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A Fast Convergence Efficiency Method of Inverse kinematics for Robot Manipulators

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Abstract: Analytical solution of the inverse kinematics has been found only for some particular structures of 6-DOF serial manipulator. We propose the Method of Sequential Retrieval by modifying the effective solution for inverse kinematics to solve the inverse kinematics for a 6-DOF robot arm which has not been analytically solved yet. We confirmed that our method can find some solutions which cannot be solved by the Effective Solution for Inverse Kinematics. In this study, we present the effectiveness of our method by comparing its convergence speed with that of the Newton-Raphson Method.

Key words: Inverse Kinematics, 6-DOF manipulator, method of sequential retrieval, numerical solution, modification weight

INTRODUCTION

At present, robots play an active role in every field but industrial use, use in hazardous environments and use for collaboration in human society are their three major application fields. In industrial applications, robots automatically perform jobs such as carrying, processing and factory assembly, thereby contributing to promotion of efficiency in production. Robots used in hazardous environments function in environments that are dangerous for humans such as rescue efforts at disaster sites, deep-sea investigation, space activities and work around nuclear reactors. Human society collaboration robots are functioning actively in mascot and entertainment fields.

Industrial use robots and hazardous environment robots are designed and manufactured to serve specific functions. A robot manipulator is indispensable to attain their tasks. Regarding robot manipulators, arm-type manipulators are considered important elements in robot systems because of their wider movement range and higher general versatility.

Robot manipulator designs are of two categories: general purpose and exclusive work. For general-purpose robot manipulators, those with more than 6 degrees of freedom (DOF) are necessary to allow free positioning and posturing of hand tips (end effectors) to cope with jobs of every type. Although, those with more than 7 DOF can handle tasks that are more complicated, their structures become increasingly more complicated with the difficulty of the task. For this reason, most general-purpose manipulators have 6 DOF. Those with less than 5 DOF are exclusive-use manipulators that are designed specifically with structures for tasks that can be performed even if restrictions are imposed on the posture and working range.

The subject of this study is 6 DOF general-purpose robot arms, which we are investigating specifically. Figure 1a shows that the general structure of a 6 DOF arm allows arbitrary length of each link and direction of joints. To move such a robot arm to do tasks of some kinds, it is necessary to obtain, from target values of hand tips, the angles of each joint corresponding to them, i.e., to solve so-called inverse kinematics. However, no analytical solution of the inverse kinematics applicable to arbitrary structure is available at present. Therefore, with most that

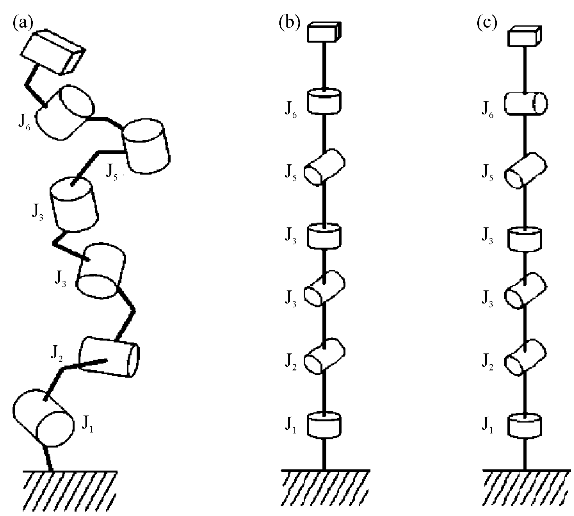

Fig. 1(a-c): Types of 6-DOF robot arm (a) Arbitrary structure, (b) PUMA type and (c) RPY type

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are currently used for industrial applications, no analytical solution of the inverse kinematics has been found yet. Inverse kinematics can be solved analytically if "all adjoining joint axes are parallel or orthogonal". Of those, the structure designated as PUMA-type as shown in Fig. 1b has been known since long ago. It is used extensively at present (Arshad *et al*., 2012; Rosales and Gan, 2003; Gan *et al*., 2005; Guo and Cherkassky, 1989; Manocha and Canny, 1994; Meggiolaro *et al*., 2000; Oyama *et al*., 2001; Papadopoulos and Poulakakis, 2000; Tejomurtulaa and Kak, 1999; Xu, 2002; Li *et al*., 2012; Zlajpah and Nemec, 2002).

