



# Journal of Applied Sciences

ISSN 1812-5654

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## Accelerated Testing of Equipment Based on the Duane Model

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**Abstract:** In most cases, the accelerated life tests aim at only one failure mode and failure mechanism and are applied to circuit boards or unit-level products for the most part. As for complex equipment, it's difficult to choose reasonable accelerate stress and accurate model, which influences the research and application of accelerated testing on equipment-level products. We introduce a new method based on the theory of reliability growth testing. By using the Duane model, the results of step-down-stress accelerated life test were evaluated and the reliability under normal stress was extrapolated. Finally, the result of the proposed method was verified through simulation.

**Key words:** Accelerated testing, reliability growth theory, duane model

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### INTRODUCTION

With the progress of technology, large numbers of high reliability products emerges for people's increasing demand of quality and reliability. For example, typically a fighter's MTBF is 3-5 h, while the MTBF design targets are mostly 15-20 h for big passenger planes and large cargo planes being researched at present. The problem that reliability testing faces is how to assess the reliability of these highly reliable products through conventional tests. That is because traditional reliability tests based on the probability and statistics usually require lots of test samples or long test time, which is high cost, low cost-effectiveness and the time is not permitted. In fact, conventional reliability test methods cannot be achieved in engineering for most product of this kind. In this situation, accelerated testing technique (Chen *et al.*, 2002) is an effective way to shorten product development cycle and reduce development cost.

For complicated products, the failure factors and mechanism are complex considering that they contain a variety of components and materials. While single-stress accelerated test is difficult to reflect actual situation, multi-stress accelerated test method is restricted by the complexity of the accelerated models, accelerated life testing research of device-level products is greatly limited. Several ideas have also been put forward in domestic with regard to device-level accelerated life testing, among them, the transformation method (Li, 2004) is a more practical approach which is used in most cases. In abroad, the performance parameters degradation method (Zhen, 2006) and some related degradation models are proposed long since, while there are few researches in domestic.

Using the theory of reliability growth (Li, 2003) to evaluate accelerated life test is a kind of new idea, there is no relevant papers abroad and the feasibility of this approach in theory remains to be further studied.

The author holds the opinion that the device-level accelerated tests should be distinguished from accelerated life tests of components or unit-level products. Before entering the validation phase, equipment should have passed environmental function test and endurance test, corrective measures should be taken to eliminate process quality and simple design flaw and it should be ensured that wear-out failure is excluded within its lifetime, namely service life of the weak link should be long enough. Given that, accidental failure is the main kind of failure appeared before wear-out. The application of accelerated tests is to find out the relation between accidental failure and equipment reliability and determining the MTBF or maintenance-free period (maintenance interval) in a relatively short period of time.

In this method, basic principles of accelerated testing methods based on reliability growth theory were studied, the results of step-down-stress accelerated life tests were evaluated by adopting the Duane model, the reliability under normal stress was extrapolated using the method of accelerated testing and the effectiveness of the proposed method was verified through simulation.

### THE RELATIONSHIP BETWEEN MTBF AND ACCELERATED TESTING

**Analyzing product failure cause using the Interference model:** In practical engineering, both the product actual stress of using conditions and product limiting stress are

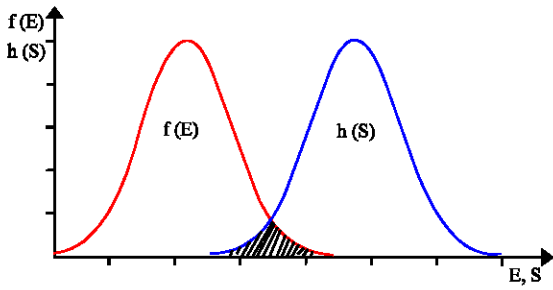


Fig. 1: Stress-intensity interference model

random variables, hence whether the product can work depends on the relationships between them. Products work well when the limit stress is greater than the operating stress, otherwise not. Both are expressed in a coordinate system in Fig. 1, where the horizontal axis shows the stress, the vertical axis indicates the probability density and functions  $f(E)$  and  $h(S)$  represent the probability density of environmental stress and working limit stress, respectively. Setting  $S$  as the product resistance to the environment and  $E$  as working stress, then only when  $S > E$  or  $S - E > 0$  can the product work normally.

Product failure occurs when environmental stress reaches or exceeds the ability the products can bear. At times when the mean value of  $S$  is greater and there is overlapping in their probability density distribution function, failure may happen in the shadow part of the two functions in diagram, for the operating limit stress may be less than the environmental stress in this area. Generally, products that are of low environmental resistance are more prone to failure.

