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Reliability-optimization Design Based on Fuzzy Entropy for Cylinder Head Bolts

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Abstract: The reliability of cylinder head bolts in work will badly affect power, dynamic balance and economy performance of the diesel engine. The study put forward a kind of design method combining entropy theory with fuzzy design and optimal design, deduced and built the mathematical model of reliability optimal design for the cylinder head bolts of diesel engine. In this way, it can effectively solve the random and fuzzy problem in mechanical products design. Taking the diesel engine cylinder head bolts for an example which are applied in the mainstream diesel engine of freight transport internal combustion locomotive, through the Matlab programs, the numerical solution can be calculated and figures can be output. The result showed that the weight could be observably reduced when reliability more than 0.9998.

Key words: Fuzzy entropy, optimal design, reliability, the cylinder head bolts of diesel engine

INTRODUCTION

DF8B is one of the freight transport internal combustion locomotive which has the largest power, fastest speed and the latest technologies in our country at present. The cylinder head bolts bear the alternating stresses which produced by diesel engine type 16V280ZJA and the maximum breakout pressure can be as high as 13.24 MPa. The bolts are the key parts to ensure air tightness of the combustion chamber. It is a requirement that they must have preloaded force of 980 N•m when they are installed. They belong to the fasten bolt under changing axial load. Their reliability will seriously affect the machinery power and dynamic balance and economy performance of diesel engine.

The conventional designs take loads, material performances, parts sizes and other design parameters as constants. It is an important factor which Chinese diesel engine has big differences from that of foreign in power per weight and the strengthened level. In engineering design, it is very common that the structure failure and safety state are fuzzy (Jiang and Chen, 2003; Ma and Wang, 2011). Most of the gradual failures are difficult to divide into safety or failure state in the way of one size to fit all, by using clear boundaries (Xu and Shi, 2011; Moller *et al*., 2003). At this point, events are described by the fuzzy failure and safety states, i.e., events are analyzed by using the generalized failure probability which contains vague. However, the position and shape parameters of the fuzzy event's membership functions are mainly given on the basis of expert's statistics (Chen, 2009).

When the number of statistical sample is little, it is difficult to obtain a scientific conclusion. To introduce entropy theory can describe scientifically and quantitatively characteristic values of subordinate function (Luo *et al*., 2010). This study applied the theory and took diesel engine 16V280ZJA for an example, designed optimally and reliably the cylinder head bolts.

PROBABILITY OF FUZZY EVENTS AND THE ENTROPY OF FUZZY EVENTS

According to fuzzy theory, the probability of fuzzy events can be written as:

$$P_{\tilde{S}}(z) = \int \mu_{\tilde{S}}(z) f(z) dz \qquad (1)$$

Here, $\mu_{\tilde{S}}(z)$ is the membership functions of fuzzy event $\tilde{S}$ and f (z) is the probability density function of random variables z.

Entropy came from statistical thermodynamics. It is the description of the disorder about physical system. Now entropy is used to characterize the uncertainties of

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random events, i.e., it indicates the statistical characteristics of the random event as a whole. And it is a kind of numerical measure about the overall average uncertainty. For fuzzy random event, its entropy can indicate the degree of event's uncertainty. The equation is:

$$H_{\tilde{S}}(z) = -\int_{-\infty}^{+\infty} P_{\tilde{S}}(z)\log[P_{\tilde{S}}(z)]dz = -\int_{-\infty}^{+\infty} \mu_{\tilde{S}}(z)f(z)\log[\mu_{\tilde{S}}(z)f(z)]dz \tag{2}$$

Here, $H_{\tilde{S}}(z)$ value is related to logarithm. Its units of measurement have relations with logarithm base. Taking 2 as the logarithm base, the unit is the bit, 10 as base, the unit is the Hartley and e for base, the unit is the NAT (Li and Lu 2010).