As described, one condition for practical application is that inverse kinematics must be solved, which imposes restrictions on selectable structures. In this study, for an RPY-type arm (Fig. 1c) which is one structure of inverse kinematics that has not been solved yet, we propose the Method of Sequential Retrieval, for which the modification weight is added to an efficient solution of the inverse kinematics to obtain a solution.

DEFINITIONS OF SYMBOLS

First, to assure the accuracy of description of the problem, this section provides definitions of symbols used for this study. The problem is then described using these symbols. Figure 2a portrays a schematic diagram of an RPY-type arm when all joint angles are 0 degree.

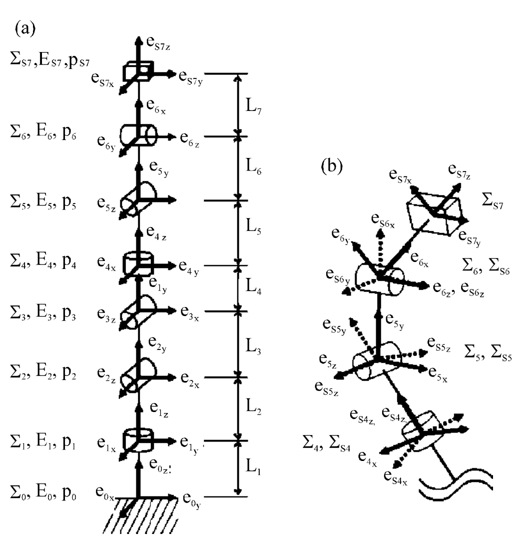

Fig. 2(a-b): Definition of coordinate systems (a) Initial state (all angle joints is 0[rad]) and (b) Σ_i (solid line) Σ_{Si} (dashed line at certain angles)

METHOD OF SEQUENTIAL RETRIEVAL

Solving inverse kinematics of an RPY-type arm analytically is difficult. For such a structure, Newton-Raphson method and numerical solutions designated as efficient solution of the inverse kinematics (hereinafter designated as efficient solution) have been proposed. The efficient solution has features such as rapid convergence and that eight or fewer solutions are obtained. Additionally, it is considered that the efficient solution is characteristic in that solutions of a particular pattern can be selected. This feature plays an important role when the hand tip target positions and posture trajectories are converted to joint angle trajectories. However, a problem area of this efficient solution is that no convergence occurs depending on the target values. In this study, an improvement of the efficient solution, which is termed as the Method of Sequential Retrieval, is proposed.

Outline of method of sequential retrieval: Using the Method of Sequential Retrieval, first, the initial value of each joint angle is given. Then two modes of processing are used, as described below.

- Process 1: θ_1, θ_2 and θ_3 are modified so that positions can become identical and
- Process 2: θ_4, θ_5 and θ_6 are modified so that postures can become identical

They are repeated to achieve convergence of θ_1,..., θ_6 values to meet with the target value. In an efficient solution, the positions and postures come to agree with the target value in Process 1 and Process 2. However, in this study, the framework that is used is to set the modification amount using the modification weight Wp, Wo in Process 1 and Process 2. When Wp = 1 and Wo = 1, the same processing as that observed in the efficient solution is made. However, by setting an appropriate modification weight, it becomes possible to achieve convergence even for a case in which convergence cannot be made by the efficient solution. These operations are presented in Fig. 3 in flowchart form. Details of Process 1 and Process 2 will be described in the next and subsequent sections (Nie and Huang, 2012).

