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Two-ordering Newsvendor Based on CVaR Decision Criteria with Information Updating

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Abstract: In this study, we investigate the two-ordering newsvendor with demand information updating. In this background, the retailers can take advantage of the demand information that gathered between the first stage and the second stage to update demand forecast with Bayesian principle. We then establish the two-stage ordering model for maximizing the conditional value at risk (denoted as CVaR) of optimal profits and use dynamic programming methods to analyze the existence and uniqueness of the optimal solution. Next, we design the optimal solution algorithm and give numerical analysis with a real case example in which we discuss that the retailer's decision-making is related to the risk coefficient. The higher the degree of the risk (the smaller of) is, the smaller the value of profits is. The research results show that risk-neutral model is only a special case of CVaR decision rule. This study extends the two-ordering newsvendor from risk-neutral case to CVaR case with information updating.

Key words: Newsvendor, two ordering, conditional value-at-risk (CVaR), information updating

INTRODUCTION

Traditionally, ordering inventory policy model mainly consider the optimization of expected profit or expected cost in a certain period, which is more appropriate for risk neutral decision makers who are insensitive to changes in profit or cost. However, the expectations criterion will be no longer appropriate when decision makers are risk averse. In real life, there are a lot of decision makers trying to avoid the risk, hoping to balance the expected profits and risks to avoid higher losses. Therefore, when decision makers are risk averse, a natural way is to introduce a measure criterion of risk aversion.

Weng (2004) studied the two-stage ordering policy of perishable products considering the scenario of one single seller and one buyer. The seller offers the buyer a chance to order before the selling season with low ordering cost. During the selling season the buyer can still order but the cost associated with the second order is higher than that associated with the first order. Yan *et al.* (2003) developed a model with two ordering alternatives for the retailer?one fast but expensive and the other cheap but slow?and studied the influence it has on profits whether there's information updating. Donohue (2000) introduced the return contract into two-stage ordering policy making the profit distribution between the supplier and the retailer more coordinating. Yao and Cao (2008) also studied the two-stage ordering policy and assumed the decision model under the circumstance of uncertain cost for the second order and the of information updating. Miltenburg and Pong (2007a, b) studied the two-stage ordering policy for multi-products considering the existing of process capacity constraints or not separately and meanwhile the impact of information updating on decisions. Choi *et al.* (2003) put forward a two-stage newsboy model with Bayesian information updating and extended the second stage model on this basis. Chen *et al.* (2006) studied the pricing and inventory joint decision problems considering information updating and random demand, assuming that the supplier repurchased the excesses from the retailer after the selling season and the retailer compensated for the supplier's overproduction before the selling season, which makes the supplier and retailer share the market risk together. Sethi *et al.* (2004) studied the adjustment of order quantity in mid-sale season. By assuming the retailer is allowed to order from other suppliers, the study found out the optimal order quantity in the beginning and mid of the selling season with information updates and analyzed the influence of the accuracy of information updating on decisions.

The method for two-stage ordering policy is usually using information updating to forecast the demand for the second stage after collecting the demand information of the first stage and then make decisions under the principle of maximizing the expected profits. However, the reality

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shows that decision makers are rarely risk neutral. Despite that information updating adds accuracy to the future demand forecasting, risks still exist. Therefore the method using expected profits as a single optimization objective apparently can't reflect the true decision-making behavior and generally speaking, most decision makers are risk averse. Thus, the difference of this study from others is that we take CVaR into the frame of decision-making (Conditional Value-at-Risk, CVaR for short) which reflects the ordering practice of decision makers more veritably and enriches the two-stage ordering model with information updating.

PROBLEM DESCRIPTION AND PARAMETERS DEFINITION

Problem description: The retailer orders and sells products with a short life cycle (perishable products) and has two opportunities to order before the selling season. Before the start of the selling season, there's a long period when the retailer has a chance to order with low cost. Updating demand forecasting before the second order based on information that is accumulated during the first stage, the retailer has a chance to have the quantity adjusted near the selling season with higher cost however. Hence, the retailer has to decide the right order quantity in two stages as to maximize the profit.
Parameters definitions:

- **r:** Retail price of unit product
- **v:** Salvage value of unit product not sold after the end of the selling season
- **b:** Shortage cost of unit product not satisfied during the selling season
- **c_i:** Production cost of unit product at stage i = 1, 2
- **q_1, q_2:** Order quantity at each stage separately (decision variable)
- **x_i:** Demand forecasted at stage i = 1, 2 (Random variable)
- **F_i:** Mean of the demand forecast made at stage i = 1, 2

In order to make the model meaningful, we assume $r>c_i>v>0$ i = 1, 2 by reference to the existing work of Song *et al.* (2011).

