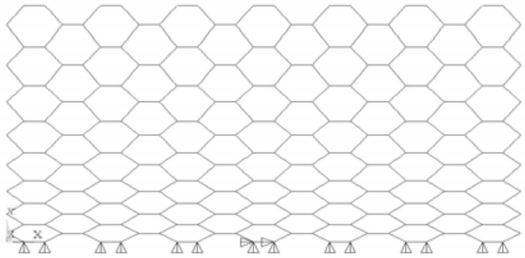


Fig. 3: FE modelling of cellular structure under uniaxial compressive loading

All Finite element modelling of cellular structures are performed by using available commercial software, ANSYS. The meshes have been created using two dimensional Timoshenko beam elements BEAM3 with circular cross-section, uniform thickness and three degrees of freedom, with translations along the global x and y directions and in-plane rotation along the z axis. Each rib of unit cell is represented by six elements. When loading along the global y direction, the bottom was subjected to a slide boundary condition except for the middle part was subjected to imposed deformation and zero in-plane rotation as in Fig. 3.

Figure 4 shows the deformation modes of conventional hexagonal structures with h/l = 2 and $\alpha = 120°$, in which the Poisson's ratio is v = 0.6. And Fig. 5 shows the deformation modes of functionally graded Poisson's ratio cellular structure, in which the Poisson's ratio varying through thickness with upper cell ($\alpha = 120°$) v = 0.6to lower cell ($\alpha = 120°$) v = 0.10073. From Fig. 4-5, it is seen that the deformation modes of two designs are different under the same loading condition, which the deformations of the two are 0.001417 and 0.00039 m.

Figure 6 shows three different configurations with symmetric variation of Poisson's ratio with respect to

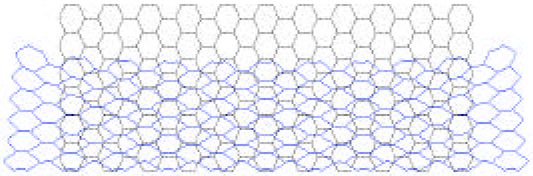


Fig. 4: Deformation of cellular structure with same conventional hexagonal cell (v = υ.σ)

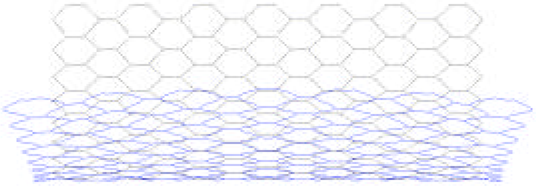
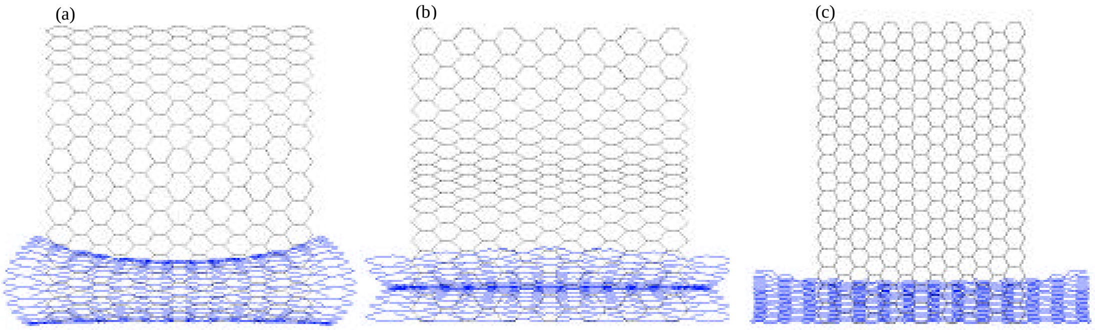


Fig. 5: Deformation of functional graded Poisson’s ratio cellular structure with variable hexagonal cell (v = 0.6→0.10073)



V (0.100723®0.6®0.110073)

V(0.6® 0.10073® 0.6)

v (0.6)

Fig. 6: Deformation of functional graded Poisson’s ratio cellular structure with variable positive values

thickness direction and a constant positive Poisson’s ratio value, in which the dashed lines represent the undeformed shape and solid blue lines represent the deformed shape. And the FE modelling results show that deformations of three cellular structures are 0.005388, 0.005334 and 0.012109 m, respectively. This suggests that cellular structures with functionally graded Poisson’s ratio have enhanced resistance.

