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## Study of New Type Herringbone Gear with Narrow Tooth Width and Beveloid Teeth

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**Abstract:** New narrow herringbone gear, a new kind of transmission, is derived from traditional herringbone gear and broadens its application range. Based on the theories of differential geometry and gear meshing principles, the author describes the tooth and the transmission characteristics of the gear and deduces the equations of tooth face and fillet. According to the theories of material mechanics and gear strength check, the author deduces the equations of gear fatigue strength calculation, contact ratio and restriction of addendum sharpening. Based on the basic theories deduced before, a kind of software is developed which can achieve optimization design, modeling and analysis. ADAMS is used to simulate the contact forces of the gear and the result is compared with the theoretical mechanical calculation to confirm the axial force of the gear is lower than the helical gear. Finite element analysis is carried out to confirm the equations of gear fatigue strength are practical and correct.

**Key words:** Gear, narrow herringbone, design, analysis

### INTRODUCTION

Herringbone gear which can bear heavy duty, transmit stable load and eliminate axial load, is widely used in mechanical transmission (Bu *et al.*, 2012) but disadvantages, such as large axial dimension, existence of tool withdrawal groove and so on (Wang *et al.*, 2010), restrict its application. Based on the characteristics of traditional herringbone gear, new herringbone gear is put forward, it has similar transmission characteristics with traditional herringbone gear but with no tool withdrawal groove, so the weight is lighter and the contact ratio is larger, moreover, it is more suitable to use in the occasions of narrow axial dimension, for example, clock, instrument, aerospace engineering, agriculture etc. Based on the theories of gear meshing principles and gear strength check, the author deduces several main equations that can reveal the main transmission characteristics and the bearing capacity of the gear (Litvin *et al.*, 2005; Jehng, 2002; Li, 2008). At last, the author develops a kind of software that can gather the optimization design, modeling and analysis of the gear together.

### MATHEMATICAL MODE OF NARROW HERRINGBONE GEAR

**Tooth faces of narrow herringbone gear:** The tooth shape of the narrow herringbone gear is different from the traditional gear tooth, the tooth space width of pinion and the tooth thickness of wheel is variable along the axial

direction. While, it has the standard involute profile on the middle end face which is the same as helical and herringbone gear.

As shown in Fig. 1, tooth faces of the narrow herringbone gear can be generated by a group of rack tooth faces. The upward tooth faces of the rack are used to generate the tooth faces of wheel while the downward tooth faces are used to generate the tooth faces of pinion.  $r_1$  represents the radius of the pitch circle of pinion and  $r_2$  represents the radius of the pitch circle of wheel. the speed of the rack is  $v$  relative to the ground while the

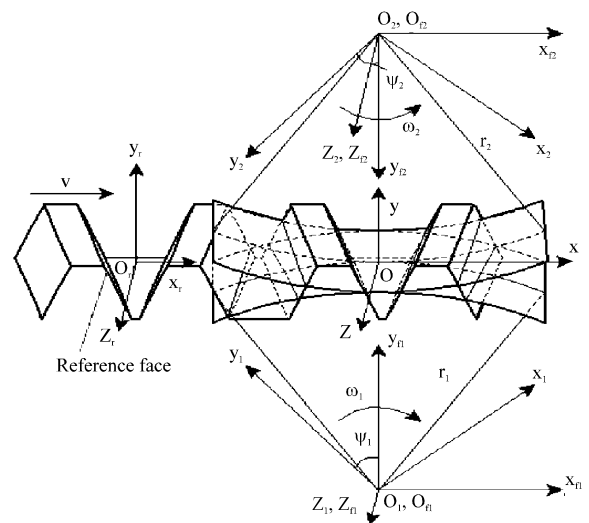


Fig. 1: Coordinate systems used in the process of generating motion

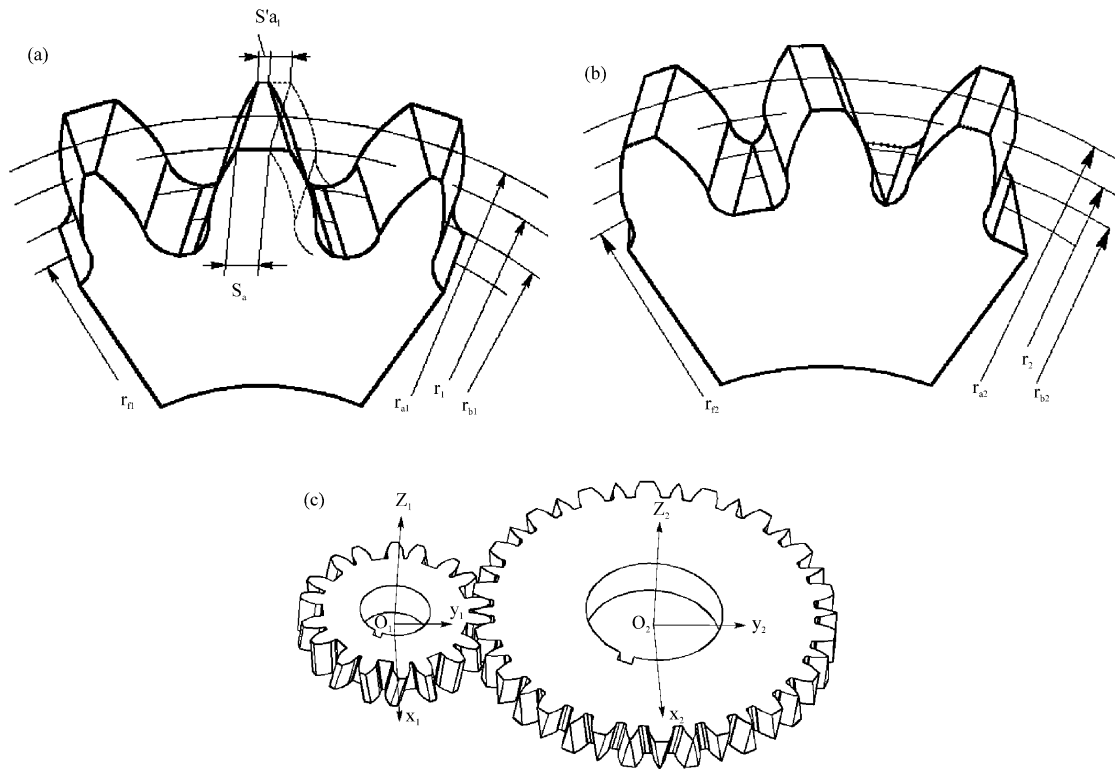


Fig. 2(a-c): Tooth shape of narrow herringbone gear (a) Tooth of wheel, (b) Tooth of pinion and (c) Model of the gear