Basic assumptions: Assumptions are as below:

- CVaR measures the mean value of the profits below the quartile α and the computations are tractable. Ignoring the profit beyond quartile level, CVaR mainly considers the average profit below the quartile, which is what decision makers are concerned. Under the circumstances of maximum profit
- We define α-CVaR related to q at the confidence level α as:

$$\begin{aligned}&\mathrm{CVaR}_{a}\left(\pi(q,x)\right)\\&=\max_{v\in R}\left\{v-\frac{1}{\alpha}E\left[-\pi(q,x)+v\right]^{+}\right\}\end{aligned}\tag{1}$$

- Demand x_i for each stage is random variable. We assume the demand distribution of each stage is subject to normal distribution with mean and standard deviation d_i, that is x_i~N (F_i, d_i). Mean F_i is also random variable which is assumed subject to normal distribution with mean F_{i-1} and standard deviation σ^2, that is μ_i~N (F_{i-1}, σ^2), i = 1, 2
- In this study, we consider the shortage cost and salvage value of products. The former is caused by the insufficient order to meet the market demands and the latter caused by excessive order that can?t be sold

MODEL

The two-stage ordering for the retailer is sequential decision process which is usually solved by of reverse solution of dynamic programming.

Second-stage ordering

During the second-stage ordering: Based on the known order quantity q_1 of the first-stage, the retailer observed market information $\hat{x}_i$ and update mean μ_1, resulting in a new distribution function of demand. Let the expected profit of the second-stage is $\pi_2(Q)$ where Q refers to the total order quantity of two stages, then we have q_2 = Q-q_1. The retailer's expected profit is:

$$\begin{aligned}\pi_2(Q)&=rE\left[Q-(Q-x_2)^{+}\right]+vE\left[Q-x_2\right]^{+}\\&-bE\left[x_2-Q\right]^{+}-c_2*\max(Q-q_1,0)-c_1q_1\end{aligned}$$

Given that the retailer is risk averse, choose α-CvaR related to Q of the retailer as the objective function where α refers to the retailer's risk level. Accordingly we have:

$$\mathrm{CVaR}\left(\pi_2(Q)\right)=\max_{y\in R}\left\{y-\frac{1}{\alpha}E\left[-\pi_2(Q)+y\right]^{+}\right\}$$

$$g(Q,y)=y-\frac{1}{\alpha}E\left[y-\pi_2(Q)\right]^{+}$$

The decision model of the second-stage is:

$$Q = \arg\max_{Q>0} \max_{y} g(Q, y)$$

By referencing to the detailed solution process from Rockafellar and Uryasev (2002) and Xu *et al.* (2006), we have:

$$Q = \frac{1}{r+b-v}\begin{bmatrix} (r-v)F^{-1}(\alpha\frac{r+b-c_2}{r+b-v}) \\ +bF^{-1}(1-\frac{\alpha(c_2-v)}{r+b-v}) \end{bmatrix}$$

For convenient, let:

$$s_1 = \alpha\frac{r+b-c_2}{r+b-v}$$

$$s_2 = 1-\alpha\frac{c_2-v}{r+b-v}$$

And:

$$A = \frac{\sigma_2}{r+b-v}\left[(r-v)\Phi^{-1}(s_1)+b\Phi^{-1}(s_2)\right]$$

First-stage ordering: when derived to the first-stage in reverse, owing to the definite total order quantity of two stages, the order quantity in the second stage is denoted as q_2 = max{0, Q-q_1} which means q_2 has two types of values 0 and Q-q_1 representing an order or no in the second stage, respectively, resulting in different expressions of expected profits:

- When q_2 = Q-q_1, that is μ_2<q_1-A, substitute this into π_2 (q_2):

$$\begin{aligned} E\pi(q_1) &= (r-c_2)\mu_2+(c_2-c_1)q_1 \\ &+(r+b-c_2)A \; -(r+b \\ &-v)\left[A\Phi(\frac{A}{\sigma_2})+\sigma_2\varphi(\frac{A}{\sigma_2})\right] \end{aligned} \tag{2}$$

- When q_2 = 0, then q_1 = Q, that is μ_2<q_1-A, substitute this into π_2 (q_2):

$$\begin{aligned} E\pi(q_1) &= (r+b-c_1)q_1-b\mu_2-(r+b-v) \\ &\left[(q_1-\mu_2)\Phi(\frac{q_1-\mu_2}{\sigma_2})+\sigma_2\varphi(\frac{q_1-\mu_2}{\sigma_2})\right] \end{aligned} \tag{3}$$

