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Free Vibration Analysis of Elastic Pipe with Crack Defects

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Abstract: Free vibration of elastic pipe with crack defects was analyzed. On the basis of Timoshenko beam theory, the mathematical modeling of the problem was formulated to integral equation. The numerical approximation of the frequency equation was derived from the integral equation. The first three natural frequencies and the relative mode shapes of the pipes were obtained by using Matlab program. The numerical results showed that both the first and the second derivatives of the mode shape functions has a sudden change at the position of the crack, the magnitude of the change would be enlarged with increase of the crack depth. The variation of the first and second natural frequencies against different depth and sector angle of the crack were discussed.

Key words: Crack identification, elastic pipe, free vibration, integral equation, timoshenko beam

INTRODUCTION

Cracks present a serious threat to proper performance of structures. Most of the failures of presently used equipments are due to material fatigue. For this reason, methods making early detection and localization of cracks possible have been the subject of numerous investigation (Hu *et al.*, 2007). And pipes are inportant engineering stuctures widely met in many applications, e.g., chemical plants, gas and oil transportation and power plants. Huge disaster would be caused once the pipes are damaged. So it is especailly important to detect and forcast the cracks in pipes.

The vibration analysis of the crack pipe is the inversion problem of the crack detection and it is the foundation of crack detection. The presence of crack could change the stiffness of the cracked region and then the vibration. So the key point of this study to investigate the vibration regular of cracked pipe and detect the crack according to the regular.

There is a number of researches have been done by researchers. Teoh and Huang (1977) presented vibration of orthogonal anisotropic cantilever beam. It was analyzed by energy method considering shear deformation and moment of inertia. Karthikeyan *et al.* (2007) modeled the beam using finite element method by Timoshenko beam theory and developed a method to detect the crack location and size; Viola *et al.* (2007) investigated the changes in the magnitude of natural frequencies and model response of a uniform Timoshenko beam using a particular member theory. The theory was demonstrated by two illustrative examples of bending. Kisa *et al.* (1998) analyzed the vibration characteristics of a cracked Timoshenko beam integrating component mode synthesis and finite element.

However, the integral equation method is rarely used in vibration of cracked pipe. The integral equation presented in this paper could be expressed in a unified form. Its integral kernel involves all the information of cracked pipe, such as the material parameters, geometric parameter, boundary conditions and crack parameters. That is why the method is of profound theoretical and practical significance.

The simulation of the crack is crucial to the modeling of the cracked beam and is the guarantee of the correctness of model. The mainly presented methods of simulating cracks are equivalent cross-section method (Dimarogonas, 1996), the local flexibility method (Irwin, 1957) and continuous modeling method (Carneiro and Inman, 2002). This study simulated the crack with the equivalent cross-section method which considered the reduction of the inertia moment and cross section.

The pipe was mostly modeled in Euler-Bernoulli beam theory. In this study the pipe was modeded in Timoshenko beam theory considering the influence of moment of inertia and shearing deformation and it is more accurate than Euler-Bernoulli beam theory.

MATHEMATICAL MODELING

Consider free vabration of a cantilever elastic pipe with a fan shaped crack shown in Fig. 1. Denode the outside radius of the pipe as b , the inside radius as a, the

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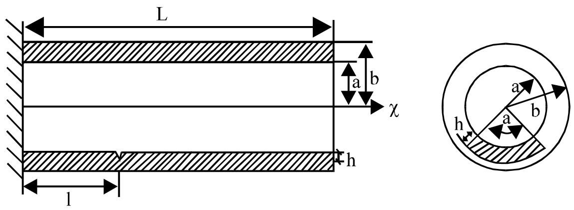

