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Studies on the Scattering Problem of Circular Cavity in Circular Area to In-plane Wave

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Abstract: The wave function expansion method, complex function method and the special function transformation technology were employed to investigate the elastic wave scattering problems of the circular cavity in the circular area under the action of the in-plane disturbance locating on the outer boundary of the circular area. The boundary of the circular area and the boundary of the inner circular cavity produce scattering waves, the superposition principle was used to give the total displacement potentials in the circular area, the boundary stress conditions and the Fourier series expansion were utilized to obtain the infinite algebraic equation group of the unknown coefficients, solve the equations and obtain the unknown coefficients, the variations and distributions of the dynamic tangential stress concentration coefficient on the circular cavity boundary about the dimensionless wave number were discussed and the numerical computation results of the example was given to illustrate the theoretical conclusions.

Key words: Circular area, circular cavity, inner-plane wave scattering, wave function expansion method, dynamic stress concentration coefficient

INTRODUCTION

Inner-plane wave scattering is an important investigation direction of the elastic wave scattering, it consists of the P-wave scattering and the SV-wave scattering and their combination and mostly the two kinds of waves are concomitant, this is the just reason of the difficulty in studying the inner-plane scattering problems in which there are straight line or plane boundary, so people generally use a circularity with large radius to approximate the straight boundary, combine the wave function expansion method to deal with the inner-plane wave scattering problems involving the circular cavity and circular liner or other heterogeneity (Zhong and Zhang, 2010; Shi *et al*., 2002; Huang *et al*., 2009). Qi *et al*. (2012) calculated the crustal velocity and depth in Anhui area with P wave travel-time by the inversion with two-layer model and three-layer model. Wang (2008) computed the time responses of two mountain apices and the free ground between them with three different incident angles of SV-waves, Liu (1997) used the curve coordinate and the complex function method and the orthogonal function to study the scattering boundary problems of canyon on the half space boundary to the steady P-waves. Yang (2010) utilized the Fourier-Bessel series expansion of wave functions to deduce the analytical solution to the two-dimensional scattering problem of incidental plane P-waves by circular-arc canyon topography with different depth-to-width ratio. but the method of approximating the straight boundary by a large circle easily always obtains an ill-posed linear algebraic equations and can not ensure the convergence preciseness, Jianwen *et al*. (2005) used the wave function expansion method to derive the analytical solution for scattering of plane P waves by a semi-cylindrical hill and verified the accuracy of truncation and discussed the effect of incident frequency and incident angle on the surface motion of the hill.

Referring to the references, we found that a lot more investigation results have been made on the inner-plane wave scattering problems in half space and complete space but little attentions have been paid to the studies about the inner-plane wave scattering problems in circular area but these problems frequently appear in the non-destructive detection and the design of the composite materials and the biomedical field. In this study, we will employ the complex function method and wave function expansion method and the special function addition method to study the scattering problem of a circular cavity in circular area to in-plane wave and give

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the theoretical solution, the method here has two important meanings, one is that it is a theoretical method which has fast convergent speed and perfect computation precision ; the second is that it can supply a thinking and method which can be directly used to deduce the Green function solution of similar problems and can be further used to solve those wave scattering problems of composite materials. the numerical results of the given example verify all the facts of the above-mentioned.

MODEL AND DEDUCTIONS

There is a circular area with radius R_0 as shown in Fig. 1, its Lame constants are λ, μ; its mass density is ρ and a coordinate system $x_0o_0y_0$ is set up. There is a circular cavity with radius $R(R<R_0)$ in this circular cavity, its boundary is denoted as Γ, the location complex number of its center o is $Z = r_0e^{i\beta}$ in the coordinate system $x_0o_0y_0$ and the local coordinate system xoy is also set up as shown in Fig. 1. Suppose that there is a distributing loading $[f(\theta), g(\theta)]e^{-i\omega t}$ acting on the outer boundary Γ_0 of the circular area. Here, ω is the disturbance circular frequency, $f(\theta)e^{i\omega t}$ and $g(\theta)e^{i\omega t}$ denotes the radial part and the peripheral part respectively.