ENTROPY OF FUZZY RELIABILITY OF STRENGTH ABOUT CYLINDER HEAD BOLTS OF ENGINE

Diesel engine's cylinder head bolts bear alternating load. Their failure forms are yield damage and fatigue fracture. Numbers of experiments showed that the bolt material's stress amplitude of yield limit and fatigue limit is normal distribution. At the moment, the occurrence probability of the fuzzy event $\tilde{s}$ (fuzzy variables are in safety state space, when the bolt's generalized strength is bigger than generalized stress) is:

$$P_{\tilde{S}} = P(s \le S) = P(z \le 0) = \int_{-\infty}^{0} \mu_{\tilde{S}}(z)f(z)dz \tag{3}$$

Here:

$$f(z) = \frac{1}{\sigma_z\sqrt{2\pi}}\exp[-(z-\mu_z)^2/(2\sigma_z^2)]$$

S is the generalized strength of bolts (the bolt's yield limit or fatigue limit); s is the generalized stress of bolts (the maximum tensile stress bolts or fatigue stress):

$$z = s - S(z \sim N(\mu_z, \sigma_z); \mu_z = \mu_s - \mu_S; \sigma_z = \sqrt{\sigma_s^2 + \sigma_S^2}; \mu_{\tilde{S}}(z)$$

is the membership function of $\tilde{s}$.

When z<<0, bolts certainly will not fail and at the moment, $\mu_{\tilde{S}}(z) = 1$. When z>>0, bolts certainly invalid, then $\mu_{\tilde{S}}(z) = 0$. When z changes from less than 0 to greater than 0 within a certain range, bolts are in the intermediate state, $0 < \mu_{\tilde{S}}(z) < 1$.

There are two kinds of membership functions can meet the above conditions. One is the down half normal distribution function and the other is linear function (Jun, 2010; Palanisamy and Selvan, 2009). And the down half normal distribution function is used most extensively. Its form is:

$$\mu_{\tilde{S}}(z) = \begin{cases} 1 & z < a \\ \exp[-(z-a)^2/b^2], & z \ge a \end{cases} \tag{4}$$

Here, a and b are distribution parameters. And a is the turning point of the downward trend for the membership function $\mu_{\tilde{S}}(z)$. b reflects how quickly the bolt's state transition from the normal working state to failure. Its value is mainly drawn by the expert's statistics. The introduction of the entropy concept, it can give a quantitative description in range for a and b, thus guide their value.

According to the definition of the fuzzy event's probability, the fuzzy reliability can be written as:

$$R_{\tilde{S}} = \int_{-\infty}^{+\infty} \mu_{\tilde{S}}(z)f(z)dz = \int_{-\infty}^{a} \frac{1}{\sigma_z\sqrt{2\pi}}\exp[-(z-\mu_z)^2/2\sigma_z^2]dz + \int_{a}^{+\infty} \exp[-(z-a)^2/b^2]\bullet\frac{1}{\sigma_z\sqrt{2\pi}}\exp[-(z-\mu_z)^2/2\sigma_z^2]dz \tag{5}$$

If $\tilde{Q}$ is used to describe the failure state space of fuzzy variables (when bolt's generalized strengths are less than the generalized stresses), then $\tilde{Q} = 1 - \tilde{s}$. In the same way, the bolt's fuzzy failure probability is:

$$R_{\tilde{Q}} = \int_{-\infty}^{+\infty} \mu_{\tilde{Q}}(z)f(z)dz = \int_{a}^{+\infty} \{1 - \exp[-(z-a)^2/b^2]\}\bullet\frac{1}{\sigma_z\sqrt{2\pi}}\exp[-(z-\mu_z)^2/2\sigma_z^2dz \tag{6}$$