Fig. 1: Diagram of cracked pipe

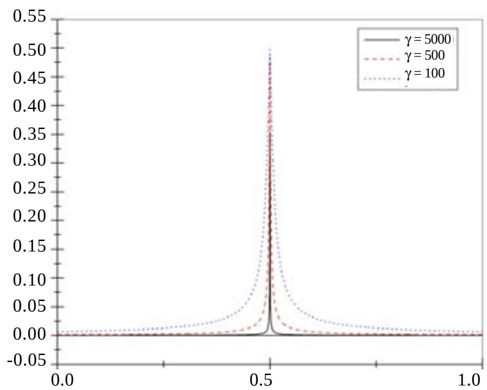

Fig. 2: Crack function

length as L, the section shape factor as k. ρ is the volume density of the pipe material, E and G is Young's and the shear modulus, respectively. l is the distance from the crack to the fixed end. The area and the inertia moment of the cross-section is denoded as A(x) and I(x), respectively which are considered as functions of variable x and dependented on the crack depth h and the sector angle α.

The pipe is modeled as Timoshenko beam, that is, the effect of shear deformation and rotary inertia are not neglected. On the basis of Timoshenko beam theory, the fundamental equations about the deflection w(x, t) and the rotation φ(x, t) of the beam are:

$$\begin{aligned} &\frac{\partial}{\partial x}\left[kGA(x)(\frac{\partial w}{\partial x}-\varphi)\right]=\rho A(x)\frac{\partial^2 w}{\partial t^2} \\ &\frac{\partial}{\partial x}\left(EI(x)\frac{\partial \varphi}{\partial x}\right)+kGA(x)(\frac{\partial w}{\partial x}-\varphi)=\rho I(x)\frac{\partial^2 \varphi}{\partial t^2} \end{aligned} \tag{1}$$

The boundary conditions are:

$$w|_{x=0}=\varphi|_{x=0}=0, \left(\frac{\varphi w}{\partial x}-\varphi\right)\Big|_{x=1}=\Big|\frac{\partial \varphi}{\partial x}\Big|_{x=L}=0 \tag{2}$$

Introduce the crack function:

$$\eta(x)=\frac{2h}{\pi(b-a)}\mathrm{ac\,cot}(\gamma\,|\,x-1\,|) \tag{3}$$

where, γ is a parameter relative to the size of the crack opening, h is the maximun of the crack depth. The crack depth is a function of variable x as H(x) = (b-a) η(*x*). As an example, the curves of the crack function η(x) f or h = (b-a)/2, l = L/2, γ = 100, 500 and 5000 is, respectively shown in Fig. 2.

The area and inertia moment of cross-section can be expressed as:

$$\begin{aligned} A(x) &= \pi(b^2-a^2)-\frac{\alpha}{2}(2aH(x)+H^2(x)) \\ I(x) &= \frac{\pi(b^4-a^4}{4}+\frac{\sin\alpha-\alpha}{8} \\ H(x) &= (2a+H(x))\left[a^2+(a+H(x))^2\right] \end{aligned} \tag{4}$$

Assuming the deflection and the rotation can be separated by variables as follows:

$$w(x,t)=Y(x)e^{i\omega t},\ \varphi(x,t)=\Phi(x)e^{i\omega t} \tag{5}$$

and substituting (4)and (5) into (1), one obtains:

$$\begin{aligned} &\frac{d}{dx}\left[kGA(x)\left(\frac{dY(x)}{dx}-\Phi(x)\right)\right]+\rho\omega^2 A(x)Y(x)=0 \\ &\frac{d}{dx}\left(EI(x)\frac{d\Phi(x)}{dx}\right)+kGA(x)\left(\frac{dY(x)}{dx}-\Phi(x)\right)+\rho\omega^2 I(x)\Phi(x)=0 \end{aligned} \tag{6}$$

