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Hermite Surface Fitting Base on Normal Vector Constrained

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Abstract: Given a polyhedron mesh, the methods and theories to problems related to normal controlled surface fitting are still not enough and suffer from similar defects. We propose a new technique for fitting surfaces from 3D data points and their associated normal vectors. Given vertices of surface and their normal vectors constraints, this study presents some schemes to construct G^2 continuous Hermite interpolation surfaces. Firstly, combined three tangent vector equation, methods to calculate tangent vector are presented. Secondly, due to the length of two endpoint tangent vectors in Hermite interpolation directly affects the shape of curve, even may produce sharp points and focal points, this study adjusted the curve surface shape by controlling the input coefficient and obtained the scope of coefficient by experiment. This study also presents a method to reconstruct some common surfaces accurately, such as spheres, columns and cones. In the end the error grid was analyzed between the experimental results and the original surfaces.

Key words: Hermite surface fitting, Normal vector constrained, three tangent vector equation

INTRODUCTION

Curved surface fitting problem is one of the basic and extensive problems in CAGD and the free curved surface fitting is relatively more difficult which has been widely research (Shen and Shi, 2013). In practical problems it may not only require interpolation curved surface through some known points, but also in some points require satisfies the requirement of the given tangent vector and curvature, etc., Parametric surface (such as Hermite interpolation curved surface) has many advantages, such as geometric invariance. Generally Hermite surface interpolation formula is through four endpoint coordinates, tangent vector direction and two parameters of mixed tangent vector at each point to fitting of a curved surface, the boundary is determined by the boundary only two endpoint, mixed tangent vector decided to internal surface shape. Recently some surface (curve) reconstruction introduces vector (tangent vector) and curvature characteristics such as control. Normal vector control method in the design and reconstruction of surface modifications of the control surface than using b-spline representation of the control polygon has more direct visual. Under the condition of only getting the coordinates and the normal vector of each point, it is usually not able to fit a satisfactory surface direct use of Hermite surface interpolation formula. For example, visual monitoring through the processing of shooting video to calculate the coordinates and the normal vector of marking points which distribute in underground tunnel walls. In this study, it determines all the necessary conditions of fitting Hermite surface through the normal vector and then fit out a good surface. Quality standards for surface fitting is not only concerned about fitting surface through the coordinate points, tangent vector (normal vector) satisfies the condition, but also to ensure the surface with the physical proximity and to make the fitting curves and surfaces with good shape (Zhang, 2009). In this study, aiming at this problem, based on general Hermite surface is put forward a solution method of vector constraints Hermite surface fitting method (Zhu *et al*., 2006; Li *et al*., 2013). Given vertices of surface and their normal vectors constraints, this study presents some schemes to construct G^2 continuous Hermite interpolation surfaces.

CUBIC HERMITE INTERPOLATION

In the parameter domain $[p_0, p_1]$, constructing Hermite curve segments meets condition:

$$H_3(0) = p_0,\ H_3(1) = p_1$$

$$H_3'(0) = p_0',\ H_3'(1) = p_1'$$

Constructed Hermite curve by these four conditions can be expressed as (1):

$$H_3(x) = p_0\alpha_0(x) + p_1\alpha_1(x) + p_0'\beta_0(x) + p_1'\beta_1(x)$$

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Among them: $x \in [0,1]$, $\alpha_0(x) = (1+2x)(x-1)^2$, $\alpha_1(x) = [1-2(x-1)]x^2$, $\beta_1(x) = (x-1)x^2$ is Hermite basis function, the matrix expression form of Hermite curve is:

$$p(u) = [\alpha_0(u), \alpha_1(u), \beta_0(u), \beta_1(u)] \begin{bmatrix} p_0 \\ p_1 \\ p_0' \\ p_1' \end{bmatrix} \quad (2)$$