Convergence judgment: In the convergence judgment, it is considered that convergence occurred when $|\Delta p_{S7}|<t_p$ and $|\Delta O_{S7}| < t_0$ are established and that $\theta_6[k]$,..... and θ_6 [k] at that point of time are considered to be solutions.

Application of numerical values (Example obtaining eight solutions): It is said that with an efficient solution, eight

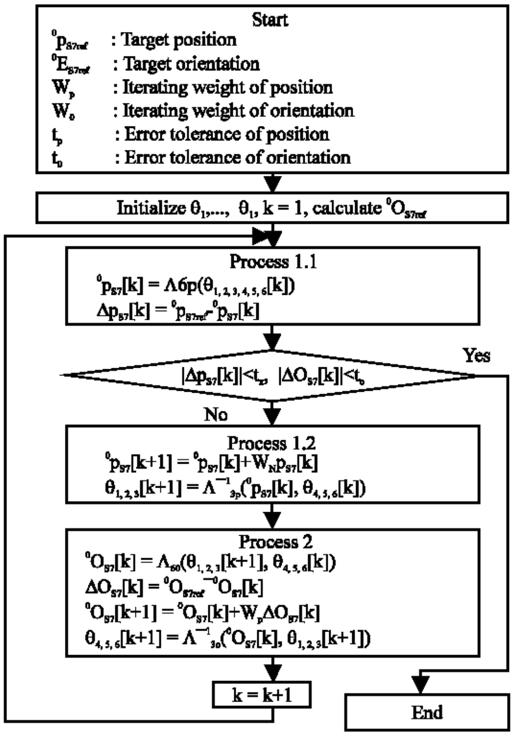

Fig. 3: Flow chart of the method of sequential retrieval

or fewer solutions are obtained for one target value. This can be verified using concrete numerical values if eight solutions are obtained, even if modification weights are used.

The length of each link is shown in Table 1.

The hand tip target position $^0P_{S7ref}$, modification weight W_p, W_o, allowable error e_p, e_o, initial joint angle $\theta_1[1],\ldots, \theta_6[1]$, target posture $^0O_{S7ZY\,Zref}$ are set as follows:

$$^0p_{S7ref} = \begin{bmatrix} -0.16 \\ -0.42 \\ 0.09 \end{bmatrix} \quad \begin{aligned} W_p &= 0.7 \\ W_0 &= 1 \\ e_p &= 0.001[m] \\ e_0 &= 0.01 \end{aligned} \quad \begin{aligned} \theta_1[1] &= 0 \\ \theta_2[1] &= 0.3183 \\ \theta_3[1] &= 0.3183 \\ \theta_4[1] &= 0 \\ \theta_5[1] &= 0.3183 \\ \theta_6[1] &= 0 \end{aligned}$$

$$^0O_{S7ZY\,Zref} = (257,178,262)[deg]$$

The allowable error of the position is 0.12% as compared with the whole arm length 0.835[m]. The initial joint angle is set to such a value avoiding peculiar posture in accordance with the Newton-Raphson method because a peculiar posture is taken when all angles are 0[rad] and because calculation of Jacobian determinant by Newton-Raphson method is not possible, as explained later.

Table 1: Link length

Link No.	1	2	3	4	5	6	7	Total
L_i[m]	0	0.09	0.28	0.2	0.085	0.1	0.08	0.835

Table 2: Angle values ([rad]) of obtained solutions

No.	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
1	2.7335	5.1566	4.9733	1.1615	2.9943	4.2970
2	2.7883	4.1525	1.3347	1.7191	2.7877	2.5952
3	5.9293	2.1287	4.9450	4.8632	2.7881	2.5894
4	5.8757	1.1270	1.3064	4.3039	2.9934	4.2941
5	2.6429	5.8709	4.3503	4.2479	0.2419	1.0979
6	2.5690	3.4925	1.9039	4.9814	0.5326	5.7971
7	5.7109	2.7906	4.3789	1.8398	0.5322	5.7968
8	5.7835	0.4098	4.9328	1.1061	0.2437	1.0959