In a similar way, the boundary conditions (2) becomes:

$$Y(0)=\Phi(0)=0,\ Y'(L)\text{-}\Phi(L)=\Phi'(L)=0 \tag{7}$$

THEORETICAL ANALYSIS

The equivalent integral equation: Integrating the two sides of Eq. 6 and using the boundary condition (7), one obtains the system of coupled integral equation as follows:

$$\begin{bmatrix} Y(x) \\ \Phi(x) \end{bmatrix}=\lambda\int_0^L \begin{bmatrix} K_{11}(x,\xi) & K_{12}(x,\xi) \\ K_{21}(x,\xi) & K_{22}(x,\xi) \end{bmatrix}\begin{bmatrix} Y(\xi) \\ \Phi(\xi) \end{bmatrix} d\xi \tag{8}$$

where, the integral kernels:

$$\begin{aligned} K_{11}(x,\xi) &= \begin{cases} C(x)P(\xi) & (0\le\xi\le x) \\ C(x)P(x) & x<\xi\le L \end{cases} \\ K_{12}(x,\xi) &= \frac{1}{\lambda}\begin{cases} 2 & (0\le\xi\le x) \\ 1 & x<\xi\le L \end{cases} \\ K_{21}(x,\xi) &= \begin{cases} C(\xi)\ \ (J(\xi)-\xi H(\xi))\ \ (0\le\xi\le x) \\ C(\xi)\ \ (J(\xi)-\xi H(x))\ \ (x<\xi\le L) \end{cases} \\ K_{22}(x,\xi) &= \begin{cases} r^2C(\xi)H(\xi) & (0\le\xi\le x) \\ r^2C(\xi)\ \ H(x)) & (x<\xi\le L) \end{cases} \end{aligned}$$

in which:

$$H(x)\int_0^x \frac{dt}{D(t)},\ J(x) = J(x) = \int_0^x \frac{tdt}{D(t)},\ P(x) = \int_0^x \frac{dt}{D(t)}$$

are known functions, $\lambda = \rho\omega^2/kG$ is the frequency parameter, $r = \sqrt{I/A}$ is radius of gyration.
C (x) = kGA(x) and D(x) = El(x) is, respectively the shearing and the bending rigidity parameter.

Up to now, the system of integral equation for the cracked elastic beam is obtained. It should be noted that the integral kernels in Eq. 8 involves all information about the beam and the crack.

Numerical discretization: In order to solve the system of integral Eq. 8 it was discreted numerically into an algebraic equation by using the collocation method. The interval [0, L] can be divided into n segment evenly:

$$0 = x_0 < x_1 < x_2 < \ldots < x_{n-1} < x_n = L$$

Then Eq. 8 can be discreted numerically as:

$$\begin{bmatrix} Y_i \\ \Phi_i \end{bmatrix} = \frac{\lambda L}{n}\sum_{j=1}^{n}\begin{bmatrix} K_{11}(x_i,\xi_j) & K_{12}(x_i,\xi_j) \\ K_{21}(x_i,\xi_j) & K_{22}(x_i,\xi_j) \end{bmatrix}\begin{bmatrix} Y_j \\ \Phi_j \end{bmatrix} \quad (9)$$

where, $Y_i = Y(x_i)$, $\Phi_i = \Phi(x_i)$ (i = 1, 2, ...,n)

Eq. 9 can be written as:

$$\left(\lambda \frac{L}{n} K - I\right) Z = 0 \quad (10)$$

where, I is 2 n-order unit matrix, the mode shape vector $Z = [Y_1 \ldots Y_n\ \Phi_1 \ldots \Phi_n]^T$ and the integral kernel matrix:

$$K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$$

Here denote submatrice:

$$K_{lk} = \left[K_{lk}\left(x_i, \xi_j\right)\right]_{n\times n}\ (l, k = 1, 2)$$

As Z is a non-vanishing vector, the natural frequency can be determined by the untrivial solution requirment of Eq. 10:

$$f(\lambda) = \det\left(\lambda \frac{L}{n} K - I\right) = 0 \quad (11)$$

Once the frequency is solved from the above equation, then substituting it into Eq. 10 obtains the mode shape.

NUMERIC EXAMPLE

Vibration mode of cracked pipe: Since mode shape is sensitivity to the crack parameters of beams (Jassim *et al.*, 2013), analysis of mode shape is meaningful for crack detection. The above numeric method was programed in Matlab and the normalized mode shape was found. As an example, consider a cantilever pipe which physical parameters and geometrical sizes are given in Table 1. The results of numeric computation are shown in Fig. 3-7.