In this part, the 3D version of cellular structure with conventional hexagonal cell is considered. The meshes have been created using 3D Timoshenko beam elements BEAM4 with circular cross-section, uniform thickness and six degrees of freedom, with translations along the global x, y and z directions and rotation along the x, y and z axis. Each rib of unit cell is represented by six elements. When loading along the global y direction, the bottom was subjected to a slide boundary condition except for the middle part was subjected to imposed deformation and zero rotation. And hexagonal unit cell is shown as in Fig. 7.

Figure 8 shows 3D functionally graded Poisson’s ratio cellular structure, in which the Poisson’s ratio varying through the thickness from v = 0.6 to v = 0.10073. And the deformation of the cellular structure is 0.004933 m.

Figure 9 illustrates configuration of 2D re-entrant cellular structure with negative Poisson’s ratio, in which three design variables are also introduced to describe the

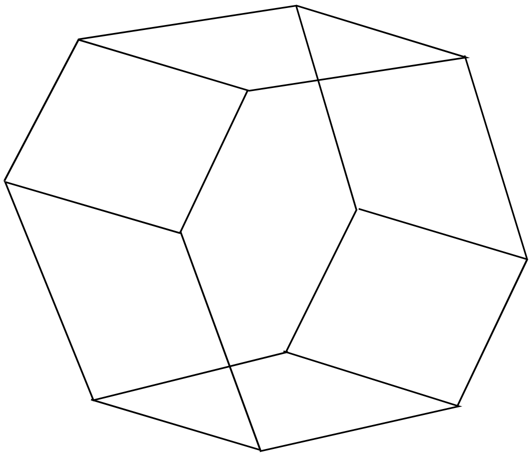


Fig. 7: 3-D unit hexagonal cell

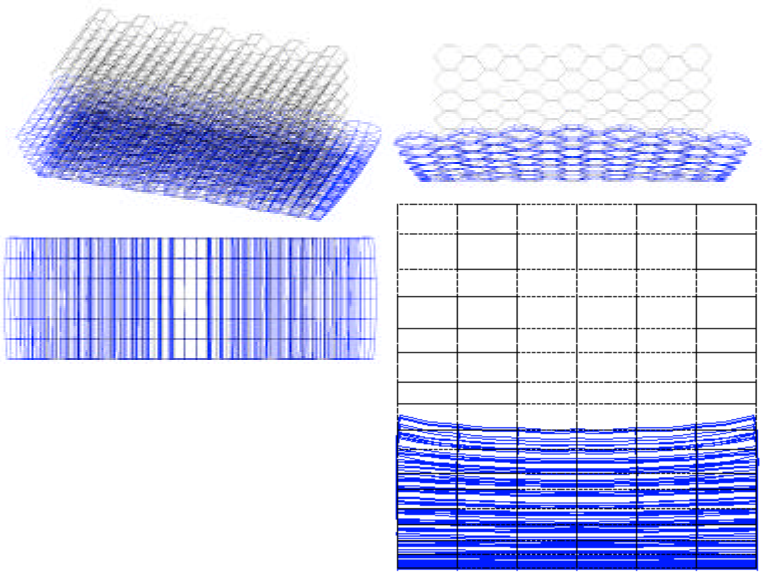


Fig. 8: Deformation of 3-D functional graded Poisson's ratio cellular structure with variable hexagonal cell

