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## Game Analysis of Enterprise Emission Rights Competition under the Background of Sustainable Development

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**Abstract:** Emissions trading as an important system innovation are becoming one of the environmental protection policies which the emerging countries care more and more. But Emission allocation between different enterprises has become a problem of government environmental policy implementation. Effective allocation of emission permits is not only can improve market allocation efficiency of environment and resources also can greatly reduce the government management of the environment cost. The article adopts evolutionary game theory analyzing the competition game process for pollution rights between the two kinds of enterprises whose emission amount is different under the restriction of the environmental capacity. When the government make pollutant discharging enterprises within the area pollution total force limit, pollutant discharging enterprises will compete fiercely for emission amount and eventually tend to dynamic steady state. Clearly, putting forward comprehensive adopting the way of market allocation resources can reduce the pollutant discharging enterprises to the possibility of pollutant emissions.

**Key words:** Evolutionary game theory, emission rights, environmental efficiency

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### INTRODUCTION

The famous North externality theory pointed out, when an individual or enterprise's own cost is not equal to social cost, their benefit is not equal to social benefits AND it will generate externalities. While the externalities are divided into two cases, the first is the so-called external economies which is that those affected receive benefit is positive, the second is the external diseconomy, namely to the affected persons because of the externality imposed costs and their benefits become negative. Under the condition of the market mechanism, according to economics usually made rational economic man hypothesis can clearly see that, rational individuals or enterprises for the sake of cost will not conscious and take the initiative to solve the external diseconomy problems. Because of individual or enterprises emission cost is far less than the social cost caused by environmental pollution, coupled with the characteristics of environmental unclear property rights, so that allowing an individual or enterprises have motivation of continually increase production and emission pollution. Pigou (1920) found out the phenomenon that an individual or enterprises environmental pollution caused by production problems and damaged to the social welfare and called it "the environment external diseconomy".

The main characteristic of external diseconomy of environmental pollution is that an individual or enterprise uses environment resources to seek their own interest but shifts the environment cost on to others and society in the process of production, the benefits of others and society are unfairly invaded. Repullo and Suarez (1999) stated that the existence of the externality diseconomy will produce two kinds of negative effects; the one is that Contaminants enterprises will consume environment resources excessively but not effectively constrained, the other is caused society unfairness. Because the contaminants enterprises pillage and overuse abuse the environment, the market mechanism will produce market failure phenomenon in adjusting environment resources availability and distribution. The literatures on enterprise emission dynamic game are mainly divided into two categories; one is that not considering the cumulative amount of pollution that is environmental capacity degree of damage which is related to the speed of pollution, that is to say the problem is static in nature. This kind of problem is mainly discussed in view of the problems of "prisoner's dilemma" cooperation in the repeated game. Such as Maler (1991) found that when the game of basic type is prisoner's dilemma, even though corporation can generate Pareto optimal results and each Contaminants enterprise once betrays cooperation or take a "free ride" in pollution reduction will face severe punishment but

corporation in order to reduce the pollution is difficult to achieve. Some scholars analyzed the prisoner's dilemma model with repeated game and found that: if the participants know each game is not just one time, they have enough patience, at the same time they don't know when the game will end, Pareto optimal results can be achieved (Fudenberg and Tirole, 1991). Some scholars studied expanding prisoner's dilemma model and found (in simple game), part of gamers limited corporation exist equilibrium (Heitzig *et al.*, 2001; Siniscalco, 1993). Batabyal (1995, 1996) elaborated the government of first mover advantage has relationship and effect with Contaminants enterprises by Stackelberg dynamic game model. Damania (2001) analyzed the effect of government environment regulations on demand uncertainly monopoly enterprise strategies based on repeated game, the result is taxing on emission will affect enterprise's financial structure and reduce the market competition of enterprises. Zenkevich and Zyatchin (2007) analyzed three different enterprises the amount of emissions distribution and optimal allocation scheme in the condition of sharing emission rights based on repeated game model. another kind is considering the accumulation of pollutants in the process of continuous damaging effects to the environment, this kind of research mainly concentrate on cross-border pollutant problems, that is acid rain game model. Such as Joregensen and Zaccour (2003) established differential game about two countries pollutant problems, starting from individual rationality to formulate corresponding environmental convention to ensure that in the process of game, any participant will get benefit from cooperation not less than default, so as to achieve the purpose of improving the environmental quality. On the basis of it, some people from the point of view of economic coordination made up two import and export trade relations and asymmetric countries environment protection problem using differential game research (Cabo *et al.*, 2006).