rotate speeds of pinion and wheel blank are  $\omega_1, \omega_2$ , by the end of generating motion, the teeth faces of pinion and wheel can be generated, respectively by the upside and downside teeth faces of a rack (Wu and Fong, 2009). In the process, several coordinate systems are needed,  $S(xyz)$  is the fixed coordinate system, the origin of  $S$  is at the pitch point on the middle end face, its  $y$  axial parallels with centre line of pinion and wheel;  $S_i(x,y,z_i)$  is fixed on the rack;  $S_1(x_1,y_1,z_1), S_2(x_2,y_2,z_2)$  are fixed on pinion and wheel, respectively. All  $xy$ -plane are situated on the middle end face.

At the end of the generation process, the teeth of pinion and wheel are generated, as shown in Fig. 2a-b and the model of narrow herringbone gear is as shown in Fig. 2c:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} \cos \psi_1 (-u \sin \alpha_1 - p_t/4 + b \tan \beta) + u \sin \psi_1 \cos \alpha_1 - r_1 \psi_1 \cos \psi_1 + r_1 \sin \psi_1 \\ -\sin \psi_1 (-u \sin \alpha_1 - p_t/4 + b \tan \beta) + u \cos \psi_1 \cos \alpha_1 + r_1 \psi_1 \sin \psi_1 + r_1 \cos \psi_1 \\ b \end{pmatrix} \quad (1)$$

$$\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = \begin{pmatrix} \cos \psi_2 (-u \sin \alpha_1 - p_t/4 + b \tan \beta) - u \sin \psi_2 \cos \alpha_1 - r_2 \psi_2 \cos \psi_2 + r_2 \sin \psi_2 \\ -\sin \psi_2 (-u \sin \alpha_1 - p_t/4 + b \tan \beta) - u \cos \psi_2 \cos \alpha_1 + r_2 \psi_2 \sin \psi_2 + r_2 \cos \psi_2 \\ -b \end{pmatrix} \quad (2)$$

**Envelope equations of tooth faces:** The change rules of pinion and wheel teeth faces are periodical, every four teeth faces are a circle, so four teeth faces of rack also a circle and the teeth faces equations of pinion and wheel can be obtained by coordinates transformation of the four teeth faces of the rack. Taking the reference tooth face for example which is shown in Fig. 1, after the generating motion, the envelopes equation of reference face of pinion and wheel are obtained, as shown in Eq. 1 and 2.

In the Eq. 1 and 2,  $u$  and  $\psi_i$  are the variable parameters, the value of  $i$  is 1 and 2, represents pinion and wheel, respectively,  $\alpha_t$  the transverse pressure angle,  $p_t$  the tooth space on the middle end face,  $r_i$  the pitch circle radius,  $b$  the face width.

The requirement of a point being a meshing point is that it must meet the equation of  $n \cdot v = 0$  (Wu, 1982;

Zhang and Wang, 2012),  $n$  is the normal vector of the meshing point,  $v$  is the relative speed of the meshing points on pinion and wheel, so the meshing equations of reference face of pinion and wheel are as follows:

$$f(u, b, \psi)_r = -u - p_i \sin \alpha_i / 4 + b \tan \beta \sin \alpha_i + r_i \psi_i \sin \alpha_i = 0 \quad (3)$$

$$f(u, b, \psi)_{2r} = -u - p_i \sin \alpha_i / 4 + b \tan \beta \sin \alpha_i + r_i \psi_2 \sin \alpha_i = 0 \quad (4)$$

### STRENGTH CALCULATION OF NARROW HERRINGBONE GEAR

**Fillet equations of narrow herringbone gear:** The gear fillet is the equidistant curve of extension epicycloids (Wu and Wang, 1989) while, because of the different tooth shape of pinion and wheel, the curvature radius of the equidistant curve is also different from the helical gear and the tooth shape of generating rack, especially the radiuses of addendum fillet of the rack, are also different from the rack of helical gear, as shown in Fig. 3, the fillet parameter equations of pinion and wheel are as follows:

$$\begin{pmatrix} x_i \\ y_i \\ \varphi \end{pmatrix} = \begin{pmatrix} m_i z \sin \varphi / 2 - \cos(a' - \varphi)(a_i / \sin a' + r_i) \\ m_i z \cos \varphi / 2 - \sin(a' - \varphi)(a_i / \sin a' + r_i) \\ (a_i \cot a' + b_i) / r \end{pmatrix} \quad (5)$$

In the equation, the value of  $i$  is 1 and 2, represents pinion and wheel, respectively,  $a_i, b_i$  are the coordinates of rack fillet center,  $a'$  can change from  $\alpha_i$  to  $90^\circ$ :

$$\begin{cases} a_i = h_a^* m_n - L \tan \alpha_t / 2 \\ b_i = \pi m_t / 4 + h_a^* m_n \tan \alpha_t + L / 2 \\ r_i = L / 2 \cos \alpha_t \\ L = m_i \pi / 2 - 2 m_n \tan \alpha_t - B \tan \beta \end{cases} \quad (6)$$