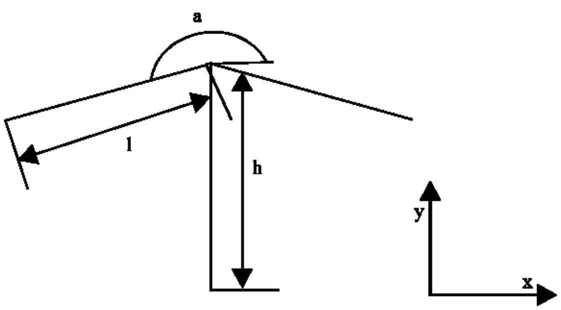


Fig. 9: Unit re-entrant cell

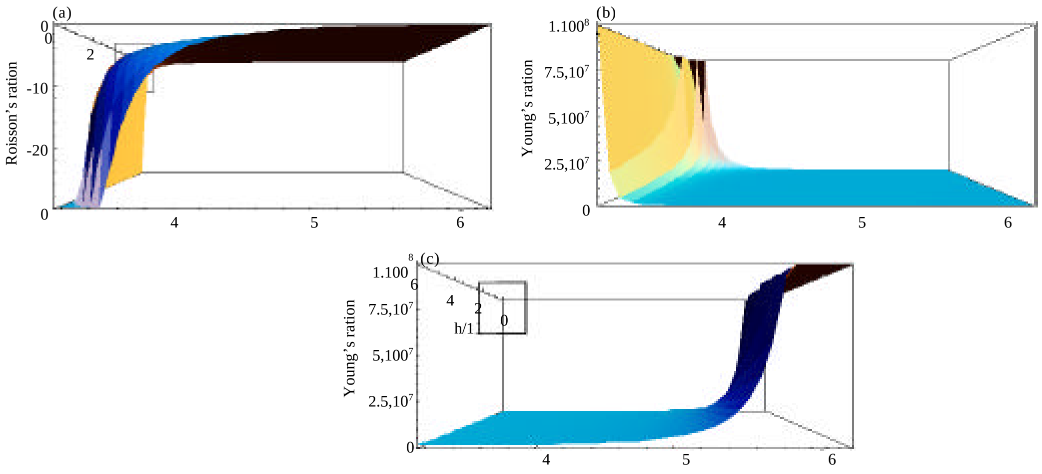


Fig. 10(a-c): Material properties of hexagonal cell with variation of h/l and α

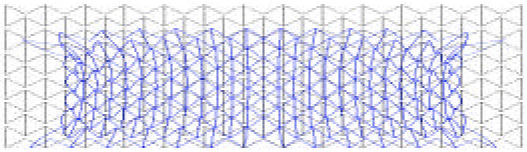


Fig. 11: Deformation of cellular structure with same re-entrant cell (v = -0.1)

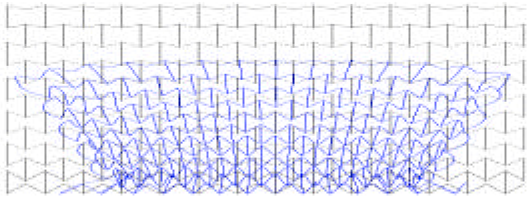


Fig. 12: Deformation of functional graded Poisson's ratio cellular structure with variable re-entrant cell (v = 06→ 0.10073)

unit cell, (0, α, π) denotes angle of the two adjacent inclined ribs with the same length l and h is horizontal rib of cell. These three design variables uniquely define geometry of unit cell. And the inclined and horizontal ribs can also be made of different materials.

Also assuming a uniform thickness of for all the ribs of the cell, effective Poisson's ratio, Young's modulus of cellular structure can be defined as:

$$v_{yx} = -\frac{\sin^2\frac{\alpha}{2}}{(h/l-\cos\frac{\alpha}{2})\cos\frac{\alpha}{2}} \qquad (4)$$

$$E_x = E(\frac{t}{l})^3\frac{\sin^2\frac{\alpha}{2}}{(h/l-\cos\frac{\alpha}{2})\cos\frac{\alpha}{2}} \qquad (5)$$

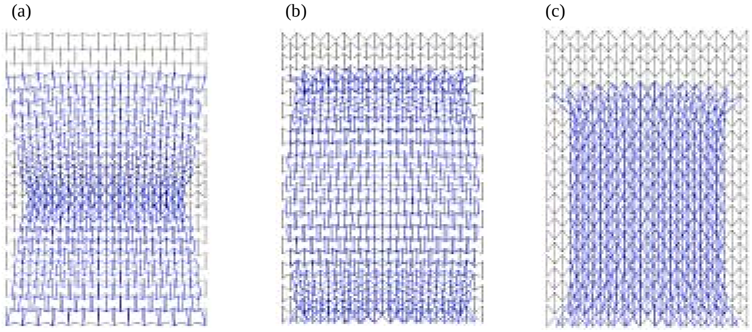
$$E_y = E(\frac{t}{l})^3\frac{(h/l-\cos\frac{\alpha}{2})}{\sin^3\frac{\alpha}{2}} \qquad (6)$$

Figure 10 illustrates how effective material properties vary with respect to the design variables h/l and angle. It can be seen that the valued of effective Poison's ratio are turned to be negative and Young's modulus decreased with angle and Young's modulus increased with angle.

Fig. 11 shows a structural with α = 240° and h/l = 2, in which Poisson's ratio is v = -1.0. And Fig. 12 shows the functionally graded Poisson's ratio cellular structure, in which Poisson's ratio varying through the thickness with upper cell (α = 240°) v = -2.07037 to lower cell (α = 240°) v = -1.0. From Fig. 11-12, it can be seen that the deformation shapes of two designs are different under the same loading condition, which the deformations of the two are 0.001818 and 0.00154 m.