Time variable, i.e., the change of strength and stress over time, is not taken into account in the analysis above, in that it is usually considered that most of the failures occurred before product longevity are accidental failures after experiencing environmental function test and endurance test, for problems exposed concerning process quality and product design are eliminated in those tests which assures that the service life of the weak links are long enough. As products' life approaching their limits, wear-out increases and the intensity distribution as a function of time drift to left, which lead to the increase in product failure rate as was shown in Fig. 2 (Carson, 1979).

**The relationship of  $\lambda_0$  and the overlapping area of the two distributions:**  $\lambda_0$  is used as the metric parameter of product reliability which was shown in Fig. 3. And  $\lambda_0$  is dominated by the overlapping area of the two probability distributions, the greater its area, the bigger the value of  $\lambda_0$ . The probability curve of products' resistance to

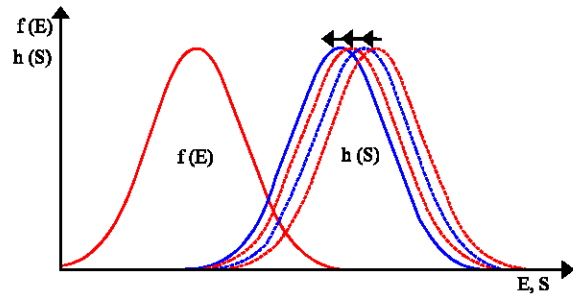


Fig. 2: Stress-intensity interference model-strength depletion drift

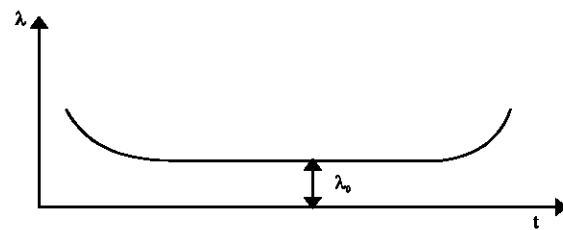


Fig. 3: Equipment reliability bathtub curve

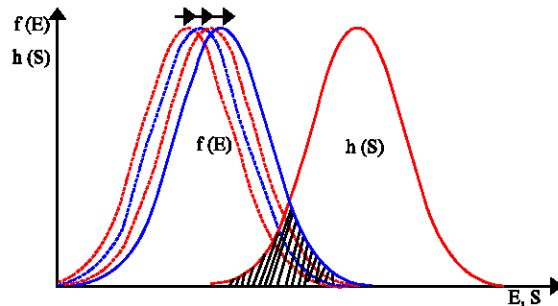


Fig. 4: Stress-intensity interference model-step stress

environment requires test results from large numbers of samples, which is usually hard to achieve. While the resistance tolerance against environmental stress of a product through a RET (Reliability Enhancement Testing) is just a point on the curve of product's capacity of resistance to environmental stress.

**The relationship between accelerated testing and product reliability parameters:** Generally there are three kinds of accelerated stress tests at present, constant stress accelerated test, step stress accelerated test and progressive stress accelerated test.

In a step stress accelerated test, gradually increasing stress was applied to test products with the time, which reduces the distance between mathematical expectation of function  $f(E)$  and  $h(S)$  and results in the increase in failure rate as can be discovered in Fig.4.

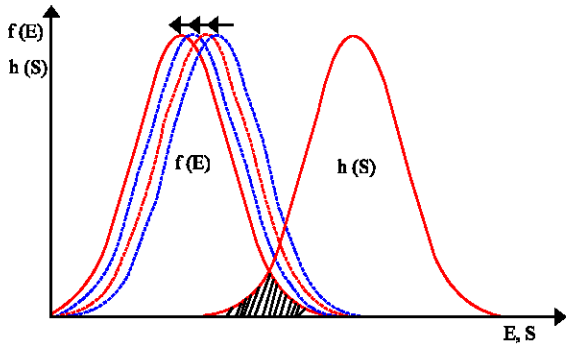


Fig. 5: Stress-intensity interference mode-step back stress

Step back stress test is carried out in reverse sequence of the above step stress test, that the highest stress level is employed at first, then the test stress gradually reduce. Apparently, failures occur easily at the beginning, but with the reduction of the stress level, the distance between mathematical expectation of function  $f(E)$  and  $h(S)$  increases and results in the decrease in failure rate as was showed in Fig. 5.

**METHOD OF RELIABILITY ACCELERATED TESTING BASED ON DUANE MODEL**

**Determining product MTBF utilizing reliability growth model:** Reliability growth test can improve product reliability by planned failures motivation, failure analysis, design improving and the verifying of improvement measures' effectiveness (Mei, 2003). Through reliability growth test,  $\lambda_0$  can be reduced and the reliability of the product can get improved. If the environment remains unchanged which means the environmental stress probability curve stay the same, then the reducing of  $\lambda_0$  actually means that the distance between the mathematical expectations of the two probability curve increases or that  $h(S)$  moves towards right.