Hermite curve has geometrical invariability which is commonly used in the design of geometric method (Ma and Xu, 2013). Using the principle of the tensor product expands the Hermite curves into one pair of cubic Hermite surface. In the parameter domain [0, 1]×[0, 1], the four corner point coordinates is denoted by p(i, j) = f(xi, yi).The direction vector of U is denoted by p_u (i, j) = f'_u (x_i, y_i), i = 0, 1, j = 0, 1. The direction vector of v is denoted by p_v (i, j) = f'_v (x_i, y_i), i = 0, 1, j = 0, 1. The mixed tangent vector of the four corners is denoted by $p_{u,v}$ (i, j) = f''_{uv} (x_i, y_i), i = 0, 1, j = 0, 1. Using these known information to write the expression of Hermite surface:

Denoted:

$$R = [F_0(u), F_1(u), G_0(u), G_1(u)]$$

$$T = [F_0(v), F_1(v), G_0(v), G_1(v)]$$

$$M = \begin{bmatrix} p(0,0) & p(0,1) & p_v(0,0) & p_v(0,1) \\ p(1,0) & p(1,1) & p_v(1,0) & p_v(1,1) \\ p_u(0,0) & p_u(0,1) & p_{uv}(0,0) & p_{uv}(0,1) \\ p_u(1,0) & p_u(1,1) & p_{uv}(0,0) & p_{uv}(1,1) \end{bmatrix}$$

Get:

$$p(u,v) = R \cdot M \cdot T \quad (3)$$

Among them $F_0(t) = 1-3t^2+2t^3$, $F_1(t) = 3t^2-2t^3$, $G_0(t) = t(1-t)^2$, $G_1(t) = -t^2(1-t)$ are basis functions, t can be valued u, v, $u \in [0,1], v \in [0,1]$.

DETERMINE THE TANGENT VECTOR AND THE EXISTEN-CE OF SOLUTION BY USING THREE TANGENT VECTOR EQUATION

By the conditions showed in formula (2), fitting a Hermite surface also lack the necessary direction tangent vector and the hybrid vector, but based on the conditions of vector and three tangent vector equation can only determine the accurate value of the tangent vector by the method of limiting parameter values. Here are introductions of the three tangent vector equation (Zhu, 2000).

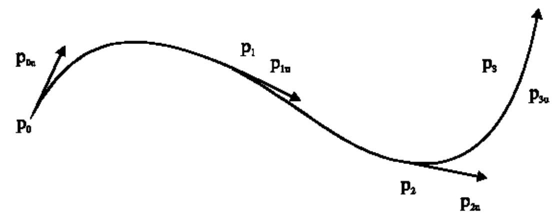

Fig. 1: A planar Hermite curve based on four points

Figure 1 is known as coordinates of four points: p_0, p_1, p_2, p_3 and known the condition of normal vector at each point: n_0, m_1, n_2, n_3, it each point: p_{0u}, p_{1u}, p_{2u}, p_{3u}.

Hypothetical, the curve is $_{c2}$ continuous which requires reached the same second derivative function at the two points of p_1, p_2 (connection point between the curve segments):

$$P_i''(1) = P_{i+1}''(0) \qquad i = 1, 2 \quad (4)$$

Among them, P_i (x), i = 1, 2, 3 represent three sections of Hermite curve segment. If give tangent vectors of boundary, then by the formula (1) and formula (3) can be obtained (Shi, 2001):

$$\begin{bmatrix} 1 & & & \\ 1 & 4 & 1 & \\ & 1 & 4 & 1 \\ & & & 1 \end{bmatrix} \begin{bmatrix} p_{0u} \\ p_{1u} \\ p_{2u} \\ p_{3u} \end{bmatrix} = \begin{bmatrix} p_{0u} \\ 3(p_2 - p_0) \\ 3(p_3 - p_1) \\ p_{3u} \end{bmatrix} \quad (5)$$

Because of space curve tangent vector at each point must be perpendicular to the normal vector at that point. So it has:

$$n_i \cdot p_{iu} = 0 \qquad i = 0, 1, 2, 3 \quad (6)$$

Obviously, p_{1u}, p_{2u} can be linear representation by using p_{0u}, p_{3u} since the formula of (4).It is only necessary to determine p_{0u}, p_{3u} and also needed to determine the six variables which are p_{0u} (i), p_{3u} (i), i = 1, 2, 3 in the three-dimensional space. The result is infinite which works out six unknown parameters by using four linear equations. However, by controlling the value of $_{p0u\,(1),\,p3u\,(1)}$ represents the only other variables and each solution can make fitting out of the surface which can meet normal vector through the coordinates of points. But the definite curved shape is below requirements. So only through the control of p_{0u} (1), p_{3u} (1) could uniquely determine a $_{c2}$ continuous Hermite curve to make the shape of curve reasonable. A Hermite curve introduced in front is determined by the length and the direction of tangent vector at the two end points. The results from the equation of three tangent vectors can directly control the length of tangent vector direction in the two end points through controlling the value of p_{0u} (1), p_{3u} (1) to achieve the adjustment curve shape, approaching it with real curves unlimited.