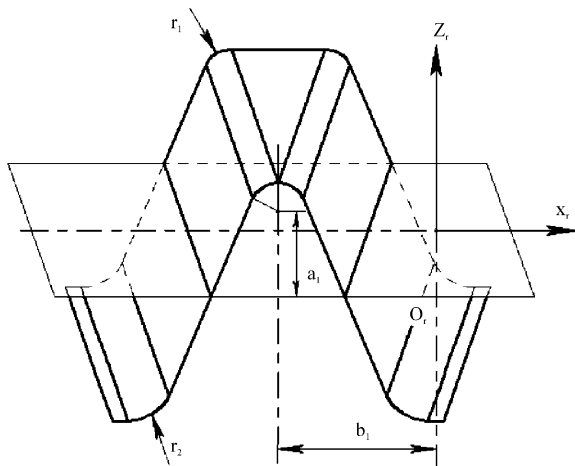


Fig. 3: Shape of rack fillet

$$\begin{cases} a_2 = (h_a^* + c^*) m_n - r_2 \\ b_2 = \pi m_t / 4 + h_a^* m_n \tan \alpha_t + r_2 \cos \alpha_t \\ r_2 = c m_n / (1 - \sin \alpha_t) \end{cases} \quad (7)$$

In the Eq. 6 and 7,  $m_n$  represents the normal module,  $m_t$  the transverse module,  $h_a^*$  the addendum coefficient,  $c^*$  the bottom clearance.

**Equations of gear strength calculation:** The dedendum section shape of narrow herringbone gear is different from the standard helical gear, so the strength calculation equations are also different. The study adopts flat section method (Fan, 2011) in material mechanics to deduce the gear strength equations of pinion and wheel and choose  $a' = 30^\circ$  for the dangerous section, as shown in Fig. 4, the equations are as follows:

$$\sigma_F = \frac{Y_{sa} Y_e Y_\beta M}{W_i} \quad (8)$$

$$\begin{cases} Y_{sa} = (1.2 + 0.16 S_{F1} / h_{F1})(S_{F1} / 2 \rho_{F1})^{S_{F1} / (1.2 S_{F1} + 2.1 h_{F1})} \\ W_1 = B \cos \beta_{c1} S_{F1}^3 / 12 X_1 \\ W_2 = \frac{B^3 (6 S_{F2a}^2 + 6 S_{F2c} S_{F2a} + S_{F2c}^2) \cos^2(\pi/2 - \beta_{c2})}{36(2 S_{F2a} + S_{F2c}) X_2 \cos \beta_{c2}} \\ \quad + \frac{B(S_{F2b}^4 - S_{F2a}^4) \sin^2(\pi/2 - \beta_{c2})}{48(S_{F2b} - S_{F2a}) X_2 \cos \beta_{c2}} \\ S_{F2c} = S_{F2b} - S_{F2a} \end{cases} \quad (9)$$

In the equation, the value of  $i$  is 1 and 2, represents pinion and wheel, respectively,  $\beta_{c1}$  the helix angle on the dangerous section,  $Y_{sa}$  the correction factor of stress,  $Y_e$  the contact ratio parameter,  $Y_\beta$  the helical angle parameter,  $M$  the twisting moment,  $W_i$  bending modulus,  $S_{F1}$  the chordal tooth thickness of the minimum life section,  $h_{F1}$  the distance from point of load to the minimum life section.

The tooth face of narrow herringbone gear is evolute helical tooth face, the meshing characteristics are similar to the helical gear, so the calculation of contact strength can borrow the equation of helical gear:

$$\sigma_H = Z_H Z_E Z_\epsilon Z_\beta \sqrt{\frac{2KT_1}{Bd^2} \cdot \frac{(u \pm 1)}{u}} \leq \sigma_{HP} \quad (10)$$

In the equation,  $Z_H$  represents the coefficient of region,  $Z_E$  the coefficient of elasticity,  $Z_\epsilon$  the contact ratio parameter,  $Z_\beta$  the helical angle parameter.

### LIMITS OF TOPPING

Because tooth thickness of wheel is variable along the axial direction, the narrow side of wheel addendum is

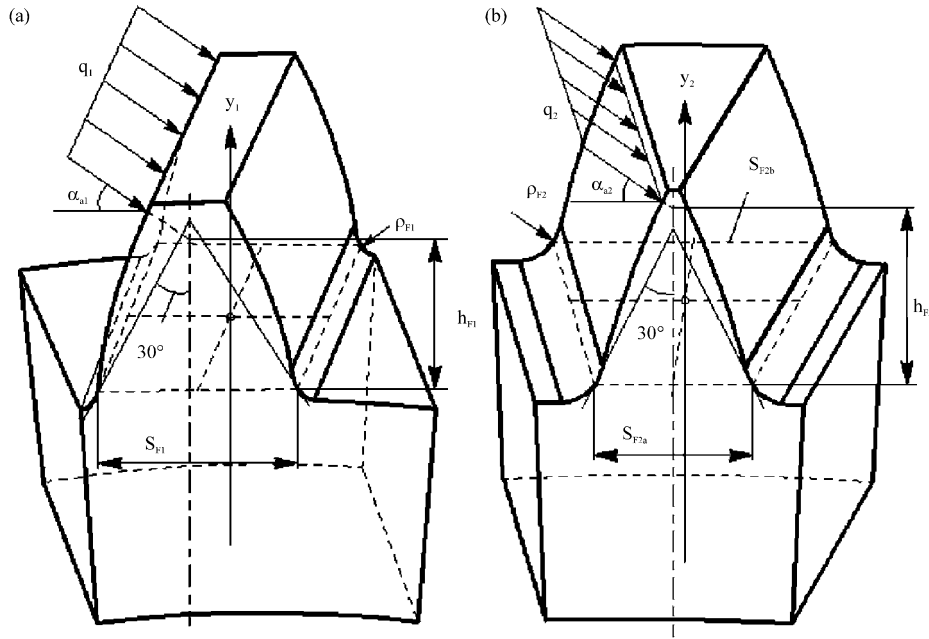


Fig. 4(a-b): Schematic diagram of bending stress calculation (a) Tooth of pinion and (b) Tooth of wheel