As is shown in stress-intensity interference model, the interference region gradually increases when stress level rises, which means that the reliability under high stress level is lower than that under low stress level and that demonstrates the principles of accelerated testing. That is, first, obtaining the reliability under high stress levels in a short time by increasing the stress level, then applying the appropriate acceleration model to extrapolate product reliability under normal stress level (actual usage).

For the size of interference region indicates the failure rate  $\lambda_0$ . If we do not consider the absolute coordinates, results of stress-intensity interference model are consistent for step back stress test and reliability growth

test. Therefore, step back tests can be described in Duane model and then be evaluated.

Apparently, failures occur easily at the beginning because high-level stress is adopted first and failures will gradually reduce with the reduction of stress level. It is clear that step-back tests can get more failure information, however, how to determine the initial stress is a problem that should be solved at first.

We can learn from the method of HASS (Highly Accelerated Stress Screening) of using RET (Reliability Enhancement Test) as a pre-test, that is using a RET as a pre-test to get product stress limit and use 80% to 85% of its stress limit as the initial stress of step back stress test. ( For example, if the temperature limits of a certain product is  $\pm 100^\circ\text{C}$ , we use  $-85^\circ\text{C}$  as the starting stress for low-temperature step back test and use  $85^\circ\text{C}$  as the starting stress for high-temperature step back test). This test can be regarded of equal effect as a time terminated reliability growth test. Given the above, mature theories and analytical methods referred to reliability growth test can be put into use in step-back tests, particularly the Duane model which is widely applied in reliability growth.

**Formulation of modified duane model:** Product cumulative failure rate is defined as the ratio of cumulative failure number  $N(t)$  (from 0 to  $t$ ) and cumulative test time  $t$ , that is:

$$\lambda_z(t) = N(t)/t \tag{1}$$

In Duane model, it is assumed that the relationship between  $\lambda_z(t)$  and  $t$  can be represented by a straight line in double-logarithm coordinate, the mathematical expression is:

$$\lambda_z = \ln \alpha - m \ln t \tag{2}$$

where  $\alpha$  is the scale parameter ( $\alpha > 0$ ) and  $m$  is the growth rate ( $0 < m < 1$ ).

Transforming formula (3) into exponential form, we can get cumulative failure rate:

$$\lambda_z(t) = at^{-m} \tag{3}$$

Thus the relationship between  $N(t)$  and  $t$  is:

$$N(t) = at^{1-m} \tag{4}$$

And Instantaneous failure rate is:

$$\lambda(t) = \frac{dN(t)}{dt} = a(1-m)t^{-m} \tag{5}$$

Unlike reliability growth testing, both the scale parameter  $m$  and shape parameter  $m$  are variables when using Duane model to evaluate the results of step-back stress accelerated test. In order to better describe the relationship between the cumulative failure number and the cumulative failure time, a modified Duane model was put forward in this paper.

$$N(t) = \alpha t^b + c \tag{6}$$

The above function is a single-variable function including three undetermined coefficients  $a, b, c$ , take the partial derivatives of residual sum of squares with respect to  $a, b, c$  separately according to the least square method, we can obtain the sets of equations of the undetermined coefficients, which can be solved by half-separated method or the 0.618 method.

According to the least square method, we make every residual sum of squares partial derivative to  $a, b, c$ . After finishing, we can obtain the equations of solving undetermined coefficient. We can use the right method or 0.618 methods to solve equations. Numerical evaluation of the parameters is carried on by MATLAB programs in this paper.

Instantaneous failure rate can be determined from Eq. (5):

$$\lambda(t) = dN(t)/dt = \alpha b t^{b-1} \tag{7}$$

Let  $t_{s1}, t_{s2}, \dots, t_{sn}$  be the test time under each test stress,  $N_{s1}, N_{s2}, \dots, N_{sn}$  be the corresponding failure number, then:

$$\sum_{j=1}^i t_{s_j}$$

is the cumulative test time and:

$$\sum_{j=1}^i N_{s_j}$$

is the cumulative test number, we can get the instantaneous MTBF:

$$\hat{\theta} \left( \sum_{j=1}^i t_{s_j} \right) = \frac{\left( \sum_{j=1}^i t_{s_j} \right)^{1-\hat{b}}}{\hat{a}\hat{b}} \tag{8}$$