For the inner-plane steady wave scattering problems, it is convenient to use the displacement potential $\Phi = \phi e^{i\omega t}$ (longitudinal wave potential) and $\Psi = \psi e^{i\omega t}$ (shearing wave potential) to study them, Here, ϕ, ψ satisfy the following Helmholtz equations:

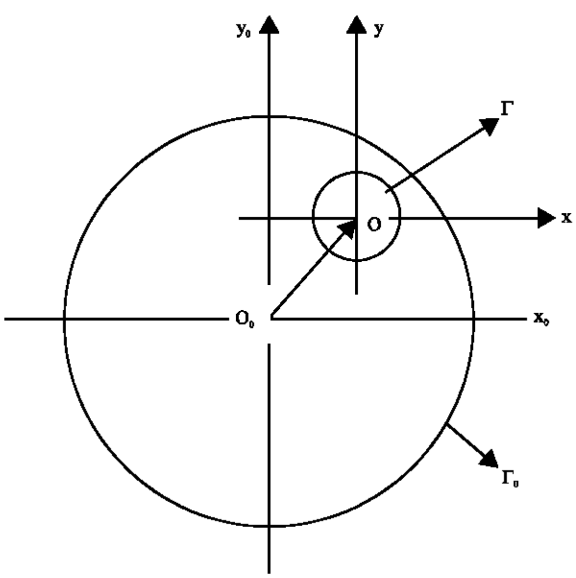

Fig. 1: In-plane dynamic mechanical response of the circular cavity in circular area

$$\nabla^2\phi + K_P^2\phi = 0,\ \nabla^2\psi + K_S^2\psi = 0 \tag{1}$$

Here, K_p, K_S is the wave number of the longitudinal wave and the shearing wave which can be denoted as $K_p = \omega/V_p$, $K_S = \omega/V_S$, respectively; V_p, V_S, is the longitudinal wave velocity and the shearing velocity, respectively which can be denoted as $V_P = [(\lambda+2\mu)\rho^{-1}]^{1/2}$ and $V_S = (\mu/\rho)^{1/2}$; Δ^2 is the Laplace operator; $\exp(-i\omega t)$ is the time factor, in the coming discussions, it will be omitted; i is the imaginary unit which is defined as $i^2 = -1$.

The differential relations in complex number form between the stress $\sigma_{rr}, \sigma_{r\theta}, \sigma_{\theta\theta}$ in the medium (the time factor $\exp(-i\omega t)$ omitted) and the potentials ϕ, ψ in the plane polar coordinate system can be expressed as:

$$\begin{cases} \sigma_{rr} - i\sigma_{r\theta} = 4\mu\Phi_{zz}\exp(2i\theta) - \Omega \\ \sigma_{rr} + i\sigma_{r\theta} = 4\mu\Psi_{\bar{z}\bar{z}}\exp(-2i\theta) - \Omega \\ \sigma_{\theta\theta} = -\Omega - 2\mu\Phi_{zz}e^{2i\theta} - 2\mu\Psi_{\bar{z}\bar{z}}e^{-2i\theta} \end{cases} \tag{2}$$

Here, $z = x+iy$ is the complex coordinate in the coordinate system xoy, $\bar{z} = x-iy$ is the corresponding conjugate complex number, $\Omega = (\lambda+\mu)K_P^2\phi$, $\Phi = \phi + i\psi$, $\Psi = \phi - i\psi$, $\Phi_{zz} = \partial^2\Phi/\partial z^2$, $\Psi_{\bar{z}\bar{z}} = \partial^2\Psi/\partial\bar{z}^2$.

Because of the excitation of the disturbance loading acting on the out boundary of the circular area, the boundary of the inner circular cavity will produce longitudinal waves and the shearing waves in the circular area, the corresponding potential functions are denoted as ϕ, ψ; Moreover, the will also produce the scattering longitudinal waves and shearing waves inside the circular area medium, the corresponding potentials denoted as ϕ_0, ψ_0 satisfy the two equations noted in equations (1) and take the following forms:

$$\phi_0(z_0,\bar{z}_0) = \sum_{n=-\infty}^{\infty} A_nJ_n(K_P|z_0|)(z_0/|z_0|)^n \tag{3}$$

$$\psi_0(z_0,\bar{z}_0) = \sum_{n=-\infty}^{\infty} B_nJ_n(K_S|z_0|)(z_0/|z_0|)^n \tag{4}$$

$$\phi(z,\bar{z}) = \sum_{n=-\infty}^{\infty} a_nH_n^{(1)}(K_P|z|)(z/|z|)^n \tag{5}$$

$$\psi(z,\bar{z}) = \sum_{n=-\infty}^{\infty} b_nH_n^{(1)}(K_S|z|)(z/|z|)^n \tag{6}$$

Here, $A_n, B_n, a_n, b_n (n = 0, \pm1, ...)$ are the unknown coefficients, $z_0 = x_0 + iy_0$ is the complex coordinate of medium point in coordinate system $x_0o_0y_0$, $\bar{z}_0 = x_0 - iy_0$ is its conjugate complex number.