Similarly, the theory of three tangent vector equation on the curve can be extended to the process of solving which knows the spatial coordinates and the direction of tangent vector at each point in the Hermite surface and the mixed tangent.

Computing the direction of tangent vector and mixed tangent for a $_{4\times4}$ grid points in the three tangent vector equation, follows these steps:

- **Step 1:** Along with the direction of v, the partial derivation of the direction of at each point in every Hermite curve is respectively calculated by control p_{i0v} (1), p_{i3v}(1), i = 1, 2, 3; where it forms eight control parameters which is p_{i0v} (1), p_{i3v}(1), i = 1, 2, 3 that uniquely identify the partial derivative of the direction of v at each point
- **Step 2:** Along with the direction of u, the partial derivation of the direction of u at each point in every Hermite curve is respectively calculated by control p_{i0u}(1), p_{i3u}(1), i = 1, 2, 3, where it forms eight control parameter is p_{i0u}(1), p_{i3u}(1), i = 1, 2, 3 that uniquely identify the partial derivative of the direction of u at each point
- **Step 3:** First along with u to the endpoint of p_{i0v}, p_{i3v}, i = 0, 3 as a "point coordinates" and through a given mixture tangent vector of two endpoints as the partial derivative of endpoint along u, calculate the other point of tangent vector after calculate the partial derivative of the direction u, v.The reason why did not determine the mixed tangent vector like the direction tangent vector is the action of the normal vector will disappear in the process of determining the mixed tangent vector. P_{i1uv}, p_{i2uv}, i = 0, 3 is respectively calculated by the three tangent vector equation which is the entire mixed tangent vector on the two boundaries of u = 0, u = 3
- **Step 4:** p_{i0u}, p_{i1u}, p_{i2u}, p_{i3u}, i = 0, 3 as the "point coordinates", along with the direction of $_{v}$, by controlling the input p_{i0uv}, p_{i1uv}, p_{i2uv}, p_{i3uv}, i = 0, 3, p_{i0v}, p_{i1v}, p_{i2v}, p_{i3v}, i = 0, 2 is calculated by the three tangent vector equation, that is all the mixed tangent vector on the two boundary of u = 1, u = 2
- **Step 5:** The tangent vector of the two direction at every point in the 4×4 grid is all determined by the first and second steps and then is determined the mixed tangent vector of the two direction at every point in the 4×4 grid though the third and fourth steps, since the mesh is form the nine small quadrilaterals and the coordinates, direction tangent vector and mixed tangent of quadrilateral four corner points have been identified, so the 9 small surfaces can be fitted by using the Hermite interpolation surface in the second chapter, according to the location of coordinates spliced the nine small surface together. As the construction process of Hermite curve is under the condition of continuous of C2, so that every border is C2 continuous. So that can reach C2 continuous structure of curved surface (Shi, 2001).

EXPERIMENTAL RESULTS

For the theory was given in section I and section II, we repeated verification to find the exact parameter ranges respectively in spherical, cylindrical and conical.

