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Block Matching Algorithms Based on Extended Mahalanobis Distance and Kullback-leiber Divergence for Motion Estimation

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Abstract: There is a growing interest in research of applications using image sequences, mainly in the fields of motion analysis and video processing. The latest include filtering operations, temporal and spatial interpolation, sampling and compression of images using the motion information whose objectives are: Improving the visual quality and convert between different video formats. While the analysis of image sequences includes some operations for extracting attributes for the purpose of reconstruction or interpretation of information from the scene. However, with the latest technology, there are many situations where degradations of the original scenes are too large for the intended application. The reasons behind this can be, for example, the harsh conditions encountered in medical imaging, astronomy, the military and many more. Sometimes it is impossible to improve the sensor. This can be due to reasons of cost or because of physical limitations. Then motion estimation of video sequences becomes necessary not only to improve the visual quality of these, but also to increase the performance of subsequent processing. In this study a method was proposed for motion estimation based on the block matching criterion through the modeling of image blocks by a mixture of Gaussian distributions and Extended-Mahalanobis distance and Kullback-Leiber divergence. The mixture parameters (weights, mean vectors and covariance matrices) were estimated by the Expectation Maximization algorithm (EM). The EM maximizes the log-likelihood criterion. The similarity between a block in the current image and the most resembling one in the search window on the reference image is measured by the minimization of Extended-Mahalanobis distance or Kullback-Leibler divergence between the clusters of mixture.

Key words: Motion estimation, Gaussian mixture, expectation maximization algorithm, extended Mahalanobis distance, kullback-leibler divergence

INTRODUCTION

In the aim to improve the mixture of Gaussian distributions in the motion estimation (Boudlal *et al.*, 2010), a new study based on Extended-Mahalanobis distance and Kullback-Leiber divergence has been proposed. Generally, the mixtures are robust and their optimization is performed by using the EM (Expectation-Maximization) algorithm based on an iterative optimization of parameters (a priori probability, mean vectors, covariance matrices) (Boudlal *et al.*, 2010; Dempster *et al.*, 1977; Neal and Hinton, 1999; Afify, 2005). However, their use in modeling the image blocks and look for the best matching between the blocks located in consecutive frames need a similarity measure. For this purpose, the Extended-Mahalanobis distance and Kullback-Leiber divergence have been used to measure the distance between two distributions based on combining the covariance matrices.

The Extended Mahalanobis distance (Younis *et al.*, 1998) is important in comparing between the clusters. This is because the information about spatial distribution of the points is incorporated in the metrics. The spatial distribution can be represented by the statistics of the data set where it should be measured by, the distance between two points. The Euclidian distance is considered a special case of the Mahalanobis distance. This is where the data would be uniformly distributed. This case corresponds to a distribution where the covariance matrices are diagonal matrices. In this sense, the Mahalanobis metrics become a general case of distance measurements and suitable to be used in researching the similarity between the blocks in the image. The advantage of the Extended Mahalanobis distance relies on the fact that it is easy to calculate. Practically, in the case of two Gaussian distributions $p_1(\mu_1, \Sigma_1)$ and $p_2(\mu_2, \Sigma_2)$ the measure between two mean vectors is defined as follows:

$$D_{\text{extrema}}(p_1, p_2) = \sqrt{(\mu_1 - \mu_2)^T (\Sigma_1 + \Sigma_2)^{-1} (\mu_1 - \mu_2)} \quad (1)$$

The Kullback-Leiber divergence (KL-divergence) (Kullback, 1968; Kullback, 1987) also known as the relative entropy, between two probability density functions $p_1(x)$ and $p_2(x)$:

$$KL(p_1, p_2) \stackrel{\text{def}}{=} \int \frac{p_1(x) \log(p_1(x))}{p_2(x)} dx \quad (2)$$

This is commonly used in statistics as a measure of similarity between two density distributions. The divergence satisfies three properties hereafter referred to as the divergence properties (Kullback, 1968):

- **Self-similarity:** $KL(p_i, p_i) = 0 \quad i = 1, 2$
- **Self-identification:** $KL(p_1, p_2) = 0$, only if $p_1 = p_2$
- **Positively:** $KL(p_1, p_2) > 0$ for all p_1, p_2

In Fig. 1, two situations are remarkable, where clusters are separated and where they are not. It's obvious that the Euclidian distance between both situations is almost the same, but in one case, a similar cluster was formed and a dissimilar cluster in the other.