easy to be sharpened, in order to avoid this happen, it is needed to make sure the length of  $S_a'$  is greater than zero (Fig. 2a), namely,  $S_a' = S_a - l > 0$ :

$$\begin{cases} l = B \cdot \tan(\beta_a) \\ S_a = m_n \cdot (z_2 + 2) \cdot [\pi / (2z_2) - (\tan \alpha_a - \alpha_a - \tan \alpha_1 + \alpha_1)] \\ S_a' = m_n \cdot (z_2 + 2) \cdot [\pi / (2z_2) - (\tan \alpha_a - \alpha_a - \tan \alpha_1 + \alpha_1) - B \cdot (z_2 + 2) / z_2 \cdot \tan(\beta)] \end{cases} \quad (11)$$

In the equation,  $\beta_a$  represent the helix angle on the addendum cylinder,  $\alpha_a$  the addendum pressure angle. As shown in Eq. 11, there are relationships between addendum sharpening restriction and face width B, wheel tooth number  $z_2$  and module  $m_n$ . The author defines the helical angle as  $\beta_{max}$  when  $S_a' = 0$  and draws the relation curve between  $\beta_{max}$  and B,  $z_2$  and  $m_n$  as shown in Fig. 5. From the Fig. 5a, one know the relationship between  $\beta_{max}$  and B is monotone decreasing; from the Fig. 5b, the relationship between  $\beta_{max}$  and  $z_2$  is monotone increasing; From the Fig. 5c, the relationship between  $\beta_{max}$  and  $m_n$  is monotone increasing, so one can avoid the addendum sharpening by decreasing the face width or increasing the tooth number and the module.

**CALCULATION OF CONTACT RATIO**

The arrangement regulation of the left and the right helical faces of narrow herringbone gear is alternative but

the helical angle is the same. If the basic parameters of narrow herringbone gear and helical gear are the same, the total length of the contact lines is same as the helical gear, it means the contact ratio is the same between narrow herringbone gear and helical gear, so the calculation equation is:

$$\varepsilon = [z_1 (\tan \alpha_{a1} - \tan \alpha') + z_2 (\tan \alpha_{a2} - \tan \alpha')] / 2\pi + B \tan \beta / (\pi m_n) \quad (12)$$

A pair of meshing teeth faces are contacted along a line, the line is called as contact line (Litvin, 2008). The contact line equation of reference face of narrow herringbone gear is shown as Eq. 13, choosing  $z_1 = 32$ ,  $z_2 = 64$ ,  $m_n = 20$ ,  $\beta = 6.5^\circ$ ,  $B = 118$  as the basic parameters, then, the contact ratio is 1.96, drawing the changing curve of total length of contact lines during a meshing period as shown in Fig. 6. From the picture, one can see the value of total length of contact lines is larger than the length of single contact line, it means there are always two teeth meshing and only in some special position, such as  $\psi_1 = [-6.9^\circ, 1.6^\circ, 12.9^\circ]$ , there is only a pair of meshing teeth, so the result of length calculation of contact lines is coincide with the contact ratio calculation by the Eq. 12:

$$\begin{cases} x_t = -u \sin \alpha_1 - p_1/4 + B \tan \beta \\ y_t = u \cos \alpha_1 \\ z_t = B \\ -u - p_1 \sin \alpha_1/4 + B \tan \beta \sin \alpha_1 + r_1 \psi_1 \sin \alpha_1 = 0 \end{cases} \quad (13)$$