Experiment 1: Fitting out effect is better repeatedly experiment when the experience value of the controlling variable in the process of determine the tangent vector in the second section, for the sphere which radius are 3 and the centered at the origin, $x^2+y^2+z^2 = 9$, among them, for the sixteen points and the normal vector coordinates on those points which on the $x, y \in \{-2, -1, 1, 2\}$ .The actual sphere and the surface of fitting out from discrete points which on the $x, y \in [-2, 2]$ all is given in below:

Experiment 2: Get the 24 data points and the normal vector on these points from the cylinder:

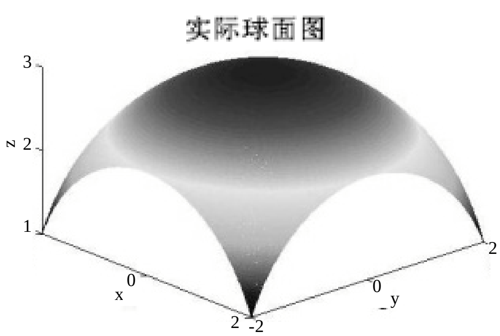

Fig. 2: Actual spherical

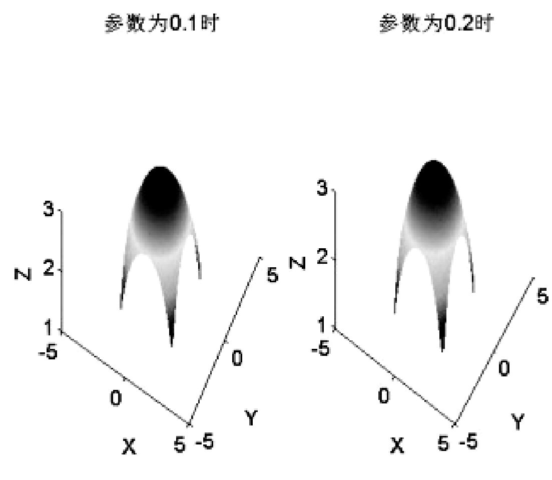

Fig. 3: The fitting spherical under the different parameters

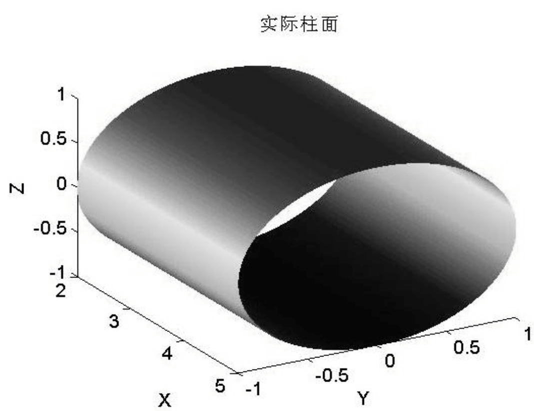

Fig. 4: Actual cylinder

$$\begin{cases} x = t \\ y = \sin(theta), \\ z = \cos(theta) \end{cases} \quad \begin{matrix} t \in [2,5] \\ theta \in [-pi/2, 3pi/2] \end{matrix}$$

The renderings as shown:

Experiment 3: Get the 24 data points and the normal vector on these points from the cone which removal the top:

$$\begin{cases} x = t \\ y = t\sin(theta), \\ z = t\cos(theta) \end{cases} \quad \begin{matrix} t \in [2,5] \\ theta \in [-pi/2, 3pi/2] \end{matrix}$$

The renderings as shown:

Table 1: The error analysis of nine small surfaces

	First block	Second block	Third block	Fourth block
error	0.0651	0.0942	0.5130	0.0983
Fifth block	Sixth block	Seventh block	Eighth block	Ninth block
1.3535e-04	0.0997	0.5254	0.0994	0.0667

Table 2: The average error of between the fitting surfaces and the actual surfaces

	sphere	Cylinder	Cone
Average error	0.1735	0.0932	0.1342

Table 3: The statistics number of coordinate points for different M in the different surface

	0.5	0.1
Parameter of the sphere is 0.1	102340	84532
Parameter of the cylinders is 0.1	115203	73289
Parameter of the cone is 0.1	120785	69034

Average error: From the fifth step of constructed step in second chapter, we obtained the average error from the camper the results of complete surfaces which composed of nine small surfaces with the actual surfaces.

The average error of sphere, cylinder and cone, (As shown in Table 2.