In reality enterprise Contaminants decision makers can only be bounded rational gamer, influenced by analytical reasoning, recognition judgment ability, information collection ability constraints and enterprises adjustment in the Contaminants strategy can only be slow influenced by such as cost' Technology? scale of enterprises, time an so on, so the ability of "learning" is relatively slow. Therefore this study analyses the game process between Contaminants enterprises groups for emission trading on the basis of the method of evolutionary game, in order to seek the possibility through the Contaminants enterprises mutual supervision to reduce the possibility of pollutant emission.

## MODEL ASSUMPTIONS

Suppose that Ais region set in a specific area,  $D = \{d_1, d_2, \dots, d_n\}$ , in this region emission rights Distribution scheme is  $\{E_1, E_2, \dots, E_n\}$ , that is the enterprises in the region,  $d_k$ , the amount of emission rights enterprise distribution is  $E_k$ , water capacity is  $E = E_1 + E_2 + \dots + E_n$ . Through the harmony of diagnostic methods to initial allocation of emission rights, analyze and judge whether this scheme can satisfy enterprises in all regions and can judge the enterprises in the region  $d_k$  relative to all other  $d_j$  ( $k = 1, 2, \dots, n, k \neq j$ ) in the relationship between the size of enterprise emission rights, notes  $S_{jk}$ , signifies emission quantity relationship between the enterprises in the region  $d_j$  and region  $d_k$ ,  $S_{jk}$  as the following three circumstances:

- If  $S_{jk} > 0$ , illustrate that enterprises emissions quantity in the region  $d_j$  is more than the region  $d_k$ , over  $S_{jk}$  emissions amount
- If  $S_{jk} = 0$  illustrate that enterprises emissions quantity in the region  $d_j$  is equal to the region  $d_k$ , should maintain the current emission amount
- If  $S_{jk} < 0$ , illustrate that enterprises emissions quantity in the region  $d_j$  is less than the region  $d_k$ , less  $|S_{jk}|$  emissions amount

The net amount of adjustment of e mission amount of enterprises in the region  $d_j$  of emission rights relative to all the other enterprises in the regions is  $S_j$ , that is  $S_j = \sum_k S_{jk}$ .

All  $S_j \geq 0$  regions called as "excessive emission rights amount area", suppose that the amount of over distributed emission rights is  $S$  in the all "excessive emission rights amount areas" but which actual emission rights amount is  $M_1$ ; all  $S_j < 0$  regions called as "emission rights amount too little area", suppose that the amount of over distributed emission rights is also  $S$  in all "emission rights amount too little areas", however, which actual emission rights amount is  $M_2$ . All enterprises in "emission rights amount too little area" are facing the same decision problem: Whether to protect their own interests by the way of conflict namely to secretly emissions, exceed the standard emission; all enterprises in "excessive emission rights amount areas" are also facing the same decision problem: Whether to give up excessive emissions rights amounts or not? Therefore, there is a game relationship between the two kinds of the Contaminants enterprises within the region.

In a certain region, influenced by constraints of the limited capacity of the environment, the initial allocation of emission rights is difficult to get a allocation scheme of

emission rights that makes all the enterprises in all the regions satisfied once in a time. Because the gamer in any region has a certain rationality, we are rational person that pursuit the maximum economic benefit, so this game will not be Nash equilibrium that one time game in perfectly rational case but rather a process of learning, communication and repeated game and ultimately to achieve a dynamic equilibrium state. In this state, the Contaminants enterprises in any region can not be willing to unilaterally change emission strategy; the equilibrium strategy is called an Evolutionary Stable Strategy (ESS). We can adjust emission rights amounts in any region according to this strategy and get a new allocation scheme that satisfies any region.

**MAIN GAME AND STRATEGY SELECTION**

In this study, the initial allocation of the enterprises emission rights in the region is divided into two game groups: The game “excessive emission rights amount” group refers to as the game A and “emission rights amount too little” group refers to as the game B “Emission rights amount too little” group hopes "excessive emission rights amount” group took the initiative to give up the emission rights of S, if the "excessive emission rights amount" group does not agree to waive the emission rights of S which will cause a strong protest in "emission rights amount too little" group, who will fight for the emission rights S by the way of the outbreak of conflict (such as secretly emissions, excessive emission). It is clear that the two main game strategy departments are selected at the same time; it is a typical dynamic game process. "Excessive emission rights amount" group has two strategies: Agree to give up the emission rights of S (referred to as the "agreement") and not agree to give up the emission rights of S (referred to as the "disagreement"); “Emission rights amount too little” group also has two strategies: “conflict” and “no conflict”.