**Extrapolation of the reliability under normal stress utilizing ALT:** High temperature can speeds up the chemical reaction inside of products such as electronic

components, insulating materials and induce products to early fail. Then the following acceleration model was put forward by Arrhinus on the basis of mass data concerning chemical reactions of this kind (Nelson, 1990):

$$\xi = A e^{E/KT} \tag{9}$$

where is a certain kind of life characteristic like the median life or the average life;  $A$  is a constant ( $A > 0$ );  $E$  is the activation energy related to materials and its unit is  $v$ , electron volt;  $k$  is the Boltzmann constant which is  $8.617 \times 10^{-5} \text{ ev}/^\circ\text{C}$ , thus the unit of  $E/K$  is temperature,  $E/K$  is also called the activation temperature;  $T$  is absolute temperature, which equals centi grade degree plus 273.

In Arrhenius model, it is shown that the life characteristics fall exponentially as temperatures rise. Take logarithm of both sides, we can get:

$$\xi = d + e/t \tag{10}$$

where,  $d = \ln A$ ,  $e = E/T$  both of which are indeterminate coefficients. Therefore, it is indicated that the logarithm to base life characteristic is the linear function of reciprocal of temperature.

We can obtain  $\hat{d}, \hat{e}$  by substituting the MTBF or  $\theta_{T_i}(t_{s_i})$  under several designed stress conditions which are calculated on the basis of modified Duane model. Given all of that, the life characteristics under normal stress  $T_0$  can be extrapolated.

### SIMULATION EXAMPLES

Assume that a certain product obeys the exponential  $\theta_1 = 2000 \text{ h}$  distribution. Choose temperature as the accelerate stress and suppose  $T_1 = 85^\circ\text{C}$  in this study. (the initial stress  $T_1$  can be determined by the operating limit and destruction limit obtained by RET) We took  $2.5^\circ\text{C}$  as an equal interval and turn to the next temperature level whenever the first failure occurs. According to the above, we got the product simulation failure data as Table 1.

According to Table 2, the life characteristic  $\theta_1$  under normal stress ( $25^\circ\text{C}$ ) can be extrapolated by  $\theta_1$  under each accelerate level  $T_i$  (35, 40, 45 and  $50^\circ\text{C}$ ) and substitute the result into formulation (10) we can get  $d = -31.12$ ;  $e = 11540$ .

And  $\theta_1$  under normal stress ( $25^\circ\text{C}$ ) is:

The result of this approach is quite consistent with the actual MTBF which is 2000 h.

What is presented is just a simulation example, in practical application the initial stress  $S_1$  should be as high

**Table 1: Cumulative failure data of a certain equipment**

$\sum_{j=1}^i N_{s_j}$	T (K)	$T_s$ (h)	$\sum_{j=1}^i t_{s_j}$ (h)
1	358	4.4	4.4
2	355.5	6.3	10.7
3	353	5.6	16.4
4	350.5	6.1	22.5
5	348	11.4	33.9
6	345.5	12.5	46.4
7	343	13.2	59.6
8	340.5	19.4	79
9	338	15	94
10	335.5	38.9	132.9
11	333	32.9	165.8
12	330.5	45.4	211.2
13	328	86	297.2
14	325.5	71.9	369.1
15	323	104.7	473.8
16	320.5	133.9	607.7
17	318	258.5	866.2
18	315.5	261.1	1127.3
19	313	266.3	1393.6
20	310.5	349.6	1743.2
21	308	810.6	2553.8

**Table2 Product MTBF under different temperatures**

Stress ( $T_i$ )	$a_i$	$b_i$	$c_i$	$t_{bi}$	$\theta_{T_i}(t_{bi})$ (h)
35□	23.66	0.09243	-27.22	473.8	566
40□	17.01	0.1162	-20.12	866.2	304
45□	11.84	0.1474	-14.5	1393.6	183
50□	7.321	0.1972	-9.373	2553.8	97

as possible if the failure mechanism has not been changed and we can get better acceleration by utilizing a larger step length or increasing the low stress level  $S_n$ .

**CONCLUSIONS**

In this study we draw conclusions as follows:

- The method that is proposed in this paper is only applicable to accelerated tests which is carried out to assess and evaluate the reliability problems concerning occasional failures
- In this study the step-back testing is estimated on the basis of the modified Duane model and the reliability under normal stress is extrapolated utilizing ALT

- From the simulation data, we can see that the result of this approach is quite consistent with the actual MTBF. Therefore, the accelerated testing approach based on Duane model provides a fresh idea for device-level accelerated testing.
- It is observed that failure data determines the evaluation accuracy of this method and the application of this approach is limited when there is no failure data or the data is insufficient

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