Table 3: Target values and results

	$^0p_{S7ref}$[m]			$^0O_{S7ZYZref}$[deg]				W_p			
No.	x	y	z	Z	Y	Z	Fig	1	0.7	0.4	Rand
1	0.11	0.10	0.31	180	288	67	4	×	∘	∘	∘
2	-0.09	0.40	0.10	330	67	197	5	×	×	∘	∘
3	-0.02	0.03	0.32	332	287	47	6	∘	×	×	∘
4	-0.07	-0.06	0.30	47	258	74	7	×	×	×	×

Using the conditions described above, the solution of inverse kinematics is obtained using the Method of Sequential Retrieval. Consequently, solutions of eight types are obtained as shown in Table 2.

Application of numerical values (Convergence possibility to solutions by modification weight values): Depending on the W_p values, convergence to a solution might or might not occur. In this section, as one example of such an instance, cases in which $W_p = 1$, $W_p = 0.7$, $W_p = 0.4$ are described and compared.

The link length is the same as those described in the preceding section (Table 1). Other parameters are set as follows:

$$W_p = 1, 0.7, 0.4 \quad W_0 = 1 \quad e_p = 0.001[m] \quad e_0 = 0.01 \tag{1}$$

In this study, the modification weight of the posture is set to $W_o = 1$; errors of the posture and convergence will not be discussed here because, as noted from Fig. 3, if $W_o = 1$, then a convergence judgment is made without adjustment after the posture is adjusted to the target value. Therefore, error e_o of the posture becomes 0 at all times.

Under the conditions described above, four target values shown in the columns of $^0p_{S7ref}$ and $^0O_{S7ZY\,Zref}$ of Table 3 were compared. The results are given, respectively, in Fig. 4, 5 and 6. Transitions of positional errors are shown, where a bold solid line means $W_p = 1$, a thin solid line means $W_p = 0.7$ and a thin broken line means $W_p = 0.4$. Its explanation is omitted here. In this study, of eight solutions, only $N_{sol} = 1$ was selected and compared for ease of comparison. In addition,

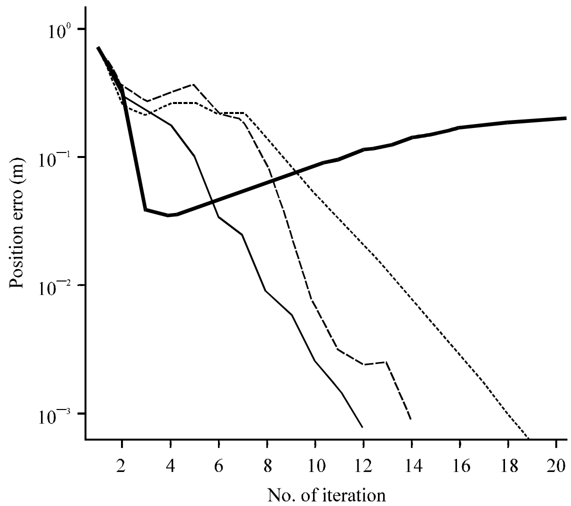

Fig. 4: Result of No.1

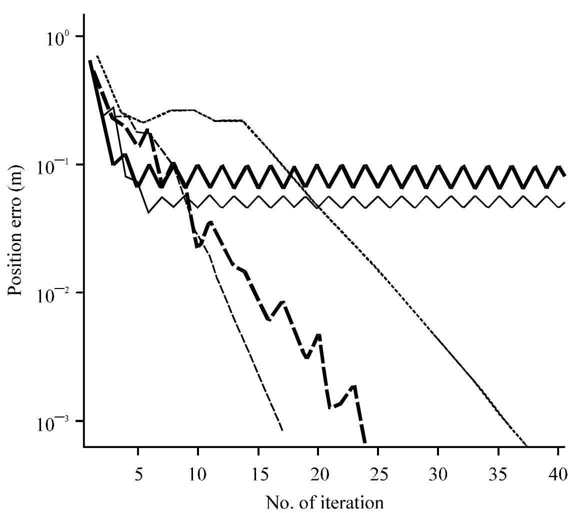

Fig. 5: Result of No.2

discontinuation of the graph on the way means that convergence is completed and that a solution is obtained at that point. Those with completed convergence are shown as [○]; those without completed convergence are shown as [×]. The results are summarized and presented in Table 3.