The KL-divergence is used in many aspects of speech and image recognition, such as determining if two acoustic models are similar (Olsen and Dharanipragada, 2003) and measuring how confusable are two words (Printz and Olsen, 2002). For two Gaussians p_1 and p_2 . The KL divergence has a closed formed expression:

$$KL(p_1, p_2) = \frac{1}{2} \left[\log \frac{|\Sigma_1|}{|\Sigma_2|} + \text{tr}(\Sigma_1^{-1} \Sigma_2) + (\mu_1 - \mu_2)^T \Sigma_2^{-1} (\mu_1 - \mu_2) - d \right] \quad (3)$$

where, d is the dimension of data space.

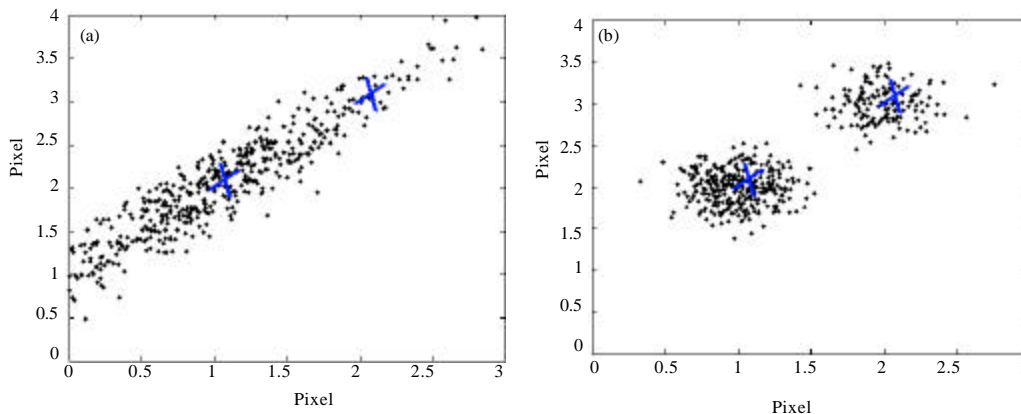


Fig. 1(a-b): The Kullback-Leiber information in two regions of distributions of different probability, (a) Region one distribution and (b) Region two distributions

Figure 1a gives a more subjective graphical interpretation of the Kullback-Leibler information. The figure in the left is a representation of a single region. p_1 and p_2 are two probability distributions of this region. The Kullback-Leibler information between p_1 and p_2 is close to 0.

Figure 1b on the right, is a representation of the two regions, p_1 is a probability distribution of the region below and p_2 is a probability distribution of the region above. The Kullback-Leibler information between p_1 and p_2 is different from 0 and therefore the information is also different from 0.

MODELING AND PARAMETER ESTIMATION OF GAUSSIAN MIXTURES

The Gaussian mixture model is as follows (McLachlan and Peel, 2000):

$$p\left(\frac{x}{\Theta_k}\right) = \sum_{i=1}^k \alpha_i p\left(\frac{x}{\theta_k}\right) = \sum_{i=1}^k \alpha_i p\left(\frac{x}{\mu_i, \Sigma_i}\right) \quad (4)$$

where, k is the number of components in the mixture model. And $(\alpha_i = 0)$ are the mixing proportions of components satisfying $\sum_{i=1}^k \alpha_i = 1$. And each component density $p(x/\theta_i)$ is a Gaussian probability density function given by:

$$p\left(\frac{x}{\mu_i, \Sigma_i}\right) = \frac{1}{(2\pi)^{\frac{n}{2}} |\Sigma_i|^{\frac{1}{2}}} e^{-\frac{1}{2} (x - \mu_i)^T \Sigma_i^{-1} (x - \mu_i)} \quad (5)$$

where, n the dimensionality of the vector is x , μ is the mean vector and Σ_i is the covariance matrix assumed to be

definite positive. For clarity, Θ_k is the collection of all the parameters in the mixture model, i.e., $\Theta_k = (\theta_1, \dots, \theta_k, \alpha_1, \dots, \alpha_k)$. Given a set of N i.i.d. samples, $x = \{x_i\}_{i=1}^N$, the log-likelihood function for the Gaussian mixture model is expressed as follows:

$$\log(p(x/\Theta_k)) = \log \prod_{i=1}^N p(x_i/\Theta_k) = \sum_{i=1}^N \log \sum_{j=1}^k \alpha_j p(x_i/\theta_j) \quad (6)$$