The purpose of the two game groups is to achieve the maximization of their own interests. Suppose that environment capacity in unit region is m. The conflict from “emission rights amount too little” group will bring a certain damage to “excessive emission rights amount” group and the damage should have a direct proportional relationship with the proportion k ( $k = S/M_2$ ) which is equal to " emission rights amount too little" group fewer emission rights allocated to S accounted for the actual amount of emission rights allocated to  $M_2$ , assume the proportion coefficient is  $\eta$ . “Emission rights amount too little” group will have the chance to achieve the emission

		<b>Game B</b>	
		$\{m (M_1-S)\} \quad m (M_1+S)\}$ <b>Conflict (<math>\phi</math>)</b>	$\{m (M_1-S)\} \quad m (M_1+S)\}$ <b>Not Conflict (<math>1-\phi</math>)</b>
<b>Disagreement (<math>1-\lambda</math>)</b> <b>GameA</b>	$[mM_2+(mS-\eta k)/2]$	$(mM_1, mM_2)$	
	<b>Agreement (<math>\lambda</math>)</b>	$[m (M_1-S)+(mS-ek)/2]$	

Fig. 1: Evolution matrix on “excessive emission rights amount” group and “emission rights amount too little”

rights amounts of S but also pay the conflict cost and the cost also has a direct proportional relationship with k, assuming that the coefficient is.

If both gamers choose “fight “as their strategy and suppose that the two sides of the game have the same probability of winning are 1/2, so their expected return will be :  $[m (M_2-S)+(mS-ek)/2]$  and  $[mM_2+(mS-\eta k)/2]$ . Suppose that in “excessive emission rights amount” group, the main game that takes “agreement”, “disagreement” strategies proportion is  $\lambda$  and  $(1-\lambda)$ ; however, in “emission rights amount too little" group the main game that takes “conflict” and “no conflict” proportion is  $\phi$  and  $(1-\phi)$ . The matrix of the sides is as shown in Fig. 1.

**EVOLUTIONARILY STABLE STRATEGY ANALYSIS**

From Fig. 1 now that when "excessive emission rights amount" group selects "agreement" and "disagreement" two strategies, they receive the expected revenue  $u_{1\phi}$  and  $u_{1\lambda}$  and average revenue  $\bar{u}_1$  are as follows:

$$u_{1\phi} = \phi * m (M_1-S)+(1-\phi)*m (M_1-S) \tag{1}$$

$$u_{1\lambda} = \phi * [m (M_1-S)+(mS-\eta k)/2]+(1-k)*m M_1 \tag{2}$$

$$\bar{u}_1 = \lambda * u_{1\phi}+(1-\lambda)*u_{1\lambda} \tag{3}$$

When "emission rights amount too little" group selects "conflict" and "no conflict" two strategies, the expected revenue  $u_{2\phi}$ ,  $u_{2\lambda}$ , average revenue  $\bar{u}_2$  are as follows:

$$u_{2\phi} = \lambda * m (M_2+S)+(1-\lambda)*[mM_2+(aS-ek)/2] \tag{4}$$

$$u_{2\lambda} = \lambda * m (M_2+S)+(1-\lambda)*mM_2 \tag{5}$$

$$\bar{u}_2 = \phi * u_{2\phi}+(1-\phi)*u_{2\lambda} \tag{6}$$

“Excessive emission rights amount” group evolutionarily stable strategy game.

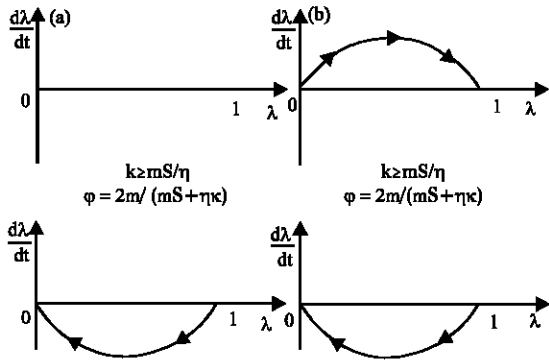


Fig. 2: "Excessive emission rights amount" group replicate dynamic

From the above Eq. 1 and 3, we can know that when "excessive emission rights amount" group selects "agreement" strategy, the replicator dynamics equation is:

$$\frac{d\lambda}{dt} = \lambda(u_{1\phi} - \bar{u}_1) = \lambda(1-\lambda) \left[ \frac{1}{2} \phi(mS + \eta k) - mS \right] \quad (7)$$