The total wave fields $\phi^{(t)}, \psi^{(t)}$ can be obtained by the superposition as the following form:

$$\phi^{(t)} = \phi_0 + \phi,\ \psi^{(t)} = \psi_0 + \psi \tag{7}$$

The boundary conditions of the problem can be expressed as:

$$\sigma_{rr}\big|_{\Gamma_0} = f(\theta), \sigma_{r\theta}\big|_{\Gamma_0} = g(\theta) \tag{8}$$

$$\sigma_{rr}\big|_{\Gamma} = 0, \sigma_{r\theta}\big|_{\Gamma} = 0 \tag{9}$$

In order to utilize the boundary conditions Eq. 8 and 9, the special function addition formula should be choosen according to the different cases:

- When the boundary condition on the boundary Γ need be used, the special function addition formula takes the following form:

$$J_n(K_P|z_0|)(z_0/|z_0|)^n = \sum_{m=-\infty}^{\infty}(-1)^m e^{i(m+n)\beta} \times J_{n+m}(K_P|Z|)J_m(K_P|z|)(z/|z|)^{-m} \tag{10}$$

$$J_n(K_S|z_0|)(z_0/|z_0|)^n = \sum_{m=-\infty}^{\infty}(-1)^m e^{i(m+n)\beta} \times J_{n+m}(K_S|Z|)J_m(K_S|z|)(z/|z|)^{-m} \tag{11}$$

- When the boundary condition on the boundary Γ_0 need be used, the special function addition formula takes the following form:

$$H_n^{(1)}(K_P|z|)(z/|z|)^n = \sum_{m=-\infty}^{\infty} e^{-im\beta} \times J_m(K_P|Z|)H_{n+m}^{(1)}(K_P|z_0|)(z_0/|z_0|)^{m+n} \tag{12}$$

$$H_n^{(1)}(K_S|z|)(z/|z|)^n = \sum_{m=-\infty}^{\infty} e^{-im\beta} \times J_m(K_S|Z|)H_{n+m}^{(1)}(K_S|z_0|)(z_0/|z_0|)^{m+n} \tag{13}$$

Introduce Eq. 3~6 into 7 and 2, we can obtain the stresses in the medium and choose appropriate formulae from Eq. 11~13 and the boundary conditions (8-9) and the Fourier transformation are employed to give the following linear equations of the unknown coefficients $A_n, B_n, a_n, b_n (n = 0, \pm 1, ...)$:

$$\begin{aligned}&0.5K_P^2\mu\{A_q[J_{q-2}(K_PR_0) + J_{q+2}(K_PR_0) - 2(\lambda/\mu+1)J_q(K_PR_0)]\\&+\sum_{n=-\infty}^{\infty} a_n e^{-i(q-n)\beta} J_{q-n}(K_Pr_0)[H_{q-2}^{(1)}(K_PR_0) + H_{q+2}^{(1)}(K_PR_0)\\&-2(\lambda/\mu+1)H_q^{(1)}(K_PR_0)] + i(K_S/K_P)^2\sum_{n=-\infty}^{\infty} b_n e^{-i(q-n)\beta} J_{q-n}(K_Sr_0)\times\\&[H_{q-2}^{(1)}(K_SR_0) - H_{q+2}^{(1)}(K_SR_0)]\} = F_q\end{aligned} \tag{14}$$

$$\begin{aligned}&0.5K_P^2\mu\{A_q[J_{q+2}(K_PR_0) - J_{q-2}(K_PR_0)] + \sum_{n=-\infty}^{\infty} a_n e^{-i(q-n)\beta}\\&J_{q-n}(K_Pr_0)\times[H_{q+2}^{(1)}(K_PR_0) - H_{q-2}^{(1)}(K_PR_0)] - i(K_S/K_P)^2\\&B_q[J_{q+2}(K_SR_0) + J_{q-2}(K_SR_0)] - i(K_S/K_P)^2\sum_{n=-\infty}^{\infty} b_n e^{-i(q-n)\beta}\\&J_{q-n}(K_Sr_0)\times[H_{q+2}^{(1)}(K_SR_0) + H_{q-2}^{(1)}(K_SR_0)]\} = i^{-1}G_q\end{aligned} \tag{15}$$