Which can be maximized to get a Maximum Likelihood (ML) (Sanjay-Gopal and Hebert, 1998) estimate of Θ_k via the following EM algorithm:

$$\alpha_i^+ = \frac{1}{n} \sum_{i=1}^N P(i/x_i) \quad (7)$$

$$\mu_i^+ = \frac{\sum_{i=1}^N x_i P(i/x_i)}{\sum_{i=1}^N P(i/x_i)} \quad (8)$$

$$\Sigma_i^+ = \frac{\sum_{i=1}^N P(i/x_i) (x_i - \mu_i^+) (x_i - \mu_i^+)^T}{\sum_{i=1}^N P(i/x_i)} \quad (9)$$

Where:

$$P\left(\frac{i}{x_i}\right) = \frac{\alpha_i p(x_i/\theta_i)}{\sum_{j=1}^k \alpha_j p(x_i/\theta_j)}$$

are the posterior probabilities. Since the EM is highly dependent on initialization, the first set of parameters selection is very important for this algorithm. If the initial parameters are not well selected, the algorithm may converge into local maxima points. The convergence properties of the EM algorithm over Gaussian mixture Model have been extensively studied by Redner and Walker (1984) and Xu and Jordan (1996).

BLOCK MATCHING METHOD BASED ON THE MINIMIZATION OF DISTANCE BETWEEN MIXTURE GAUSSIAN

Considering a video sequence containing moving objects, the displacement vector of each object is estimated in the image plane by a technique called the full search block matching algorithm (Zhu *et al.*, 2004; Xu *et al.*, 1999). The current frame is divided into a matrix of "macro blocks" that are then compared with the corresponding block and its adjacent neighbors in the

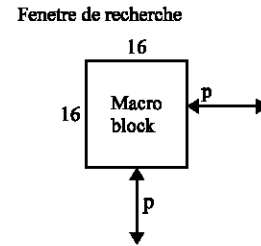


Fig. 2: Block Matching a macro block of side 16 pixels and a search parameter p of size 3 pixels

previous frame. This enables to create a vector that stipulates the movement of a macro block from one location to another in the previous frame. This movement, calculated for all the macro blocks included in a frame, represents the motion estimated in the current frame. The search area for a good macro block match is constrained up to p pixels on all four sides of the corresponding macro block in the previous frame. This "p" stands for the search parameter. Larger motions require a larger p and the larger the search parameter is, the more computationally expensive the process of motion estimation becomes. The idea is depicted in Fig. 2. The matching of one macro block with another is based on the output of a cost function. The macro block that results in the least cost is the one that closely matches to current block (Packwood *et al.*, 1997).

Cost function: For simple scenes or objects which appear to be mono-colored, a small number of components in a Gaussian mixture model is suggested, like one or two components. For complex scenes, a larger number of components in a Gaussian mixture model is suggested, say starting with 3. The maximum number is important if computation time and system efficiency are considered. In general, more components do have the potential for further improvement. In this study, the number of components of the mixture is chosen to be 2 or 3. The cost function is defined by the Extended-Mahalanobis distance and Kullback-Leibler divergence weighted by the weight of Gaussian distributions components. This distances are applied between the following components: Gaussian inter-blocks (reference/Current), the components of strong weights, the components of medium weights and the components of weak weights. Between the components of strong weights (d_1), the components of medium weights (d_2) and the components of weak weights (d_3). For weights see Fig. 3a. The distances d_1 and d_2 are defined as follows:

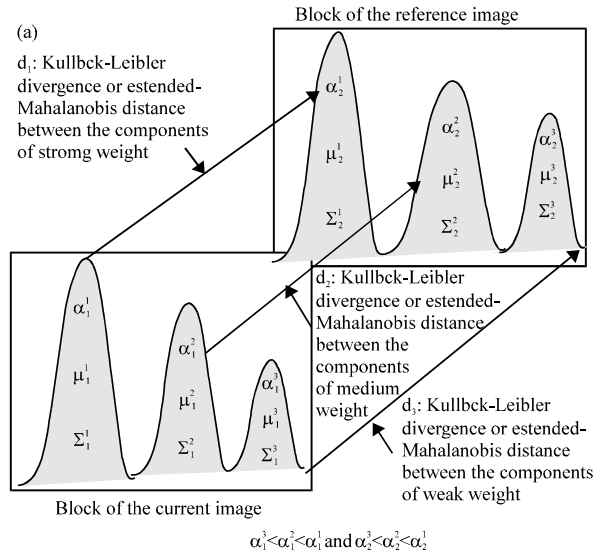


Fig. 3a: Extended-Mahalanobis distance or Kullback-Leibler divergence

• **Using Extended-Mahalanobis distance:**

$$d_1 = (\alpha_1^1 \mu_1^1 - \alpha_2^1 \mu_2^1)^T (\alpha_1^1 \Sigma_1^1 + \alpha_2^1 \Sigma_2^1)^{-1} (\alpha_1^1 \mu_1^1 - \alpha_2^1 \mu_2^1) \quad (10)$$