By Eq. 5 and 6, if a and b are known, the fuzzy reliability and failure rate of bolts will can be obtained. Using entropy, a and b can be guided to select scientifically. According to Eq. 2 and 5, the entropy to describe the degree of uncertainty for the fuzzy reliability of bolts can be written as:

$$\begin{aligned} H_{\tilde{S}}(z) &= -\int_{-\infty}^{+\infty} \mu_{\tilde{S}}(z)f(z)\log[\mu_{\tilde{S}}(z)f(z)]dz \\ &= \int_{-\infty}^{a} \left\{\frac{1}{\sigma_z\sqrt{2\pi}}\exp[-(z-\mu_z)^2/2\sigma_z^2]\right\}\bullet \log\left\{\frac{1}{\sigma_z\sqrt{2\pi}}\exp[-(z-\mu_z)^2/2\sigma_z^2]\right\}dz + \\ &\int_{a}^{+\infty} \left\{\exp[-(z-a)^2/b^2]\bullet\frac{1}{\sigma_z\sqrt{2\pi}}\exp[-(z-\mu_z)^2/2\sigma_z^2]\right\}\bullet \log\left\{\exp[-(z-a)^2/b^2]\bullet\frac{1}{\sigma_z\sqrt{2\pi}}\exp[-(z-\mu_z)^2/2\sigma_z^2]\right\}dz \end{aligned} \tag{7}$$

Also, in accordance with Eq. 2 and 6, the entropy to describe the degree of uncertainty for bolt's fuzzy failure probability can be written as:

$$H_{\tilde{Q}}(z) = -\int_{-\infty}^{+\infty} \mu_{\tilde{Q}}(z) f(z) \log[\mu_{\tilde{Q}}(z) f(z)] dz = \int_a^{+\infty} \left\{ \{1 - \exp[-(z-a)^2 / b^2]\} \bullet \frac{1}{\sigma_z \sqrt{2\pi}} \exp[-(z-\mu_z)^2 / 2\sigma_z^2] \right\} \bullet \log \left\{ \{1 - \exp[-(z-a)^2 / b^2]\} \bullet \frac{1}{\sigma_z \sqrt{2\pi}} \exp[-(z-\mu_z)^2 / 2\sigma_z^2] \right\} dz \quad (8)$$

OPTIMIZATION DESIGN FOR FUZZY RELIABILITY OF CYLINDER HEAD BOLTS

Determine the decision variables: The connection design of cylinder head bolts groups need to be sure the bolt's nominal diameter d, length L and number n, in which the length L should be determined by the specific structure of connected components. So, d and n were determined as the decision variables of optimal design and is denoted as:

$$x = (d, n)^T = (x_1, x_2)^T \quad (9)$$

To determine the objective function: Taking the lightest bolts group's total weight as principle of optimal design, for a selected material, bolts density is constant, so the lightest total weight is equivalent to the smallest total volume. No considering the length, the objective function can be equivalent to:

$$F(x) = x_1^2 \bullet x_2 \to \min \quad (10)$$

Constraints: For cylinder head bolts group's design, it is necessary to ensure that the bolts meet fatigue and the fuzzy reliability of yield strength's requirements. Cylinder sealing requirements and the bolts meet convenient to disassemble. And the bolt's nominal diameter d, number n meet the upper and lower boundary conditions:

Fuzzy reliability's constraints of bolt's fatigue strength and yield strength: For cylinder head bolts group's design, it is necessary to ensure that the bolts meet fatigue and the fuzzy reliability of yield strength's requirements. Cylinder sealing requirements and the bolts meet convenient to disassemble. And the bolt's nominal diameter d, number n meet the upper and lower boundary conditions.