$$\begin{aligned}&0.5K_P^2\mu\{\sum_{n=-\infty}^{\infty} A_n e^{i(n-q)\beta} J_{n-q}(K_Pr_0)[(-1)^{2-q}\times J_{2-q}(K_PR)\\&+(-1)^{-2-q}J_{-2-q}(K_PR) - 2(\lambda/\mu+1)(-1)^q J_{-q}(K_PR)]\\&+a_q[H_{q-2}^{(1)}(K_PR) + H_{q+2}^{(1)}(K_PR) - H_q^{(1)}(K_PR)]\\&+i(K_S/K_P)^2\sum_{n=-\infty}^{\infty} B_n e^{i(n-q)\beta} J_{n-q}(K_Sr_0)\times\\&[(-1)^{2-q}J_{2-q}(K_SR) - (-1)^{-2-q}J_{-2-q}(K_SR)]\\&+i(K_S/K_P)^2 b_q[H_{q-2}^{(1)}(K_SR) - H_{q+2}^{(1)}(K_SR)]\} = 0\end{aligned} \tag{16}$$

$$\begin{aligned}&0.5K_P^2\mu\{\sum_{n=-\infty}^{\infty} A_n e^{i(n-q)\beta} J_{n-q}(K_Pr_0)[(-1)^{-2-q}\times J_{-2-q}(K_PR)\\&-(-1)^{2-q}J_{2-q}(K_PR)] + a_q[H_{q+2}^{(1)}(K_PR) - H_{q-2}^{(1)}(K_PR)]\\&-i(K_S/K_P)^2\sum_{n=-\infty}^{\infty} B_n e^{i(n-q)\beta} J_{n-q}(K_Sr_0)\times\\&[(-1)^{-2-q}J_{-2-q}(K_SR) + (-1)^{2-q}J_{2-q}(K_SR)]\\&-i(K_S/K_P)^2 b_q[H_{q+2}^{(1)}(K_SR) + H_{q-2}^{(1)}(K_SR)]\} = 0\end{aligned} \tag{17}$$

Here, $F_q, G_q (q = 0, \pm 1, ...)$ are the Fourier expansion coefficients of the functions $f(\theta_0)$ $g(\theta_0)$, respectively.

Solve the Eq. 14~17 and obtain the unknown coefficients $A_n, B_n, , a_n, b_n, (n = 0, \pm 1, ...)$, we can determine the wave fields such as the displacement potential fields and the stress fields.

In this study, the given numerical computation example is about the variations of the dynamic stress concentration factor λ (denoted as DSCF) about the dimensionless wave number K_PR which can be defined as:

$$\gamma = \left|\sigma_{\theta\theta} / \max_{\theta\in[0,2\pi]}\sqrt{f^2(\theta) + g^2(\theta)}\right|\Big|_{\Gamma} \tag{18}$$

Here, $\sigma_{\theta\theta}$ can be determined from the third formula in the Eq. 2 or it can be determined by the following formula considering of the stress free conditions on boundary of the circular cavity:

$$\sigma_{\theta\theta} = -2K_P^2(\lambda+\mu)\phi^{(t)} \tag{19}$$

RESULTS OF EXAMPLE AND DISCUSSION

Suppose the known parameters of the numerical example given here $\lambda/\mu = 1.0\times10^{-6}$, $r_0 = 4(m)$, $\mu = 2.52\times10^9$

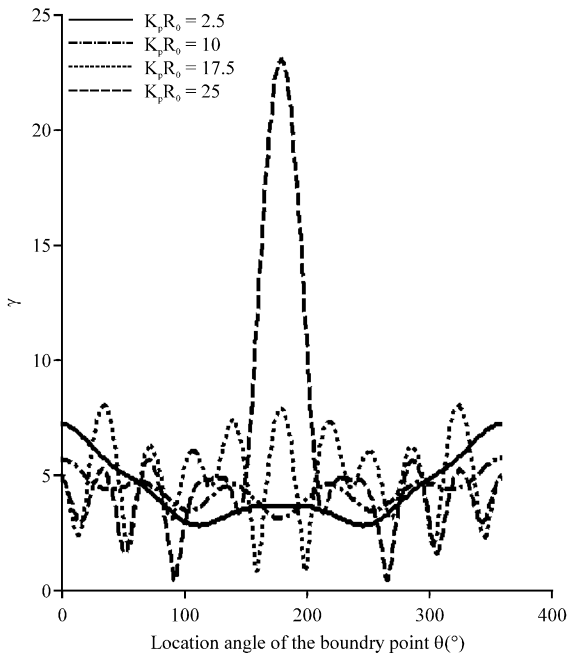

Fig. 2: Variation curve of the tangential dynamic stress concentration coefficient of the circular cavity boundary point