Figure 3-5 are the first three mode shapes of pipes with cracks of different depth at the location 2L/5. They are in accordance with the theoretical ones. The mode

Table 1: Physical parameters and geometrical sizes of the cracked pipe

a	b	L	E	G	ρ	α
15 mm	21 mm	765 mm	203 GPa	160GPa	775 kg m^3	π/2

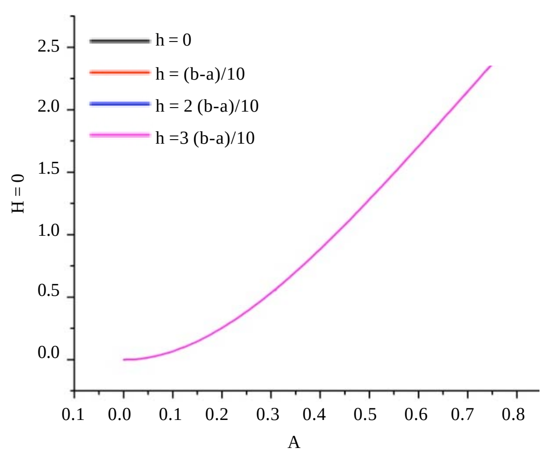

Fig. 3: First mode shapes of cracked pipes with different crack depth

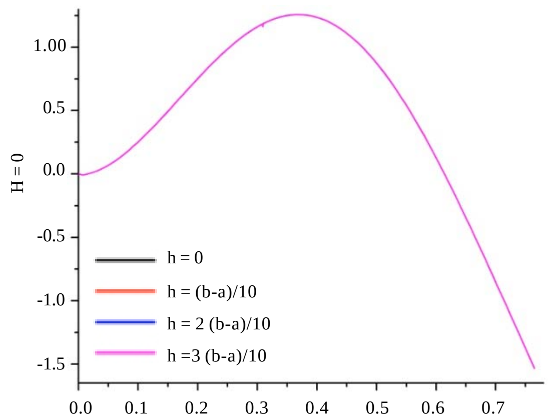

Fig. 4: Second mode shapes of cracked pipes with different crack depth

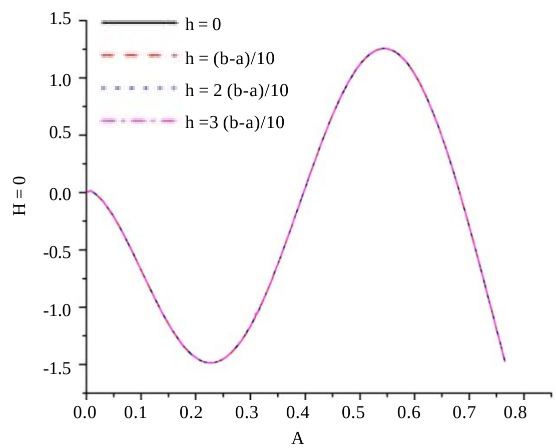

Fig. 5: Third mode shapes of cracked pipes with different crack depth

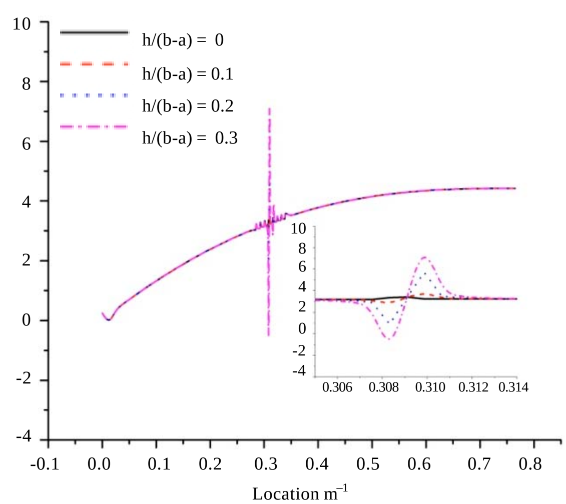

Fig. 6: First derivative of the first mode and the partial enlargement at the cracked location