In the Table 1 and 2, we take the equation 7 as the standard of determine the magnitude of the average error, (Assuming an image with M*N points, the xy domain of two image whether same is judging, we respectively use the z and Z to represent the ordinate of each point on the two image) (Zhang *et al*., 2011).

$$\varepsilon = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}(z(i,j) - Z(i,j))^2 \tag{7}$$

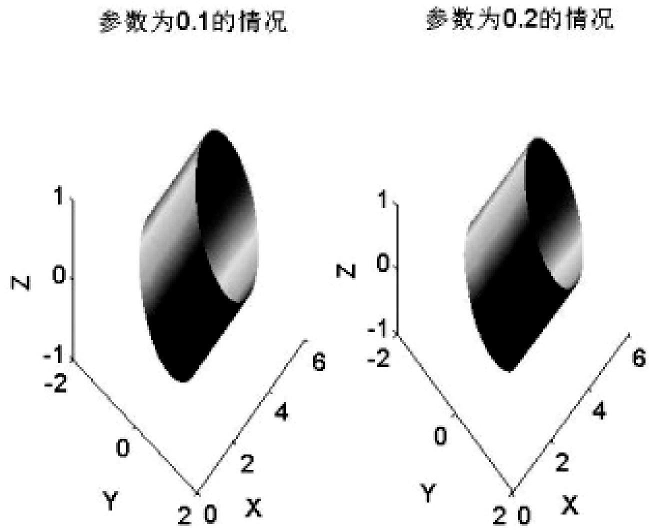

Fig. 5: The fitting cylinder under the different parameters

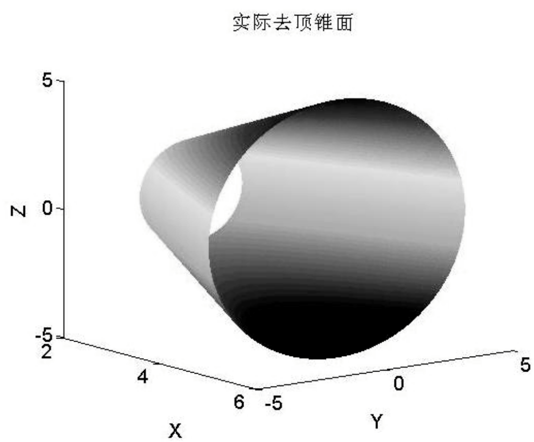

Fig. 6: Actual cone

The pixel point statistics under the limit of maximum error F (x, y) is original surface, G (x, y) is fitting surface, in the x_i, y_i meet the requirements:

$$|F(x_i, y_j) - G(x_i, y_j)| \leq M \tag{8}$$

For spherical, cylindrical and cone were respectively taken M = 0.5 and M = 0.1, then statistics the number of coordinate points which meet the Eq. 8.

When programming in MATLAB, drawing each surface according to the 401×401 coordinates, as shown in the above figure is the number of coordinates that meet the requirements in each surfaces when M = 0.5 and M = 0.1.We can easy to see the difference is very small between the vast majority of

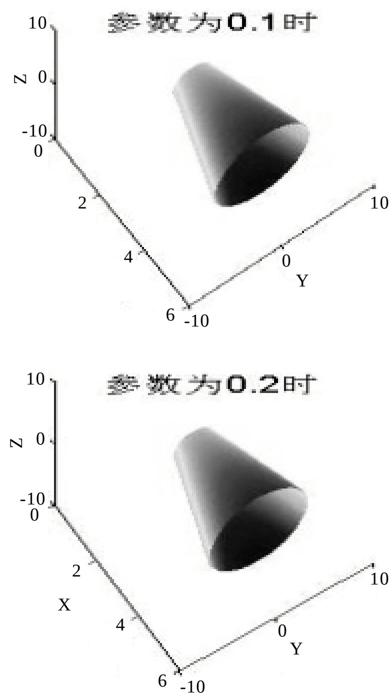

Fig. 7: The fitting cone under the different parameters

point and the point of original surface, so the surface of fitting out is meet the requirements.

CONCLUSIONS

We have presented an approach for impicit surface fitting with Hermite interpolation by directly making use of surface normal vectors at sample points. Fitted surfaces can be G^2 continous, interpolating, approximating to object surface commendably. Compared with the widely used heuristic methods, our method avoids of introducing manufactured off-surface points and can reconstruct implicit surfaces effectively and robustly. As future work, we intend to determine the theoretical range of parameters for other surfaces.

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