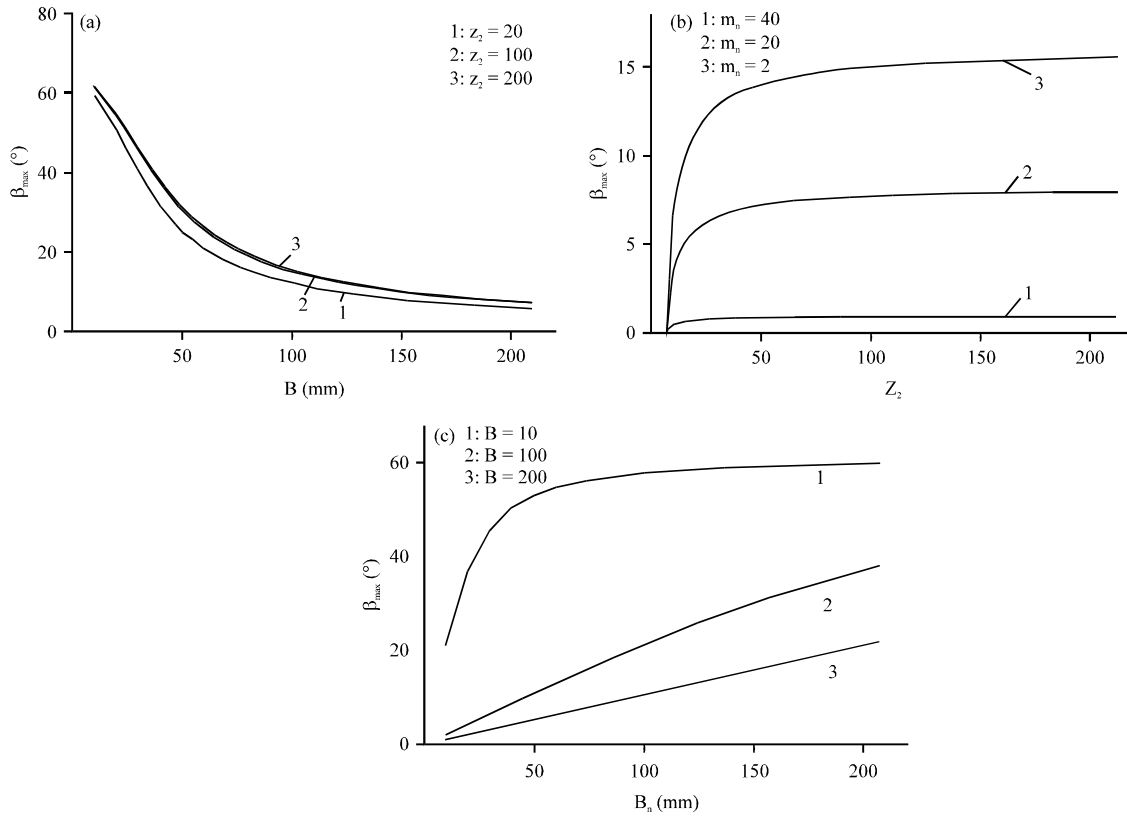


Fig. 5(a-c): Influences of basic parameters upon  $\beta_{max}$  (a)  $\beta_{max}$ -B, (b)  $\beta_{max}$ - $Z_2$  and (c)  $\beta_{max}$ - $m_n$

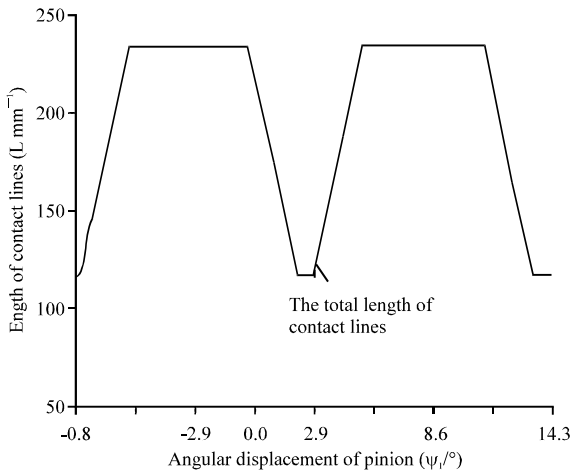


Fig. 6: Curve of total length of contact line

Drawing the relation curve between  $\epsilon$  and module  $m_n$ , face width  $B$  and teeth number  $z$  as shown in Fig. 7, from the Fig. 7a, one know the relationship between  $\epsilon$  and  $m_n$  is monotone decreasing; from the Fig. 7b, the relationship between  $\epsilon$  and  $B$  is monotone increasing;

from the Fig. 7c, the relationship between  $\epsilon$  and  $z$  is monotone increasing.

When one design the herringbone gear, the restriction of addendum sharpening and contact ratio should be taken in to account, so as to enlarge the contact ratio under the restriction of addendum sharpening, only by this way, can one make sure the proper transmission and decrease the axial force.

In the Fig. 7,  $m_n$  represents the normal module,  $B$  the breadth of tooth,  $z_1$  the tooth number of pinion and  $z_2$  the tooth number of wheel.

### SOFTWARE FOR THE DESIGN OF NARROW HERRINGBONE GEAR

Based on the basic theories of narrow herringbone gear design and optimization design, the author develops the design software platform of the narrow herringbone gear, as shown in Fig.8 which can meet the request of design, modeling and finite element analysis of the gear.

The study puts narrow herringbone gear into the practice of marine gearbox design mentioned in the

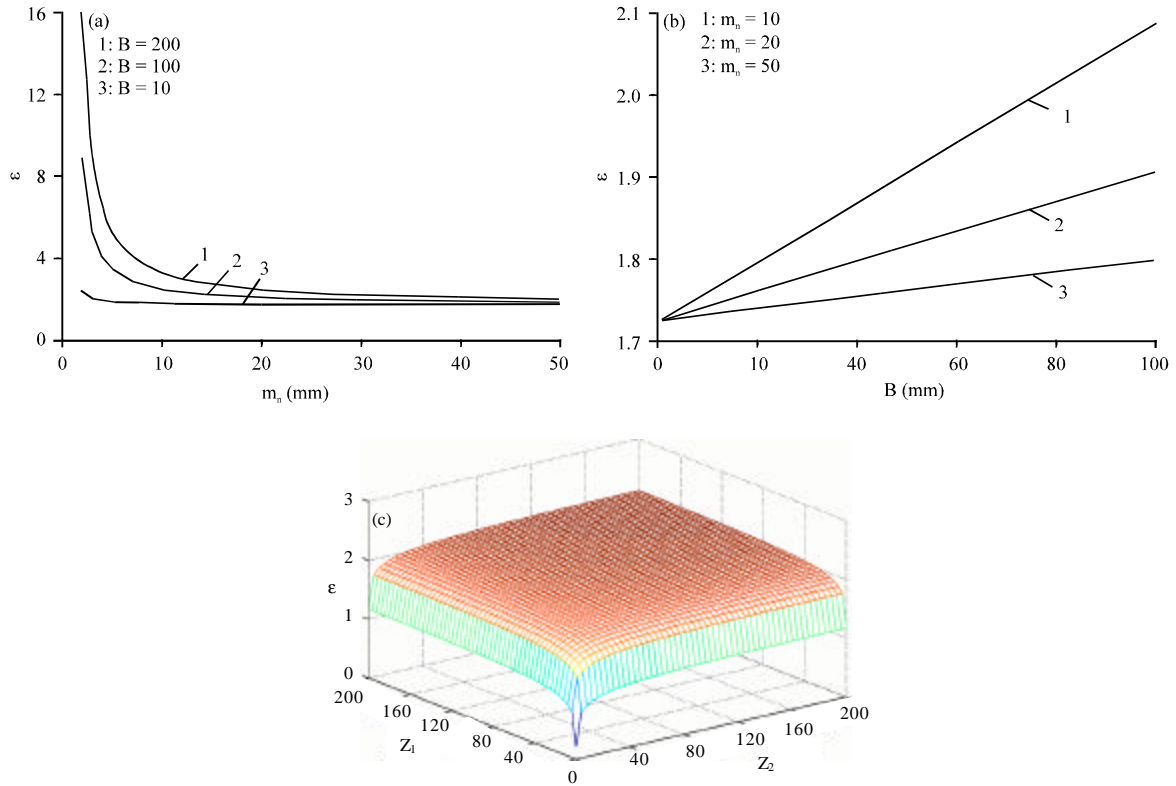


Fig. 7(a-c): Influences of basic parameters upon  $\epsilon$  (a)  $\epsilon$ - $m_n$ , (b)  $\epsilon$ -  $B$  and (c)  $\epsilon$ -  $z$