If attention is devoted to the example of No. 1 (Fig. 4), as noted from the graph, in the case of W_p, reduction in positional errors was not observed on the way. Although, repeated up to 500 times since then, further reduction in the error was not confirmed. Meanwhile, in cases of $W_p = 0.7$ and $W_p = 0.4$, the errors decreased nearly monotonously; convergence occurred at the 12th and 19th repetitions.

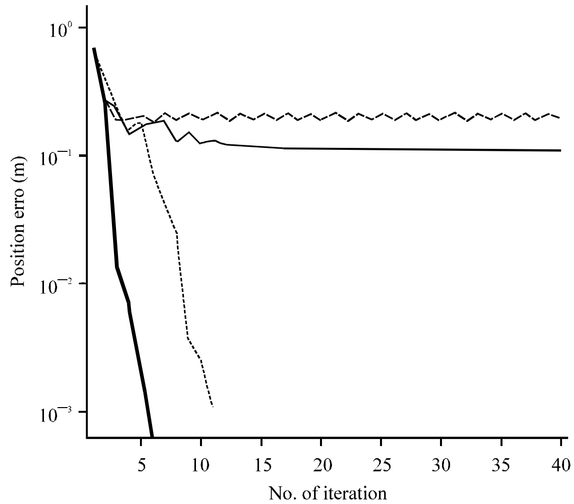

Fig. 6: Result of No.3

Regarding the case of No. 2 (Fig. 5), convergence did not occur for cases $W_p = 1$ and $W_p = 0.7$, although convergence occurred for case $W_p = 0.4$.

For the case of No. 3 (Fig. 6), convergence resulted only for $W_p = 1$.

It is known from the above that no such case exists in which convergence does not occur with a certain value but occurs with another value, depending on the value of modification weight W_p.

Modification weight values are changed at random: Convergence is expected to be possible for a target value with which convergence to a solution is not possible with constant modification weight if an optimum modification weight can be selected for the target value and initial value. However, it is difficult to find the optimum modification weight at the present stage. Therefore, a method for changing W_p value at random within a certain range for every group is investigated in this study.

Regarding the range of random numbers, because there is such a case, it is better to set the W_p value at a lower level, as is true of No. 2 of Table 3, the range was set wider and variation was made between 0.1 and 1. The obtained results are shown as bold broken lines of Fig. 4-6, and in [Rand] column of Table 3. As noted from the results thus obtained, when W_p is set randomly, convergence occurs in a case where a random weight is used, even for cases in which convergence to a solution does not occur with a certain weight.

However, for No. 4, (Fig. 7) did not result in convergence even when the method described herein was used. However, the same result was not obtained every