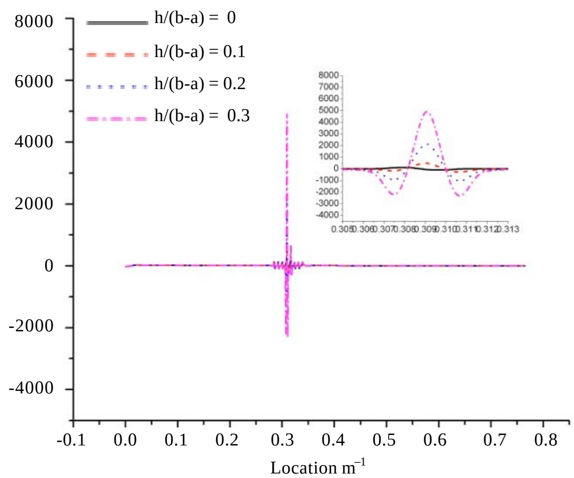

Fig. 7: Second derivative of the first mode and the partial enlargement at the cracked location

Table 2: First natural frequency $f=\sqrt{\lambda kG/4\pi^2\rho}$ against crack depth ratio h/(b-a) and sector angle α(Hz)

h/(b-a)	α = π/2	α = π	α = 3π/2	α = 2π
0	57.271	57.271	57.271	57.271
0.1	57.265	57.237	57.207	57.201
0.2	57.260	57.200	57.133	57.119
0.3	57.254	57.159	57.044	57.017
0.4	57.248	57.112	56.931	56.886

Table 3: Second natural frequency $f=\sqrt{\lambda kG/4\pi^2\rho}$ against crack depth ratio h/(b-a) and sector angle α (Hz)

h/(b-a)	α = π/2	α = π	α = 3π/2	α = 2π
0	356.249	356.249	356.249	356.249
0.1	356.359	356.322	356.286	356.359
0.2	356.432	356.396	356.286	356.469
0.3	356.542	356.469	356.286	356.506
0.4	356.652	356.542	356.212	356.506

shape of pipe with difference crack depth coincides with each other and they are so smooth as not able to detect the crack parameters.

Figure 6 and 7 are, respectively the first and the second derivatives of the first mode shape of pipe with different crack depth. All curves in the two figures have sudden change at the location of the crack and the variation is positively correlated with the crack depth. So the crack location and depth can be easily detected base on the derivative of the mode shape. Moreover, the method can be easily realized in engineering and is economical.

Frequency analysis against different crack location and depth: Table 2 is fundamental frequency against different crack depth and sector angle when the crack is at L/5. It shows that: the frequency gose down as the increase of the sector angle and depth ratio. Table 3 is the second natural frequency when the crack is at L/5. It shows that the frequency has the Minimum when |α is 3|π/2. The frequency goes up when á is less or more.

The crack depth and sector angle can be determined by the frequencies according to Table 2 and 3 when the crack location was found by the method in the above section.

CONCLUSIONS

The frequency equation of the elastic pipe was presented from the system of integral equation. Its kernel contains the information about the material parameters, geometric parameter, boundary conditions and crack parameters. The numerical calculation of the equations was obtained by discreting the integral equation using the collocation method.

The first three mode shapes of the cracked pipe were presented and the first and second derivatives of the first mode shape were obtained. The results show that the mode shape of the cracked pipe is smooth; the first and second derivatives of the first mode shape have sudden change at the location of crack and the change enlarges with the increase of the crack depth. The depth and location of a cracked pipe can be detected on the basis of this result.

The variation of the first and second natural frequencies agaist different depth and sector angel of the crack were discussed. The datum of the first two natural frequencies against the crack depth and sector angle were presented.

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