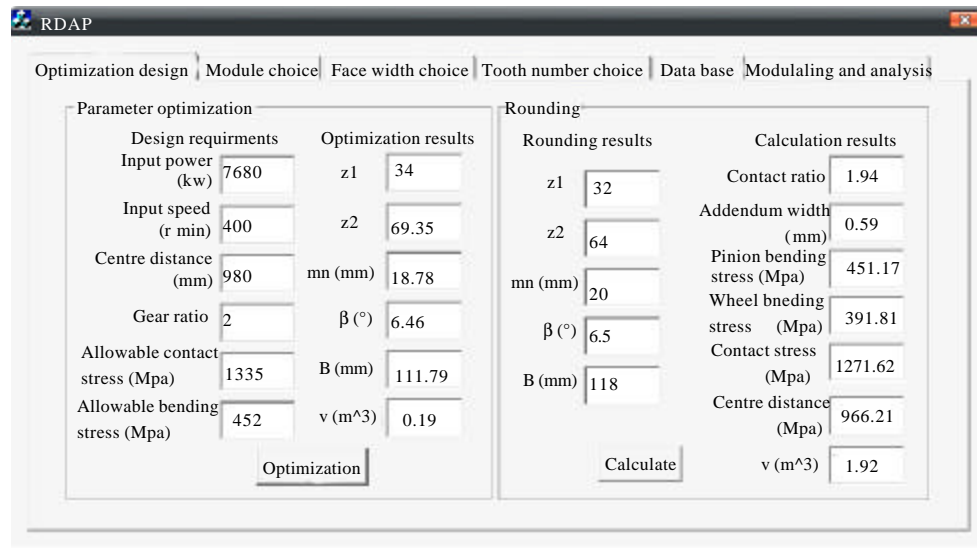


Fig. 8: Design platform of narrow herringbone gear

document (Zhu *et al.*, 2010), the maximum continuous power of the gearbox is  $P = 7\ 680$  kW, the maximum input rotate speed is 400r/min, the material is 20CrMnMo, the design variables are the basic

parameters. In order to make the gearbox more compact, the platform use the volume function for its target function, the volume function as shown in Eq. 14:

Table 1: Basic parameters of narrow herringbone gear

Project	Z <sub>1</sub>	Z <sub>2</sub>	β(°)	m <sub>n</sub> (mm)	B(mm)	Center distance a(mm)	Contact ratio ε	Volume v(m <sup>3</sup> )
Original	27	54	12.4	20	175	829.3	2.3	0.2100
Final	32	64	6.5	20	118	966.0	2.0	0.1923

$$\begin{cases} x=[x_1, x_2, x_3, x_4, x_5]=[z_1, z_2, m_n, \beta, B] \\ \min V=(m_1 z_1/2)^2 \pi B+(m_1 z_2/2)^2 \pi B \end{cases} \quad (14)$$

Constraints consist of the inequalities of contact and bending strength requirements, contact ratio larger than two, no addendum sharpening of wheel and center distance requirements:

$$\begin{cases} \sigma_{F1}=Y_{sa} Y_e Y_\beta M/W_1 < [\sigma_{FP}] \\ \sigma_{F2}=Y_{sa} Y_e Y_\beta M/W_2 < [\sigma_{FP}] \\ \sigma_H=Z_H Z_E Z_\beta Z_\epsilon \sqrt{2KT_1(u+1)/uBd_1^2} \leq [\sigma_{HP}] \\ S'_a=m_1 \cdot (z_2+2) \cdot [\pi/(2z_2) - (\tan \alpha_a - \alpha_a - \tan \alpha_i + \alpha_i)] \\ -B \cdot (z_2+2)/z_2 \cdot \tan(\beta) > 0 \\ \epsilon=[z_1(\tan \alpha_{a1} - \tan \alpha') + z_2(\tan \alpha_{a2} - \tan \alpha')]/2\pi \\ +B \tan \beta / (\pi m_n) - 2 > 0 \\ a=(z_1+z_2)m_1/2 - 980 < 0 \end{cases} \quad (15)$$

Input the relevant parameters, the basic parameters of the input stage gear of marine gearbox can be obtained, as shown in Table 1.

### FORCE ANALYSIS OF NARROW HERRINGBONE GEAR

In the ideal situation, the load is evenly distributed along the contact lines (Zhu and Zhongkai, 1992), because of the alternative contact lines, as shown in Fig. 9a and the variable total length of contact lines, the uniformly distributed load q is also alterable and the axial force F<sub>a</sub> is fluctuated, as shown in Fig. 9b, the value of F<sub>a</sub> can be obtained by the following equation:

$$F_a = q(l_1 - l_2) \sin \beta_b \quad (15)$$

In the formula, l<sub>1</sub> represents the length of left helical face contact line, l<sub>2</sub> the right helical face contact line.