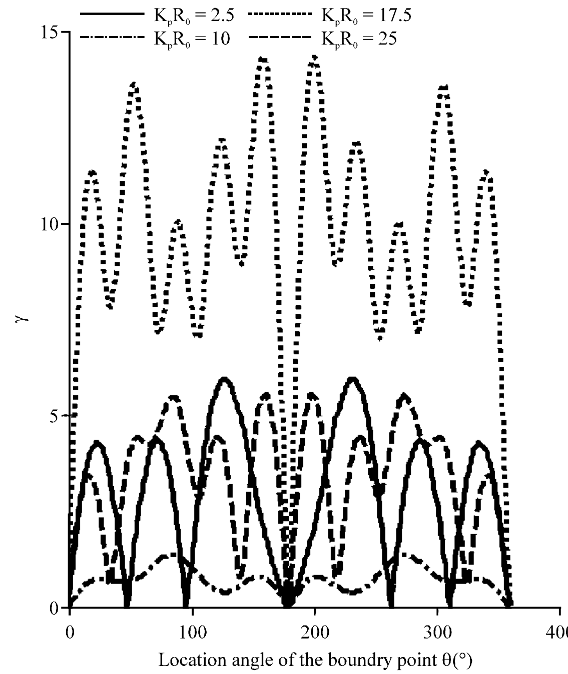

Fig. 3: Variation curve of the tangential dynamic stress concentration coefficient of the circular cavity boundary point

(N/m^2), $\rho = 1900$ (kg m^{-3}), $R_0 = 10$(m), $R = 5$(m), $K_P = 0.05 \sim 0.50$, $\beta = 0$, $f(\theta_0) = 1$ (N/m), $g(\theta_0) = 0$(N/m), $K_S/K_P = 1.5$, the computation results is shown in Fig. 2; only $f(\theta_0) = 0$(N/m), $g(\theta_0) = 1$(N/m) the other conditions not changed, the computation results is shown in Fig. 3; When $f(\theta_0) = 0$(N/m), $g(\theta_0) = 0$(N/m), and the other conditions are not changed, the computation results is shown in Fig. 4, here $0 \le \theta_0 \le 2\pi$.

According to the results shown in Fig.2 and 3, because of the symmetry of the loading distribution with the location of the circular cavity, the computation results of the dynamic stress concentration factor of the boundary point of the circular cavity also has the symmetry about the line of 0°-180° and some points have severe dynamic stress concentration. In Fig. 4, because the loading distribution has not symmetry, the profile of the dynamic stress concentration on the boundary of the circular cavity does not have the symmetry and the stress concentrations of some points are large numbers. In order to illustrate the effectiveness (including the convergent speed, computation precision) of the method here, MATLAB 7.0 software was also directly used to compute the above-mentioned example, the computation process and results show that the convergent speed of MATLAB 7.0 is clearly slower than that of the method introduced in this study and MATLAB 7.0 has poor computation precision.

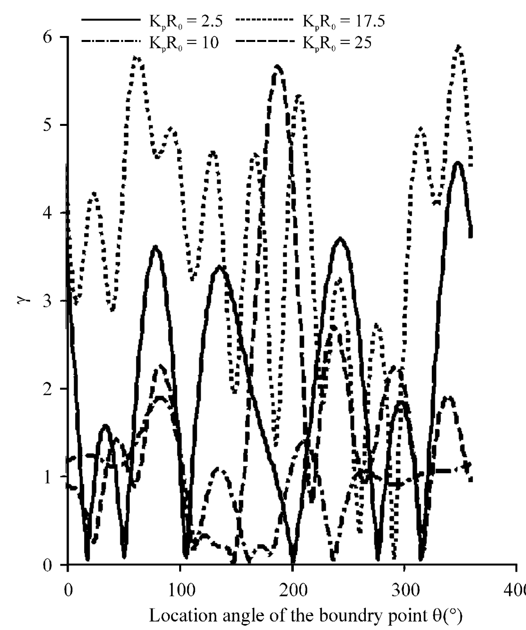

Fig. 4: Variation curve of the tangential dynamic stress concentration coefficient of the circular cavity boundary point

CONCLUSION

The wave function expansion method and the complex function method and the special function addition formula were employed to study the dynamic response problem of the circular cavity in the circular area to the in-plane disturbance on the boundary of the circular area, the variations of the dynamic hoop stress concentration factor on the boundary of the circular cavity about the dimensionless wave number were given, the given numerical example is only presented for the illustration about the theoretical conclusions. According to the computation results, the stress concentration on the boundary of the circular cavity under the in-plane disturbance loading on the boundary of the circular area are very fearful, it should be paid enough attention on the structure design. In addition, the method used here can also be employed to discuss the inner-plane scattering problems with the disturbance acting on the circular cavity boundary. Besides, when the disturbance load is linear impulse load, the theoretical conclusions given here are the Green functions of the corresponding problem.

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