Let:

$$F(\lambda) = \frac{d\lambda}{dt}$$

by first derivation, we can get that:

$$F(\lambda) = (1-2\lambda) \left[ \frac{1}{2} \phi(mS + \eta k) - mS \right] \quad (8)$$

Then let  $F(\lambda) = 0$ , according to 7, get that two possible steady state points  $\lambda^* = 0$  and  $\lambda^* = 1$ , then may have the following situations:

- When  $\phi - 2mS/(mS + \eta k) < 0$  (only when  $0 \leq 2mS/(mS + \eta k) \leq 1$  namely  $k \geq mS/\eta$ , there is always  $F(\lambda) = 0$ , meaning all levels are stable states. Under these conditions, the replicator dynamics equations of "excessive emission rights amount" group will be shown in Fig. 2a. From Fig. 2a, we can clearly understand that when the "emission rights amount too little" group chooses "conflict" in the level of  $2mS/(mS + \eta k)$ , the "excessive emission rights amount" group selects "agreement" or "disagreement" strategy, the benefit is the same, that is to say, all  $\lambda$  levels are "the stable state of the groups
- When  $\phi > \phi - 2mS/(mS + \eta k)$ ,  $\lambda^* = 0$ ,  $\lambda^* = 1$  are two possible steady state point of  $\lambda$  and  $F'(\lambda) < 0$ , so  $\lambda^*$  is the evolutionarily stable strategy of game, The

replicated dynamic equation of "excessive emission rights amount" group will be shown in Fig. 2b. When the "emission rights amount too little" group chooses "conflict" in a greater level than  $2mS/(mS + \eta k)$ , the strategy of "excessive emission rights amount" group will be gradually shifted from "disagreement" to "agreement". That is to say, the "agreement" strategy is the group evolutionarily stable strategy

- When  $\phi < \phi - 2mS/(mS + \eta k)$ ,  $\lambda^* = 0$  and  $\lambda^* = 1$  are two possible steady state point of  $\lambda$ . Because  $F'(0) < 0$   $F'(1) > 0$ ,  $\lambda^* = 0$  is the stable strategy of the game. The replicated dynamic equation of "emission rights amount too little" group will be shown in Fig. 2c. When the "emission rights amount too little" group chooses "conflict" in a less lever than  $2mS/(mS + \eta k)$ , the strategy of "excessive emission rights amount" group will be gradually shifted from "agreement" to "disagreement". That is to say, the "disagreement" strategy is the group evolutionarily stable strategy
- When  $2mS/(mS + \eta k)$ , namely to  $k < mS/\eta$  and  $\lambda^* = 0$  and  $\lambda^* = 1$  are two possible steady state points of  $\lambda$ . Then put  $\lambda^* = 0$  and  $F'(0) < 0$   $F'(1) > 0$  into Eq. 8 then we can get  $\lambda^* = 0$ , so  $\lambda^* = 0$  is the evolutionary stable strategy of the game and the replicator dynamic Fig. of "excessive emission rights amount" group is shown in Fig. 2d which means "disagreement" strategy is an evolutionary stable strategy of the group

**Emission rights amount too small group evolutionarily stable strategy game:** By the above 4 and 6 emission rights amount too little group replicated dynamic equation when choosing "conflict" strategy is:

$$\frac{d\phi}{dt} = \phi(u_{2\phi} - \bar{u}_2) = \phi(1-\phi)(1-\lambda)(mS - \epsilon k) / 2 \quad (9)$$

Let:

$$F(\phi) = \frac{d\phi}{dt}$$

we can get the first derivative of  $F(\phi)$  about  $\phi$  as follows:

$$F'(\phi) = (1-2\phi)(1-\lambda)(mS - \epsilon k) / 2 \quad (10)$$

Then let  $F(\phi) = 0$ , according to 9, we can get  $\phi = 0$  and  $\phi = 1$  are two possible evolutionary stable state points of the game. May have the following situations:

- When  $\lambda^* = 1$ , so  $F(\phi)$  is permanent establishment, that is to say, for all level  $\phi$  is stable. In this case,