Derive the expected profits in the second stage to the first stage, then in the first stage we have:

$$\begin{aligned} \pi_1(q_1) &= \int_{-\infty}^{q_1-A} E\pi(q_1)f(\mu_2)d\mu_2 \\ &+\int_{q_1-A}^{+\infty} E\pi(q_1)f(\mu_2)d\mu_2 \end{aligned}$$

Accordingly the α-CVaR model related to order quantity q_1 at the confidence level α turns out to be:

$$\max_{q_1} CVaR(\pi_1(q_1)) = \max_{q_1}\{y-\frac{1}{\alpha}E[-\pi_1(q_1)+y]^+\} \tag{4}$$

MODEL ANALYSIS

Lemma: When ordering in the first stage, model:

$$\max_{q_1}\{y-\frac{1}{\alpha}E[-\pi_1(q_1)+y]^+\}$$

is a concave function of order quantity q_1 with only one optimal solution.

Proof: For π_1 (q_1)is a subsection function of q_1

When μ_2<q_1-A, from Eq. 2 we easily obtain that Eπ (q_1) is a linear function of q_1

When:

$$f(q_1) = (q_1-\mu_2)\Phi(\frac{q_1-\mu_2}{\sigma_2})+\sigma_2\varphi(\frac{q_1-\mu_2}{\sigma_2})$$

Then Eq. 3 can be rewritten as:

$$\begin{aligned} E\pi(q_1) &= (r+b-c_1)q_1-b\mu_2-(r+b-v)f(q_1) \\ \frac{\partial^2 E\pi(q_1)}{\partial q_1^2} &= -(r+b-v)\frac{\partial^2 f(q_1)}{\partial q_1^2} \end{aligned}$$

When:

$$x = \frac{q_1-\mu_2}{\sigma_2}$$

Then we have:

$$\begin{aligned} f(q_1) &= \sigma_2[x\Phi(x)+\varphi(x)] \\ \frac{\partial f(q_1)}{\partial q_1} &= \sigma_2[\Phi(x)+x\varphi(x)-x\varphi(x)] = \sigma_2\Phi(k) \\ \frac{\partial^2 f(q_1)}{\partial q_1^2} &= \sigma_2\varphi(x) > 0 \end{aligned}$$

Therefore:

$$\frac{\partial^2 E\pi(q_1)}{\partial q_1^2} < 0$$

And $E\pi$ (q_1) is a concave function of order quantity q_1. It's easy to know π_1 (q_1) is a concave function of order quantity q_1 from the convexity-preserving of functions.

Given that:

$$\max_{q_1}\{y - \frac{1}{\alpha}E[-\pi_1(q_1) + y]^+\}$$

in Eq. 4 is also a concave function of order quantity q_1, there must exist the only maximum.

ALGORITHM DESIGN

When solving q_1, Define a function by reference to the idea of Eq. 3 using the definition of CVaR mentioned above. Let:

$$\begin{aligned} g(q_1, y) = y - \frac{1}{\alpha}\int_0^{q_1 - A}\{y - (r + b - c_1)q_1 + b\mu_2 + \\ (r + b - v)[(q_1 - \mu_2)\Phi(\frac{q_1 - \mu_2}{\sigma_2}) + \sigma_2\varphi(\frac{q_1 - \mu_2}{\sigma_2})]\}^+ dF(\mu_2) \\ - \frac{1}{\alpha}\int_{q_1 - A}^{\infty}\{y - (r - c_2)\mu_2 + (c_2 - c_1)q_1 + (r + b - c_2)A \\ -(r + b - v)[A\Phi(\frac{A}{\sigma_2}) + \sigma_2\varphi(\frac{A}{\sigma_2})]\}^+ dF(\mu_2) \end{aligned}$$

where, the value of q_1 corresponding to the maximum g (q_1, y) is the optimal solution required. Though it is difficult to find out the explicit expression after observation, specific examples can be given through numerical examples.

First of all, set a value of q_1, g (q_1, y) and it's easy to figure out the maximum of g (q_1, y) for it's a single variable function of y. Let q_1 change to find the maximum of g (q_1, y) and the corresponding q_1 is the optimal solution required.

Given that it's difficult to realize the Integral form in expression g (q_1, y), sampling is adopted for computation using:

$$\int_a^b h(x)dF(x) = \frac{\sum_{i=1}^{N} h(x_i)}{N} \qquad (N \to \infty)$$

to convert integrals into sums, where $x_1, x_2, \cdots, x_N$ are N, samples from F (x).