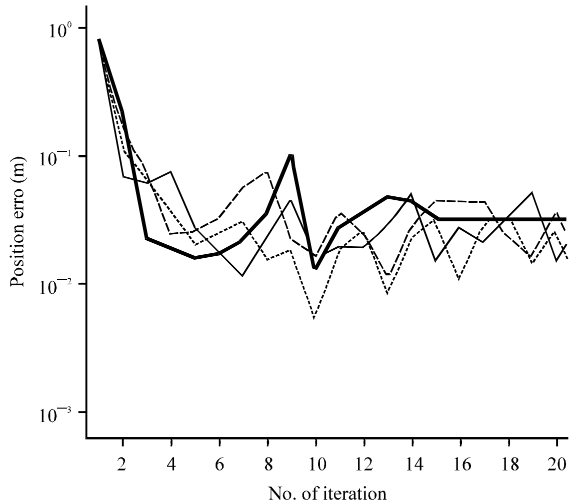

Fig. 7: Result of No.4

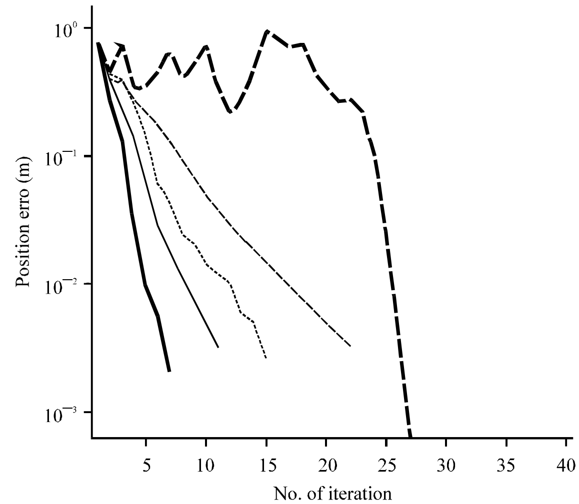

Fig. 8: Position error

time because of the use of random numbers. Convergence occurred rarely after it was applied to the same target value several times.

COMPARISON BETWEEN METHOD OF SEQUENTIAL RETRIEVAL AND NEWTON-RAPHSON METHOD

Conversion speeds when using the Newton-Raphson method, which is a typical method available at present and by the Method of Sequential Retrieval are compared to discuss the convergence speed of the Method of Sequential Retrieval.

Outline of newton-raphson method: First, the outline of Newton-Raphson method is explained. For this study, six variables of hand tip position coordinates ${}^{0}P_{S7}$ and hand tip posture ${}^{0}Q_{S7}$ were selected. The expression stated in Section 2.4 was used here for expression of the hand tip posture. If these variables are expressed collectively as the following:

$$\Omega = \begin{bmatrix} {}^{0}P_{S7} \\ {}^{0}O_{S7} \end{bmatrix} \qquad (2)$$

then modification of the joint angle using Newton-Raphson method is expressed as Eq. 37using the inverse-Jacobian determinant J^{-1} (6×6) and weight.

$$W_{NR}\ \theta[k+1] = \theta[k] + W_{NR} J^{-1}(\Omega_{ref} - \Omega[k]) \qquad (3)$$

The Jacobian determinant is a function of $\theta_1, ..., \theta_6$. Its detailed mathematical formulae are omitted here because of space limitations.

Comparison of convergence speeds: The link length, the initial joint angle, modification weight and allowable error are the same as those of Section 3.5. The hand tip target position ${}^{0}p_{S7ref}$ and target posture $Q_{S7Zy\ Zref}$ were set as described below.

Using the Newton-Raphson method, calculations were made for several types of modification weight W_{NR}. Of those, the fastest convergence occurred when $E_{NR} = 0.9$ and transitions of errors of the position in this case were compared. Results are presented in Fig. 8 where a bold chain line shows the Newton-Raphson method. It is understood from these results that the convergence speed of Method of Sequential Retrieval is faster than that of the Newton-Raphson method but slower than that of the efficient solution of inverse kinematics. Using the Newton-Raphson method, convergence begins but errors decrease slightly immediately after the start. Using the Method of Sequential Retrieval, reduction in errors was noted immediately after the start, thereafter moving to convergence.

For target values in many cases other than this example, the Method of Sequential Retrieval was confirmed as excellent in terms of convergence speed.

CONCLUSION

To obtain solutions of inverse kinematics of an RPY-type robot arm, the Method of Sequential Retrieval is proposed in this study. The following results were obtained:

- It was confirmed that a maximum of eight solutions are obtained using the Method of Sequential Retrieval

- Such an example demonstrates that a solution is not obtained with W_p at a certain value but a solution is obtained using another value
- By varying the W_p values randomly for every repetition, solutions are obtainable for more target values
- Rapidity of convergence using the Method of Sequential Retrieval is revealed through comparison of convergence speed using the Newton-Raphson method

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