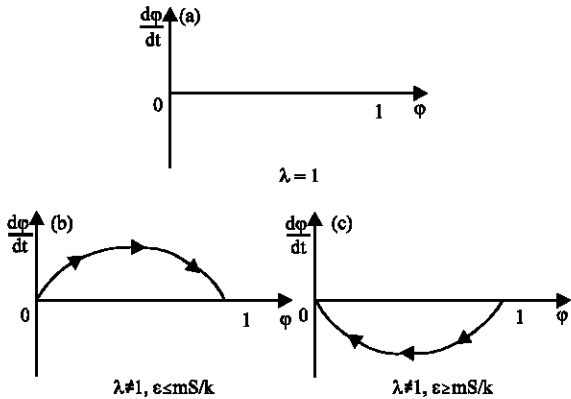


Fig. 3: Dynamic figure of "emission rights amount too little" group

"emission rights amount too little" group replicated dynamic equation is as shown in Fig. 3a. When the game decision makers of "excessive emission rights amount" group chooses "agreement" as the strategy, "emission rights amount too little" group selects "conflict" and "no conflict" benefit gained by the two strategies is the same, that is to say, for all levels  $\phi$  are the steady state of the groups

- When  $\lambda \neq \lambda^*$ , namely to  $\lambda \neq 1$  and then we can obtain  $\phi' = 0$  and  $\phi' = 1$  are two possible evolutionary stable state points of the game. Put  $\phi' = 0$  and  $\phi' = 1$  into Eq. 10, then get information as follows:

$$F'(0) = (1-\lambda)(mS-\epsilon k)/2$$

$$F'(1) = -(1-\lambda)(mS-\epsilon k)/2$$

If  $mS-\epsilon k \geq 0$ , then  $\epsilon \leq mS/k$ , so  $F'(1) < 0$ . And then  $\phi = 1$  is the evolutionary stable strategy of the group when  $\lambda \neq 1$ , equation is shown in Fig. 3b. If  $mS-\epsilon k < 0$ ,  $\epsilon > mS/k$ , so  $F'(0) < 0$  and then,  $\phi' = 0$  is the evolutionary stable strategy of the group when  $\lambda \neq 1$ , Equation is shown in Fig. 3c.

From the analysis above, we can clearly find that, when the "excessive emission rights amount" group exist policymakers adopt "disagreement" to their strategy, "emission rights amount too little" group evolutionary stable strategy has a close relationship with the cost of conflict  $\epsilon$ . When  $\epsilon$  is relatively small, that is  $\epsilon < mS/k$ , you select "conflict" is the stability strategy of the group; when  $\epsilon$  is relatively large, that is  $\epsilon > mS/k$ , they select "no conflict" as the evolutionary stable strategy of the group.

### RESULTS AND DISCUSSION

Through the above analysis, the study draws the following conclusions:

- When  $\epsilon < mS/k$ , namely "emission rights amount too little" group are assigned to less emission  $S$  account for  $M$  less than  $mS/k$ . Bounded rationality two main game through long-term repeated game, learning and strategy adjustment, ultimately make most of the main game converge to the strategy portfolio of "disagreement" and "no conflict"
- When,  $\epsilon \geq mS/k$  namely "emission rights amount too little" group is assigned to less emission  $S$  account for  $M$  more than  $mS/k$ . Bounded rationality two main game through long-term repeated game, learning and strategy adjustment, ultimately make most of the main game converge to the strategy portfolio of "agreement" and "conflict". When  $\epsilon \geq mS/k$  and "emission rights amount too little" group than percentage of players in the "conflict" as its strategy. The proportion of the population about "emission rights amount too small" group chooses the "conflict" as their strategy is above the game side  $2mS/(mS+\eta k)$ , then "agreement" is the evolutionarily stable strategy of "excessive emission rights amount" group. And for  $\phi = 2mS/(mS+\eta k)$ , so the greater is the  $k$ , the smaller  $\phi^*$  is. This turns out that when  $k$  is larger, as long as there is a small amount of "emission rights amount too little" group chooses "conflict" as the strategy, this will be able to make "excessive emission rights amount" group shifts strategy from "disagreement" gradually to "agreement"
- When the "excessive emission rights amount" group exists an individual drawing "disagreement" strategy, the evolutionarily stable strategy of "emission rights amount too little" group depends on the size of the conflict cost  $\epsilon$ . When  $\epsilon$  is small, namely to  $\epsilon \leq mS/k$ , the "conflict" is the stability strategy of "emission rights amount too little" and so when  $\epsilon \leq mS/k$ , "no conflict" is the stable strategy of the group

### CONCLUSION

Through analyzing the evolution of the game between enterprises, we can find out when government makes pollutant discharging enterprises within the area pollution total mandatory limits, polluters will be contested on emission rights amount and eventually tends to be dynamic steady state.

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