Set a value of q_1 and generate N random numbers μ_{21}, μ_{22}, μ_{23}, $\mu, \cdots$, μ_{2N} conforming to μ_2 distribution. By comparing each μ_{2i} with q_1-A, if $\mu_{2i} \le q_1$-A, substitute μ_{2i} into f_m; otherwise substitute μ_{2I} into f_n where:

$$\begin{aligned} f_m = (r + b - c_1)q_1 - b\mu_2 - (r + b - v) \\ [(q_1 - \mu_2)\Phi(\frac{q_1 - \mu_2}{\sigma_2}) + \sigma_2\varphi(\frac{q_1 - \mu_2}{\sigma_2})] \end{aligned}$$

$$\begin{aligned} f_n = (r - c_2)\mu_2 - (c_2 - c_1)q_1 - (r + b - c_2)A \\ + (r + b - v)[A\Phi(\frac{A}{\sigma_2}) + \sigma_2\varphi(\frac{A}{\sigma_2})] \end{aligned}$$

Sort the N values of function in ascending order and we have $f_1 \le f_2 \le f_3 \le \cdots \le f_N$. Thus, g (q_1, y) can be written as a piecewise linear function of y. If $N\alpha$ is an integer, we have:

$$\max g(q_1, y) = N\alpha - \frac{N^2\alpha^2 - \sum_{i=1}^{N\alpha} f_i}{N\alpha}$$

Otherwise the maximum of the function is achieved in $[N\alpha]$ or $[N\alpha]+1$. Afterwards we can use any one dimensional research to find the optimal q1 according to the optimal g (q_1, y) obtained from every the previous q_1.

NUMERICAL ANALYSIS

Let the price of product r = 120 yuan per unit; the retailer's production costs in the first and second stage are c_1 = 23 yuan per unit and c_2 = 26 yuan per unit; the shortage cost of unit product not satisfied during the selling season is b = 60 yuan per unit; the salvage value of unit product not sold after the end of the selling season is v = 20 yuan per unit.

The demand in the first stage is x_1 satisfying $x_i \sim N$ (F_i, σ_1). Let μ_1 = 100, σ_1 = 9 and the updating market information observed between two orders is $\hat{x}_1$ = 144.

Results are shown as follows after corresponding generation.

As shown in Table 1, when risk level α = 1.0, the retailer is risk neutral with the maximum ordering quantity. The smaller α is, the more risk averse the retailer is, with a consequence of ever-educed optimal order quantity and profit. In fact, few decision makers are risk neutral, which means most decision makers try to avoid risk. Thus risk should be taken into account given that profit as the only decision goal cannot reflect the accurate real demand.

Table 1: Influence of d risk level on the retailer's decision and profit

α	1.0	0.8	0.6	0.4	0.1
q_1	168.09	166.53	164.93	163.01	161.98
Q	317.79	282.46	240.25	205.27	187.48
E (Q)	11264.00	10982.00	10493.00	9783.30	8325.80

Table 2 : Influence of changing on order quantity and profit

R	130	120	100	80	60
Q_1	164.97	164.93	164.86	164.79	164.72
Q	236.10	240.25	250.02	262.52	279.12
E (Q)	11662.00	10493.00	8146.40	5783.30	3395.40

Then, process sensitivity analysis of parameters under the condition of $\alpha = 0.6$.

As shown in Table 2, the reduction of r causes a reduction of the first order quantity and total profit and an increase of total order quantity, which makes it clear that market price has apparent influence on the retailer's decision.

Similarly we get conclusions as follows: The increase of c_1 will result in the reduction of the first order quantity and profit, which demonstrates the retailer will reduce the order quantity to decrease the uncertainty. The decrease of c_2 will cause reduction of not only order quantity of two stages but also profit. The increase of b will result in the reduction of the first order quantity and profit and the increase of total order quantity and the range of the former is less apparent than that of the latter. As with the decrease of salvage v, not only the first and total order quantity has varying degrees of reduction but also the profit decreased gradually.

CONCLUSIONS

This study finds out he retailer's optimal ordering policy by studying the two-stage ordering model considering decision-maker's attitude towards risk with demand forecasting updates.The research shows that the decision results change with the given level of risk and variables such as market price, stage cost, salvage value and shortage cost will affect decision-making. This study is aimed to allow decision makers to make appropriate decisions based on their own different level of risk as not mentioned in previous two-stage models where all decision makers are assumed risk neutral which is obviously not in accordance with personal attitude towards risk. The method CVaR used in the model reflects the decision maker's psychological feelings more veritably in comparison with traditional ordering models.

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