The mathematical model of the fuzzy reliability's constraints of the cylinder head bolts as follow:

$$R_i(X) - R_0 \geq 0 \qquad (i = 1, 2, \cdots) \quad (11)$$

Here: $R_i(X)$ is the fuzzy reliability of static strength, fatigue strength; and R_0 is the fuzzy reliability of the design requirements:

- **To calculate the mean of limit stress amplitude and standard deviation:** The mean of limit stress amplitude as follow:

$$\bar{\sigma}_{al} = \frac{\bar{\sigma}_{-1l} \varepsilon k_u k_m}{k_\sigma} \quad (12)$$

Here: $\bar{\sigma}_{-1l}$ is the mean of smooth specimen tensile fatigue strength; å is the size coefficient; k_m is manufacturing process factor; k_u is discontinuity coefficient of thread and $k_б$ is the concentration factor of thread stress

The standard deviation of limit stress amplitude as follows:

$$S_{\sigma_a} = C_{\sigma_{al}} \times \bar{\sigma}_{al} \quad (13)$$

Here: $C\sigma_{al}$ is variation coefficient of the limit stress amplitude

- **To calculate the mean of the work stress amplitude and standard deviation:** The bolt's mean of the work stress amplitude is:

$$\bar{\sigma}_a = \frac{2F \bullet (\frac{c_b}{c_b + c_m})}{\pi d_1^2} = \frac{2F_a \bullet (\frac{c_b}{c_b + c_m})}{n\pi d_1^2} = \frac{2F_a \bullet (\frac{c_b}{c_b + c_m})}{\pi x_1^2 x_2} \quad (14)$$

Here, F is single bolt work load, N; F_a is total work load of bolt, N:

$$\frac{c_b}{c_b + c_m}$$

is the relative stiffness of the bolt. d_1 is bolt trail, mm

The standard deviation of the bolt's working stress amplitude as follow:

$$S_{\sigma_a} = C_{\sigma_a} \times \bar{\sigma}_a \quad (15)$$

Here, $C_{\sigma a}$ is the variation coefficient of working stress amplitude

- **Fuzzy reliability constraints:** If it is known that the reliability of design requirements is R, the corresponding reliability index can be calculated. So the fuzzy reliability constraints about bolts' fatigue strength and yield strength of can be expressed as:

$$G_1(x) = (\bar{\sigma}_{-1l} - \bar{\sigma}_{al}) / \sqrt{S^2_{\sigma_{-1l}} + S^2_{\sigma_{al}}} - u_{R_1} \geq 0 \quad (16)$$

$$G_2(x) = (\bar{\sigma}_s - \bar{\sigma}_a) / \sqrt{S^2_{\sigma_s} + S^2_{\sigma_a}} - u_{R_2} \geq 0 \quad (17)$$

Here: $S_{\sigma\text{-}1l}$ is the standard deviation of tensile fatigue strength. $\bar{\sigma}_s$ is the standard deviation of yield strength u_{R1} and u_{R2} are, respectively the reliability index of anti-fatigue and yield

Connection tightness constraints: For important connections, such as tightness of the pressure vessel and other high demanding, in order to ensure uniform sealing pressure and to prevent leakage, the bolt spacing is not greater than t. When the working pressure $p \leq 1.6$ MPa, then $t \leq 7d$. When the working pressure $1.6 \leq p < 10$ MPa, then $t \leq 4.5$ day. And when the working pressure $10 \leq p < 26$ MPa, then $t \leq 4$ day. So, the constraints have:

$$G_3(x) = t - \frac{\pi \bar{D}_0}{x_2} \geq 0 \quad (18)$$

Here: $\bar{D}_0$ is the mean of bolts distributing circle diameter, mm.

Constraints of wrench space: To ensure adequate wrench space for disassembling bolts, bolts' spacing is not less than 3 day, i.e.:

$$G_4(x) = \frac{\pi \bar{D}_0}{x_2} - 3x_1 \geq 0 \quad (19)$$

Constraints of cylinder drilling bolts' holes: In order to make it convenient for hours of degrees and crossed distribution on the circumference of cylinder, the number of bolts on the same circumference should take an even number as 4, 6, 8 and so on:

$$G_5(x) = x_2 / 2 - \text{round}(x_2 / 2 + 0.5) \geq 0 \quad (20)$$

Decision variable's upper and lower boundary constraints: Based on experience, the best number of single-cylinder bolts should be not less than four and not more than 16. And the diameter of the bolts' shank is non-negative, then:

$$G_6(x) = x_2 - 4 \geq 0 \quad (21)$$

$$G_7(x) = 16 - x_2 \geq 0 \quad (22)$$

$$G_8(x) = x_1 \geq 0 \quad (23)$$

Combined with the above-mentioned formulas, you can draw the mathematical models of optimal design as:

$$\begin{aligned}
&\min \quad F(x) = x_1^2 \bullet x_2 \\
&\text{s.t.} \\
&G_1(x) = u_{R_1} - (\bar{\sigma}_{-1l} - \bar{\sigma}_{al}) / \sqrt{S^2_{\sigma_{-1l}} + S^2_{\sigma_{al}}} \leq 0 \\
&G_2(x) = u_{R_2} - (\bar{\sigma}_s - \bar{\sigma}_a) / \sqrt{S^2_{\sigma_s} + S^2_{\sigma_a}} \leq 0 \\
&G_3(x) = \frac{\pi \bar{D}_0}{x_2} - cx_1 \leq 0 \\
&G_4(x) = 3x_1 - \frac{\pi \bar{D}_0}{x_2} \leq 0 \\
&G_5(x) = \text{round}(x_2 / 2 + 0.5) - x_2 / 2 \leq 0 \\
&G_6(x) = 4 - x_2 \leq 0 \\
&G_7(x) = x_2 - 16 \leq 0 \\
&G_8(x) = -x_1 \leq 0
\end{aligned} \quad (24)$$

Here, c 's value should be selected in according to the working pressure.

DESIGN EXAMPLE

By using of the above-mentioned optimization model, taking cylinder head bolts of diesel engine 16V280ZJA for an instance, the reliability optimization design had been done based on fuzzy entropy. Using the Matlab computing and graphics capabilities, the target of the lightest total weight had been realized, when the bolt group's connections reliability R>0.9998.

In the model, the first and the second constraint, i.e., the constraints of fuzzy reliability for bolts' fatigue strength and yield strength, had relations with the membership function $\mu_{\tilde{s}}(z)$. By entropy, this study give quantitative description to the range of the distribution parameters a and b scientifically.

In this study, a<0, b>0. To limit artificially $a = -3\sigma_z \sim 0$ and $b = 020\ \mu_z$, it can be made that the curve $a - b - R_{\tilde{s}}$ and the curve $a - b - H_{\tilde{s}}$ by applying Matlab, as shown in Fig. 1 and 2. These curves are consecutively curves $R_{\tilde{s}}$ or curves $H_{\tilde{s}}$ which changed with the change of b, when $a = -0.1\sigma_z$, $a = -0.6\sigma_z$, $a = -\sigma_z$, $a = -1.5\sigma_z$, $a = -2\sigma_z$ and $a = -3\sigma_z$. It can be obtained $R_{\tilde{s}} = 0.7101$ and $H_{\tilde{s}} = 3.488$, when $a = -\sigma_z$ and $b = 2\mu_z$, as shown in Fig. 1 and 2.

FIGURES ANALYSIS

In information theory, entropy indicates the sizes of uncertainty degree of events. The greater the entropy, the greater the degree of uncertainty and thus that the smaller entropy is we want. Therefore, when fuzzy reliability is designed, the membership function values were selected through fuzzy entropy calculation. It can be known