$$d_2 = (\alpha_1^2 \mu_1^2 - \alpha_2^2 \mu_2^2)^T (\alpha_1^2 \Sigma_1^2 + \alpha_2^2 \Sigma_2^2)^{-1} (\alpha_1^2 \mu_1^2 - \alpha_2^2 \mu_2^2) \quad (11)$$

$$d_3 = (\alpha_1^3 \mu_1^3 - \alpha_2^3 \mu_2^3)^T (\alpha_1^3 \Sigma_1^3 + \alpha_2^3 \Sigma_2^3)^{-1} (\alpha_1^3 \mu_1^3 - \alpha_2^3 \mu_2^3) \quad (12)$$

• **Using Kullback-Leibler divergence:**

$$d_1 = \frac{1}{2} \left[\log \frac{|\alpha_2^1 \Sigma_2^1|}{|\alpha_1^1 \Sigma_1^1|} + \text{tr}(\alpha_2^1 (\Sigma_2^1)^{-1} \alpha_1^1 \Sigma_1^1) + (\alpha_1^1 \mu_1^1 - \alpha_2^1 \mu_2^1)^T \alpha_2^1 (\Sigma_2^1)^{-1} (\alpha_1^1 \mu_1^1 - \alpha_2^1 \mu_2^1) - d \right] \quad (13)$$

$$d_2 = \frac{1}{2} \left[\log \frac{|\alpha_2^2 \Sigma_2^2|}{|\alpha_1^2 \Sigma_1^2|} + \text{tr}(\alpha_2^2 (\Sigma_2^2)^{-1} \alpha_1^2 \Sigma_1^2) + (\alpha_1^2 \mu_1^2 - \alpha_2^2 \mu_2^2)^T \alpha_2^2 (\Sigma_2^2)^{-1} (\alpha_1^2 \mu_1^2 - \alpha_2^2 \mu_2^2) - d \right] \quad (14)$$

$$d_3 = \frac{1}{2} \left[\log \frac{|\alpha_2^3 \Sigma_2^3|}{|\alpha_1^3 \Sigma_1^3|} + \text{tr}(\alpha_2^3 (\Sigma_2^3)^{-1} \alpha_1^3 \Sigma_1^3) + (\alpha_1^3 \mu_1^3 - \alpha_2^3 \mu_2^3)^T \alpha_2^3 (\Sigma_2^3)^{-1} (\alpha_1^3 \mu_1^3 - \alpha_2^3 \mu_2^3) - d \right] \quad (15)$$

However, this measure creates a singularity for singular covariance matrix. In practical problems it often appears in learning such models mixture. The acquired covariance matrix is not always conditioned and their inversion creates a problem. In this implementation, the inverse of the singular covariance matrix is replaced by its pseudo-inverse. Singular value decomposition is used for

the calculation of the pseudo-inverse. The Round off errors can lead to a singular value not being exactly zero even if it should be. Tolerance parameter places a threshold when comparing singular values with zero and improves the numerical stability of the method with singular or near-singular matrices.

Steps of the proposed method: The proposed method is based on three steps design:

- (1) Each block in the reference image or the current image is modeled by a mixture of three Gaussian distributions. This modeling consists on estimating the parameters of the mixture (weight, mean vectors and covariance matrix)
- (2) The Parameters are sorted based on their weights in mixture. This allows the identification of the components of weak weights, the components of medium weights and the components of strong weights
- (3) Research of minimal inter-blocks distance (reference/current)

(a) The Extended-Mahalanobis distance and Kullback-Leibler divergence between a block of the current image and all blocks in a search window $[-1, +1]$ in the reference image are stored in the matrices M_1 , M_2 and M_3

- The matrix M_1 contains the values of the Extended-Mahalanobis distance and the Kullback-Leibler divergence distances between the components of weak weights
- M_2 matrix contains the values of the Extended-Mahalanobis distance and the Kullback-Leibler divergence distances between the components of medium weights
- M_3 matrix contains the values of the Extended-Mahalanobis distance and the Kullback-Leibler divergence distances between the components of strong weights

(b) The value of the minimal distance of the three matrices M_1 , M_2 and M_3 corresponds to the most similar block in reference image

Practical considerations: The matrices M_1 , M_2 and M_3 show the Extended-Mahalanobis distance or Kullback-Leibler divergence distances between a block of the current image and all the blocks in a search window $[-1, +1]$ (Fig. 3b) in the reference image of the Foreman sequence.