Figure 13 shows three different configurations under the consideration of symmetric variation of Poisson's ratio with respect to thickness direction and a constant negative Poisson's ratio value, in which the dashed lines represent the undeformed shape and solid blue lines represent the deformed shape. And the FE modelling results indicate that deformations of three types of cellular structures are 0.002451, 0.002572 and 0.003082 m, respectively.

In this part, the 3D version of cellular structure with conventional hexagonal cell is considered. In FE



v (-2.07037 ®-1.0®-2.07037)   v (-1.0 ®-2.07037®-1.0)   v (-1.0)

Fig. 13: Deformation of functionally graded Poisson’s ratio cellular structure with variable negative values

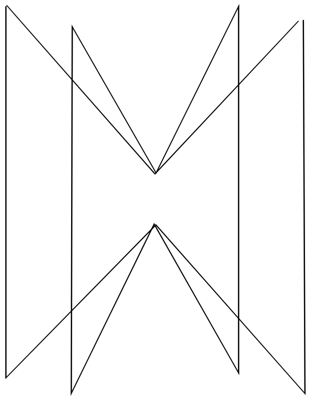


Fig. 14: 3-D FE model of unit re-entrant cell

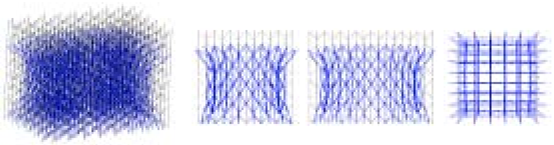


Fig. 15: Deformation of 3-D cellular structure with same re-entrant cell

modelling, ribs of structure are simulated by BEAM4 element. And the configuration of hexagonal unit cell is shown as in Fig. 14.

Figure 15 shows the design of three-dimensional cellular structure, in which the Poisson’s ratio is v = -1.0 and the deformation of cellular structure is 0.001219 m.

CONCLUSION

The functionally graded Poisson’s ratio cellular structure is introduced in this study. By considering both conventional hexagonal and re-entrant configurations of the unit cell, the deformations of the two and three-dimensional cellular structures are simulated by FE modeling. The results show that: (1) The functionally graded Poisson’s ratio cellular structures exhibit better deformation behavior than structures with constant Poisson’s ratio values for both PPR and NPR structures; (2) The functionally graded negative Poisson’s ratio cellular structure can be stiffed under compression; (3) The material properties of the functionally graded cellular structures can be enhanced by design and optimize the cell configuration.

REFERENCES

Ajdari, A., H. Nayeb-Hashemi and A. Vaziri, 2011. Dynamic crushing and energy absorption of regular, irregular and functionally graded cellular structures. Int. J. Solids Struct., 48: 506-516.

Ajdari, A., P. Canavan, H. Nayeb-Hashemi and G. Warner, 2009. Mechanical properties of functionally graded 2-D cellular structures: A finite element simulation. Mater. Sci. Eng: A, 499: 434-439.

Babaee, S., B.H. Jahromi, A. Ajdari, H. Nayeb-Hashemi and A. Vaziri, 2012. Mechanical properties of open-cell rhombic dodecahedron cellular structures. Acta Materialia, 60: 2873-2885.

Covaciu, M., M. Walczak and J. Ramos-Grez, 2011. A method for manufacturing cellular metals with open and close-type porosities. Mater. Lett., 65: 2947-2950.

Greaves, G.N., A.L. Greer, R.S. Lakes and T. Rouxel, 2011. Poisson's ratio and modern materials. Nat. Mater., 10: 823-837.

Ju, J. and A.D. Summers, 2011. Compliant hexagonal periodic lattice structures having both high shear strength and high shear strain. Mater. Des., 32: 512-524.

Kumar, R.S. and D.L. McDowell, 2009. Multifunctional design of two-dimensional cellular materials with tailored mesostructure. Int. J. Solids Struct., 46: 2871-2885.

Pal, S., S. Maiti and G. Subhash, 2010. Effect of microscopic deformation mechanisms on the dynamic response of soft cellular materials. Mech. Mater., 42: 118-133.

Schraad, M.W., 2007. The influence of dispersity in geometric structure on the stability of cellular solids. Mech. Mater., 39: 183-198.

Taylor, C.M., C.W. Smith, W. Miller and K.E. Evans, 2012. Functional grading in hierarchical honeycombs: Density specific elastic performance. Compos. Struct., 94: 2296-2305.

Velea, M.N. and S. Lache, 2011. In-plane effective elastic properties of a novel cellular core for sandwich structures. Mech. Mater., 43: 377-388.