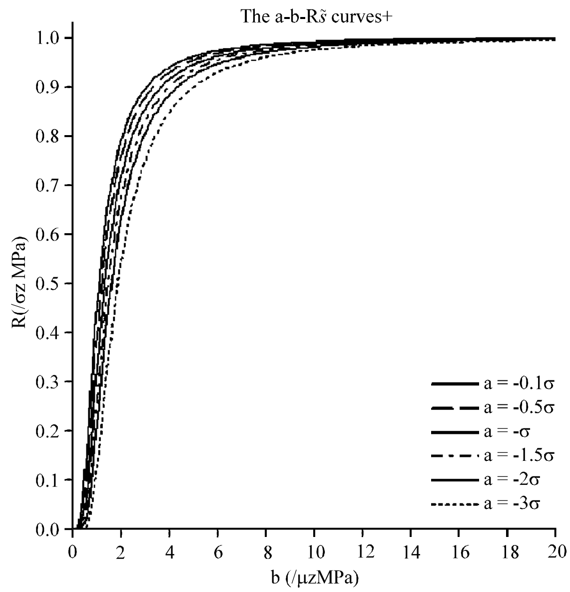

Fig. 1: $a-b-R_{\tilde{s}}$ curves

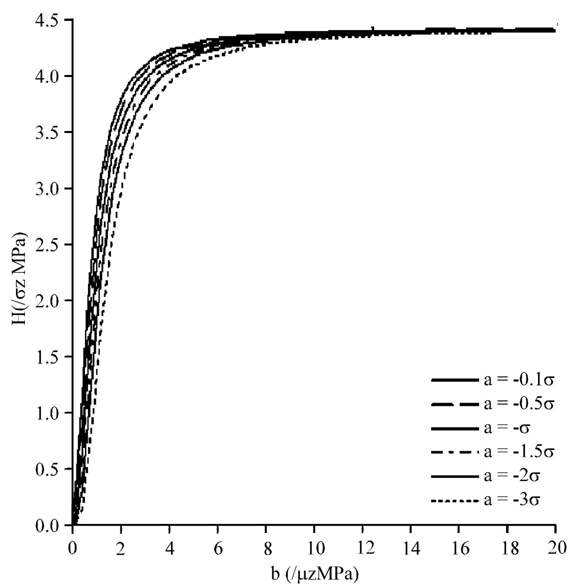

Fig. 2: $a-b-H_{\tilde{s}}$ curves

whether it is reasonable to estimate the selection of characteristic values in the fuzzy membership function. As the above calculation, taking $a = -0.1\sigma_z$ and $b = 10\ \mu_z$, the entropy is smaller and the higher likelihood, so we can believe that it had the more reasonable.

By Eq. 4, using Matlab's characteristic values of the membership function (especially fuzzy boundary), can be determined. When R>0.99995, then $a = -\sigma_z$ and when z = 0, then $b = 31.4265752\mu_z$.

Similarly, using Matlab, according to design requirement, the reliability index can be calculated, which is corresponding reliabilityR.

For example, when $a = -\sigma_z$, $b = 35\mu_z$ and $z = \mu_z/9$, they can be obtained as $u_{R1} = 3.89$, $u_{R2} = 3.89$.

Using Matlab, to solve reliability optimization design model, the optimal solution can be gotten as x* = (15.7080, 16.0000) and F (x*) = 3947.8. According to the table of bolts nominal diameter series, then d = 16, n = 16, F (x) = 4096.

The original design is d = 16, n = 16, F (x) = 9216, their values were reduced at present:

$$\frac{F_0 - F}{F_0} = \frac{48\times 4 - 16\div 16}{48\times 4} = 55.56\% \tag{25}$$

The optimization results are quite obvious. And in the original design, d = 48, n = 4, which could not to ensure that the cylinder bolts can meet to the maximum cylinder pressure of the outbreak as 13.24 MPa. And at the moment, the sealing requirements could not to meet about the pitch and it is easy to cause a cylinder leakage. So, that the economy of diesel engine power reduced.

CONCLUSIONS

Randomness and fuzziness are two inevitable uncertainty about the current mechanical design, so taking the entropy theory into the two kinds of uncertainty reliability of the design and to design the diesel engine cylinder bolts, it can be improved that the design level and performance of the diesel engine . Also it provided a reference for the product reliability design of similar mechanical.

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