Based on the final basic parameters as shown in Table 1, the model of the narrow herringbone gear is established and the theoretical force analysis is carried out. The result of theoretical force analysis, as shown in the Fig. 10a, can be obtained by using the Eq. 15 which needs calculating the total length of contact lines and the uniformly distributed load q on the lines. From the Fig. 10a, one can see the theoretical force is fluctuant and the amplitude comes to its maximum when the total length of contact lines is minimum. Input the model into ADAMS

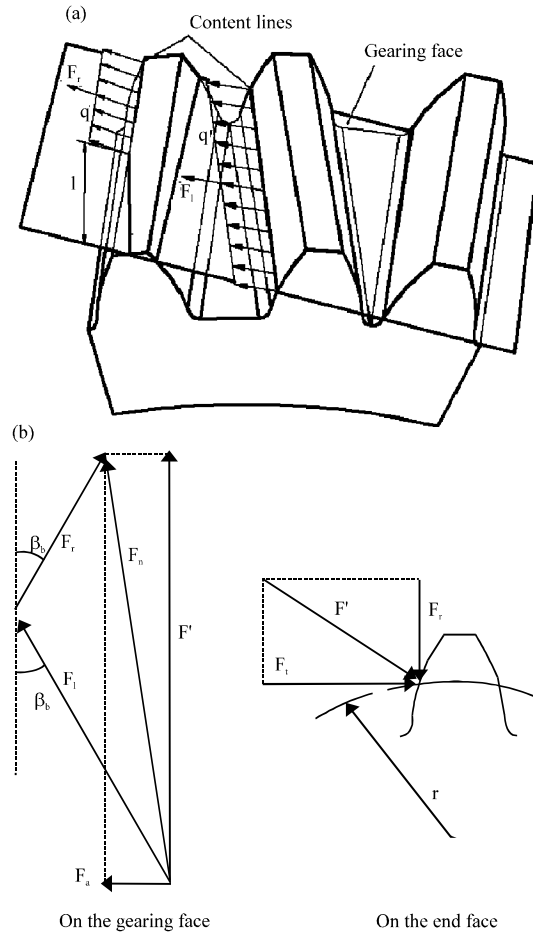


Fig. 9(a-b): Contact force analysis of narrow herringbone gear (a) Contact lines on the gearing face and (b) Sketch map of force analysis

and simulate the contact force during a meshing period (Mao *et al.*, 2012; Wang, 2011), when the rotation angle is at -7.9°, the reference face start to engage in while at 13.9°, the reference face engage out. The simulation result is shown in Fig. 10c, from the figure, one can see the axial force of narrow herringbone gear is fluctuated but its value is lower than the helical gear which has the same parameters with the narrow herringbone gear and the change rule is similar with the theoretical analysis. If one increase the contact ratio to make at least two teeth meshing during a meshing period, the maximum of axial force will be decreased, conversely, decrease the contact ratio, the axial force will be increased. In order to confirm this, another instance that has the different basic



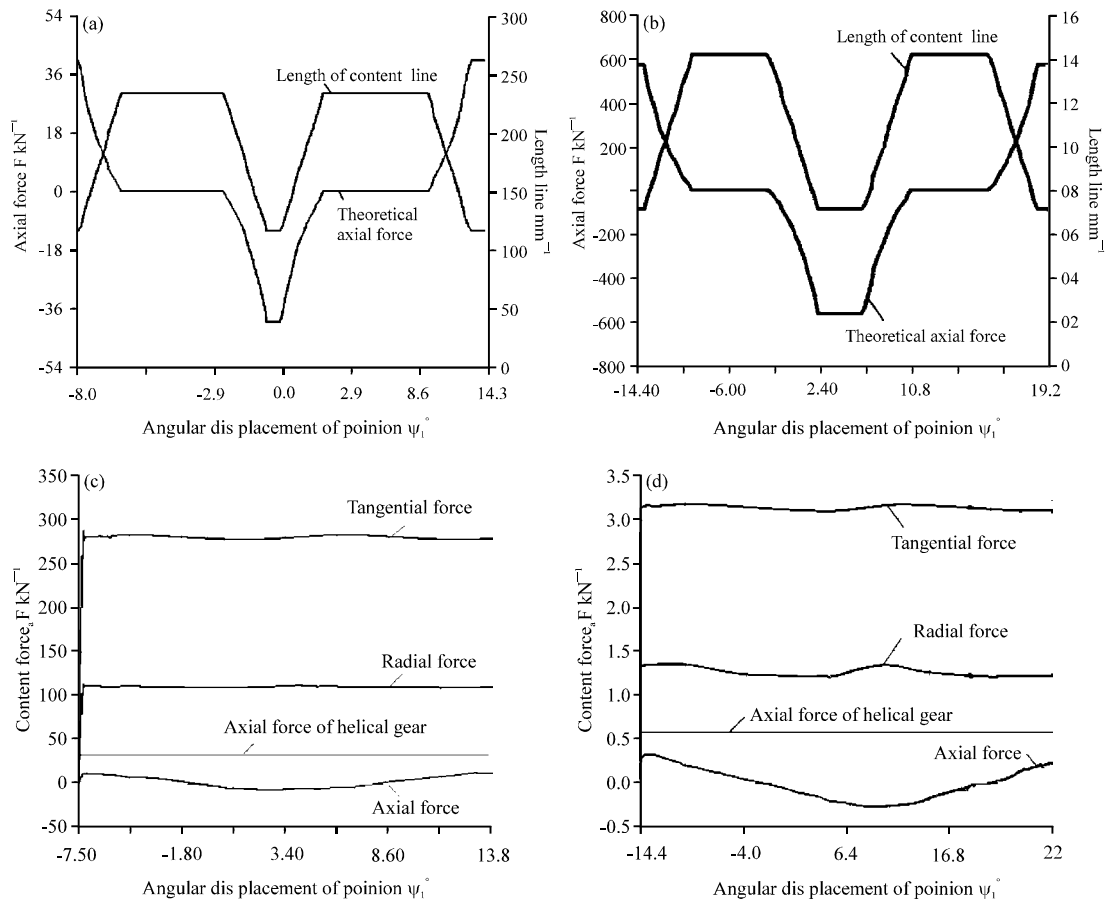


Fig. 10(a-d): Force analysis of narrow herringbone gear (a) Theoretical analysis of instance 1 (b) Theoretical analysis of instance 2, (c) Simulation analysis of instance 1 and (d) Simulation analysis of instance 2

parameters, as  $Z_1 = 18$ ,  $Z_2 = 36$ ,  $m_n = 1.75$  mm,  $\beta = 10^\circ$ ,  $B = 7$  mm,  $\epsilon = 1.89$ ,  $M = 100$  N.m, is carried out. Figure 10b is the theoretical analysis of the instance and Fig. 10d is the simulation analysis result. One can see from the Fig. 10b, d that the value of axial force is increased and the fluctuation is intensified. From the Fig. 10, one can also get that the value of tangential force and the radial force of narrow herringbone gear is the same with the force of helical gear.