The value of the minimum distance of the three matrices M_1 (Table 1), M_2 (Table 2) and M_3 (Table 3), is

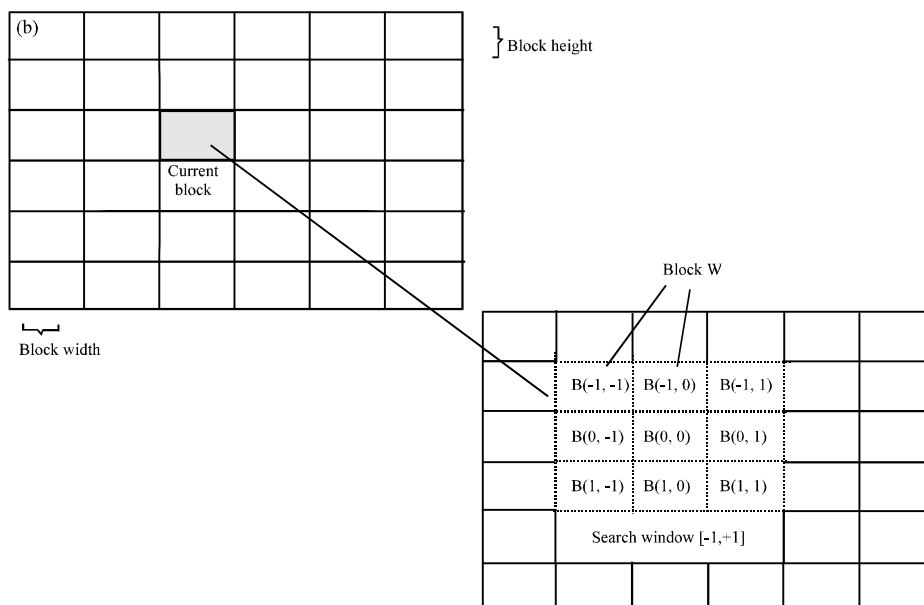


Fig. 3b: Search windows block matching

Table 1: Extended-Mahalanobis and Kullback-Leibler divergence distances between the components of weak weights (matrix M_1)

Distance Block index	Extended-Mahalanobis distance			Kullback-Leibler divergence		
	-1	0	1	-1	0	1
-1	00051	00056	32768	00050	00106	32768
0	00047	00022	32768	00020	00017	32768
1	32768	32768	32768	32768	32768	32768

Table 2: Extended-Mahalanobis and Kullback-Leibler divergence distances between the components of medium weights (matrix M_2)

Distance Block index	Extended-Mahalanobis distance			Kullback-Leibler divergence		
	-1	0	1	-1	0	1
-1	00013	00009	32768	00037	00012	32768
0	00012	00012	32768	00017	00008	32768
1	32768	32768	32768	32768	32768	32768

Table 3: Extended-Mahalanobis and Kullback-Leibler divergence distances between the components of strong weights (Matrix M_3)

Distance Block index	Extended-Mahalanobis distance			Kullback-Leibler divergence		
	-1	0	1	-1	0	1
-1	00016	00050	32768	00100	00249	32768
0	00012	00442	32768	00059	00738	32768
1	32768	32768	32768	32768	32768	32768

equal to 9. This is from the Extended-Mahalanobis distance corresponding to the first line and second column indices of matrix M_2 . This distance is equal to 8 from the Extended Kullback-Leibler divergence corresponding to the second line and second column indices of matrix M_2 . These indices correspond to the most similar blocks in the reference image.

Motion estimation in homogeneous regions in the luminance sense: For the video sequences with homogeneous regions in the luminance sense, we use the

motion estimation method which is proposed by the Gaussian Mixture Model with three components "GMM3" based on two criteria. These two criteria are the Mahalanobis generalized distance and the Kullback-Leibler divergence. They are not discriminatory and do not distinguish the blocks from each other. These ambiguities can be removed by adding other criteria. As an example, Fig. 4 is a sequence of type "Hand". This is a homogeneous region. It is estimated by the method proposed "GMM3" and by applying the criterion of the generalized Mahalanobis distances and