**FINITE ELEMENT ANALYSIS OF NARROW HERRINGBONE GEAR**

Based on the basic parameters as shown in Table 1, the three teeth gear models are established. Input the models into ANSYS and calculate the contact and bending stress in a meshing period (Wu and Tsai, 2008) while, the study take seven interpolated calculation positions- $\psi_1 = [-6.9^\circ, -4.8^\circ, -1.6^\circ, 1.4^\circ, 6.5^\circ, 9.7^\circ, 12.9^\circ]$ -as

the whole meshing process,  $\varnothing_1$  represents the rotation angle of pinion. The simulation results, when  $\psi_1 = [-6.9^\circ, 1.6^\circ, 12.9^\circ]$ , are shown in Fig. 11.

Figure 11a represents the finite element analysis result when the rotation angle of pinion  $\psi_1$  is  $-6.9^\circ$ , Fig. 11b the finite element analysis result when  $\psi_1$  is  $-1.6^\circ$ , Fig. 11c the finite element analysis result when  $\psi_1$  is  $12.9^\circ$ . Take the maximum stress on the contact faces at each calculation position as its contact stress and take the maximum stress on the fillet at each calculation position as its bending stress, then, the parallel table of the contact and the bending stress at the seven positions is shown as Table 2.

From the Table 2, one can get that the calculation results of contact and bending stress at the seven positions are lower than the theoretical values, it means that the gear designed by the software is available and the equations of strength calculation is practicability. One can also get that when  $\psi_1 = [-6.9^\circ, 1.4^\circ, 12.9^\circ]$ , the contact and

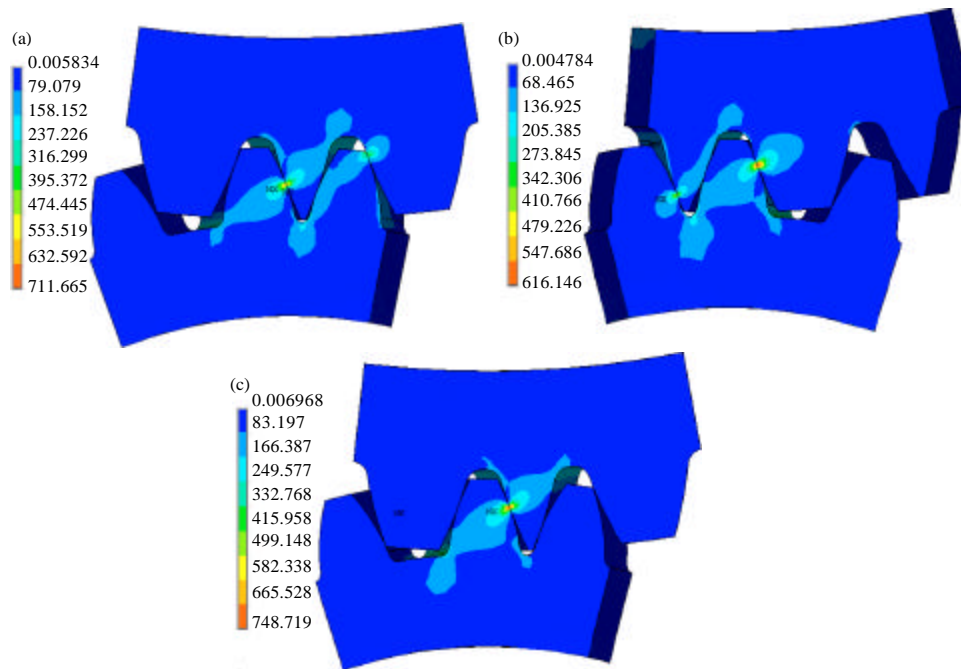


Fig. 11(a-c): Finite element analysis of narrow herringbone gear (a)  $\psi_1 = -6.9^\circ$ , (b)  $\psi_1 = -1.6^\circ$  and (c)  $\psi_1 = 12.9^\circ$

Table 2: Parallel table of the contact and the bending stress (units: MPa)

Project	-6.9°	-4.8°	-1.6°	1.4°	6.5°	9.7°	12.9°	Theoretical
Contact stress	712	630	616	712	627	608	749	825
<b>Bending stress</b>								
Pinion	298	308	292	332	313	304	345	376
Wheel	224	226	213	217	226	211	240	287

the bending stress are larger than the other positions, it is because the length curve of contact lines touches its bottom at these positions as shown in Fig. 6 and the value of uniform distributed load  $q$  reaches its maximum, it means load is mainly imposed on one tooth, so the stress is largest. In order to weaken this kind of stress fluctuation, one had better to make the contact ratio of the gear larger.

### CONCLUSION

The study focuses on the research of new narrow herringbone gear and discusses about its tooth face features and transmission principles. The equations of tooth face and fillet are deduced and the bending strength equations are deduced based on the  $30^\circ$  section method. Restriction of addendum sharpening is discussed and the relationships between the restriction and the basic parameters are revealed. The calculation method of contact ratio is proposed and the relationships between the contact ratio and the basic parameters are discussed.

Applying the interface technologies connecting Visual C++ and MATLAB, UG, Access and ANSYS, the author develops a kind of software that can gather the optimization design, modeling and analysis of the gear together and put it into the practice of the marine gearbox design. Rigid body dynamics analysis and finite element analysis are carried out to testify the practicability of the gear which is designed by the software.

### ACKNOWLEDGMENTS

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