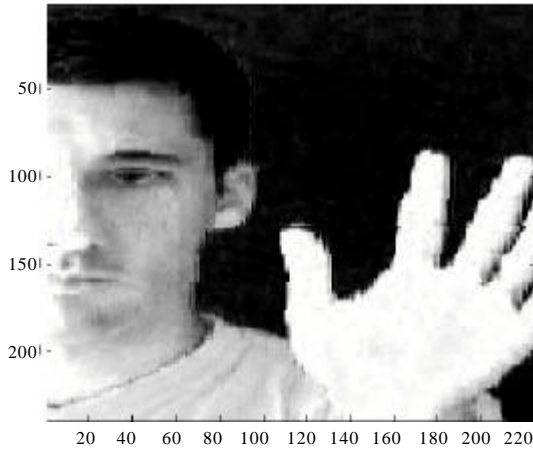


Fig. 4: Sequences "Hand" estimated by the method "GMM3" after regularization of covariance matrices

after regularization of covariance matrices. A matrix of Kullback-Leibler distances between the components of the mixture of strong weight of a block in the current image and all the blocks of a 3x3 search window in the image reference and was obtained during motion estimation of the sequence type "Hand". The matrix has two types of data:

- A value of identical distance equal to 3.2768×10^4 between the components of mixtures modeling the blocks, then the matrix M is defined as follows:

$$M = 10^4 \begin{pmatrix} 3.2768 & 3.2768 & 3.2768 \\ 3.2768 & \text{NaN} & \text{NaN} \\ 3.2768 & \text{NaN} & \text{NaN} \end{pmatrix}$$

The reports of the determinants from the covariance matrices of Eq. 13 are strictly positive. Also all the blocks of the search window of the reference image are identical to the block of the current image. As well, the reports of the determinants of covariance matrices from the homogenous region of Eq. 13 are strictly positive. The indication "NaN" means an indeterminate form obtained by an illegal operation set in a homogeneous region. The determinants of the covariance matrices of Eq. 13 are strictly negative. In this case the motion estimation cannot be accomplished by applying the criterion of the Kullback-Leibler.

RESULTS

The parameters of motion estimation used for comparing and evaluating the quality of the obtained results from the proposed methods are:

Table 4: PSNR_{avg} of sequences of images from different characteristics

Sequences	PSNR _{avg} (dB) by		Difference dB
	distance Mahalanobis	divergence Kullback-Leibler	
Football 240x336	25.53	25.08	0.45
Soccer 335x270	25.00	24.99	0.01
Cones 256x256	22.90	22.77	0.13
coastguard 240x240	24.73	24.49	0.24
Hand 255x255	23.92	-	-
Foreman 175x256	29.60	29.40	0.20

PSNR_{avg}: Peak Signal Noise Ratio average, dB: decibel

- **Method:** exhaustive block-matching (full search), is the most obvious candidate for a search technique in finding the best possible weight in the search area
- **Method proposed criterion:** Minimization of Extended-Mahalanobis distance or Kullback-Leibler divergence between mixture three Gaussian distributions ("GMM3")
 - Precision: pixel
 - Block size: 16x16
 - Search area: [-1, +1]

Simulation results without noise influence and not homogeneous in the sense of luminance: The proposed approach was evaluated using a standard measure: the PSNR (Peak Signal Noise Ratio) defined as:

$$20 \log_{10} \left(\frac{255}{\text{MSE}} \right)$$

where, MSE represents mean square deviation between the reference sequences and the estimated sequences. The proposed study was evaluated by using a standard measure, which is the average PSNR (Peak Signal to Noise Ratio), given as:

$$\text{PSNR}_{\text{avg}} = \frac{1}{F} \sum_{i=1}^F \text{PSNR}_i \tag{16}$$

where, PSNR_i is the measured PSNR for frame i and F is the total number of frames. Two methods shall be compared, which compare are the Extended-Mahalanobis distance and Kullback-Leibler divergence in motion estimation based Gaussian mixture models.

As an example, Table 4 shows that six test image sequences have different characteristics. While "Foreman" represents the characteristics of slow motion image sequences. "Hand" and "Soccer" are fast motion image sequences and "coastguard" is global motion image sequence. Football and Cones contain multiple objects moving.

Table 4 shows that the performance of motion estimation based on the Extended-Mahalanobis distance is better than the motion estimation based on Kullback-Leibler divergence.

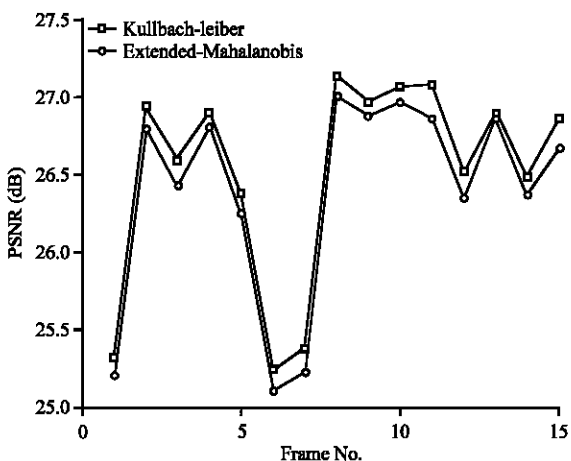


Fig. 5: PSNR comparisons of motion estimation based Gaussian mixture models using Extended-Mahalanobis distance and Kullback-Leibler divergence for the "Soccer" sequence without influence of noise

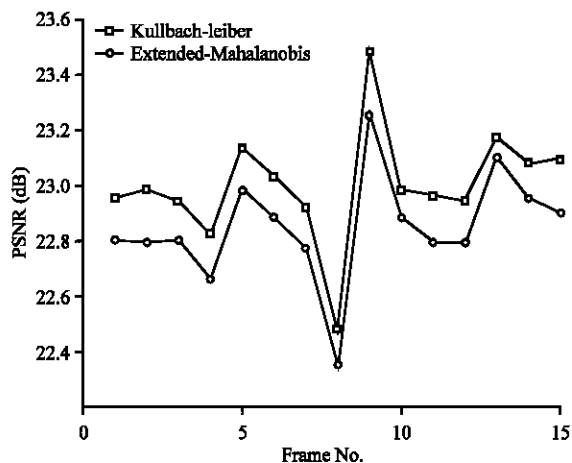


Fig. 6: PSNR comparisons of motion estimation based Gaussian mixture models using Extended-Mahalanobis distance and Kullback-Leibler divergence for the "Cones" sequence without influence of noise

The estimation of motion by the criterion of the Kullback-Leibler divergence of the sequence "Hand" can not be accomplished, because this sequence has homogeneous regions.

Another performance comparison is made among the first 15 frames of each sequence. As an example, Fig. 5 and 6 show the performance comparison for the first 15-frames of "Soccer" and "Cones" sequences. The PSNR comparison shows that the motion estimation based on

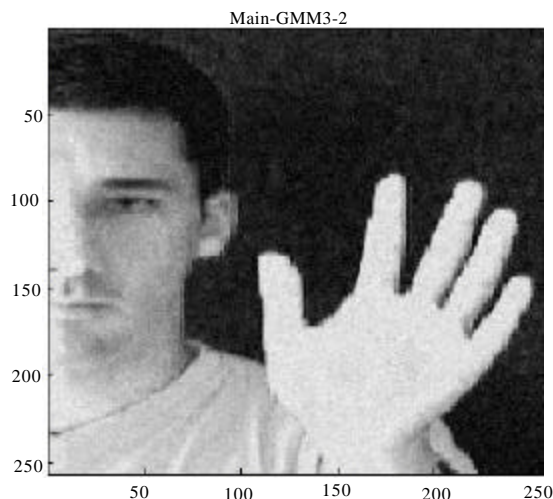


Fig. 7: Sequence "Hand" estimated by the method "GMM3" with the criterion of the Kullback-Leibler after contamination by a correlated Gaussian noise with zero mean and standard deviation equal to 10

the Extended-Mahalanobis distance usually perform better than the motion estimation based on Kullback-Leibler divergence.

Simulation results under influence of noise

Case of not homogeneous sequences: Additive Gaussian noise with standard deviation equal to 10 and uniform noise with standard deviation equal to 20, degraded the video sequences. The motion estimation by the Extended-Mahalanobis distance and Kullback-Leibler divergence on Soccer and Cones sequences was applied. The results are summarized in Fig. 8 and 9. The PSNR comparison shows that the motion estimation based on the Kullback-Leibler divergence is better than the motion estimation based on Extended-Mahalanobis distance.

Case of homogeneous sequences: The motion estimation by the criterion of the Kullback-Leibler is sensitive to homogeneous regions in the sense of luminance and the matrix of Fig. 5 shows that the motion estimation of the sequence "Hand" can not be accomplished.

On the same sequence, a correlated Gaussian noise with zero mean and standard deviation equal to 10 was introduced. This is always the criterion of comparison the Kullback-Leibler. The operation of motion estimation was performed with sucked (Fig. 7). The presence of noise has made these areas less homogenous. However, the results are less efficient than with the standard of Extended-Mahalanobis distance.

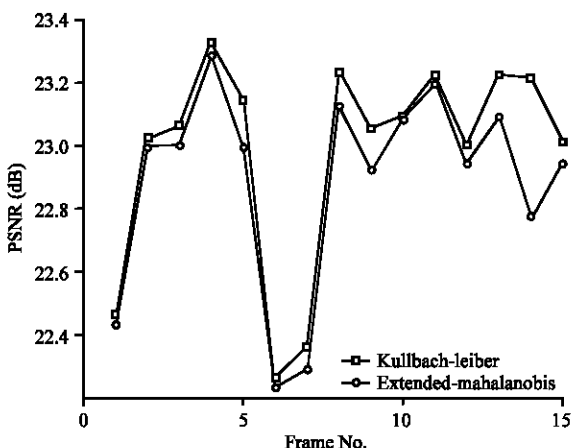


Fig. 8: PSNR comparisons of motion estimation based Gaussian mixture models using Extended-Mahalanobis distance and Kullback-Leibler divergence for the “Soccer” sequence with influence of Gaussian noise for standard deviation equal to 10

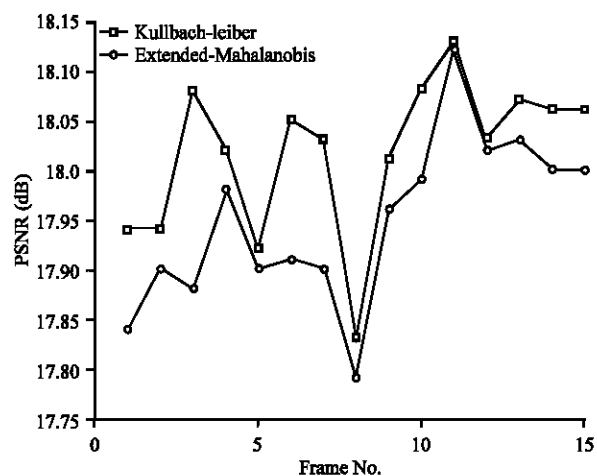


Fig. 9: PSNR comparisons of the estimation of motion based on the Gaussian mixture models using Extended-Mahalanobis distance and Kullback-Leibler divergence for the “Cones” sequence with influence of Uniform noise for standard deviation equal to 20

Another experiment of motion estimation was carried out without considering the component of weak weight as a test of similarity. It can be assumed that this component represents a Gaussian noise, the results are even better. And Table 5 summarizes the values of PSNR_{avg} sequence “Hand”, characterized by homogeneous regions. The motion estimation was performed by two criteria: the Extended-Mahalanobis distance and the Kullback-Leibler. This two criteria were applied to each method-GMM3 and GMM3-2.

Table 5: PSNR_{avg} of sequence “hand” characterized by homogeneous regions with influence of Gaussian noise of standard deviation equal to 10

Sequences hand	PSNR _{avg} (dB) by distance Extended-		Difference dB
	Mahalanobis	Kullback-Leibler divergence	
GMM3	21.50	19.92	1.58
GMM3-2	21.90	20.01	1.89
Diff' erence [dB]	0.40	0.09	

Table 6: Classification of types of image sequences according on motion estimation methods and applied criteria

Sequences	Motion estimation	
	adaptd method	Applied criterion
Not noisy	GMM3	Extended-Mahalanobis distance
Noisy	GMM3 GMM3-2	Kullback-Leibler divergence
		Extended-Mahalanobis distance
Homogeneous	GMM3	Extended-Mahalanobis distance

GMM3-2: Method of motion estimation by modeling the mixture of three components regardless of the low weight component. In the motion estimation, this component is considered as a Gaussian noise. Through this comparative study the types of video sequences were classified based on motion estimation methods adapted and the criteria. Table 6 summarizes this classification.

CONCLUSION

The results prove that both metrics are capable to measure the similarity and to search the best matching between two windows spaces (or blocks) located in consecutive frames. The motion estimation of the non-noisy image sequences based on the criterion of the Extended Mahalanobis distance has showed more appropriate. In case with influence of noise, using the Kullback-Leibler divergence proves to be most effective. One last experience in the estimation of motion was performed by the method GMM3-2. That is to say, regardless of the component of low weight as a test of similarity, It can be assumed that this component represents a correlated Gaussian noise. The results are more efficient than those produced by the method GMM3 (consider the component of low weight as a test of similarity). In the end, through this comparative study, the types of video sequences are classified based on the methods of estimating motion adapted and on the applied criteria.

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