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Two Types of the MacWilliams Identities of the $F_p+uF_p+\cdots+u^{k-1}F_p$-Linear Codes

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Abstract: Error-correcting coding theory is an important theoretieal basis of information security. And the MacWilliams identity of the code is an important branch of error-correcting coding theory. In recent years, the research interest of many scholars engaged in coding theory have been transferred to the finite ring. Researches on MacWilliams identities over finite rings have not only important theory meanings but also important practical value. Many achievements about the weight distribution of the code over the ring have been made. Let $R = F_p+uF_p+\cdots+u^{k-1}F_p$. In this study, the MacWilliams identities of the R-linear codes are discussed. Firstly, the complete weight enumerator and the symmetrized weight enumerator of R-linear codes are defined. Secondly, the complete weight MacWilliams identity and the symmetrized weight MacWilliams identity are given by using a special variable t. Finally, an example are given to show the use of two types of MacWilliams identities. This study improves the error-correcting coding theory of the ring R and promotes its actual application.

Key words: Linear code, complete weight enumerator, symmetrized weight enumerator, MacWilliams identity

INTRODUCTION

The weight distribution of the code is an important branch of coding theory. In recent years, the research interest of many scholars engaged in coding theory will be transferred to the finite ring. And a lot of achievements about the weight distribution of the code over the ring have been made (Hammons *et al*., 1994; Shiromoto, 1996). Various weight distribution of MacWilliams identities of the linear codes over the ring Z_4 were studied by Wan (1997). The generalized MacWilliams identities of Z_4-linear codes were given by Cui and Pei (2004). Various weight distribution of MacWilliams identities between the F_2+vF_2-linear codes and its dual codes were obtained in Shi *et al*. (2008). The Lee weight MacWilliams identities of the $F_2+uF_2+u^2F_2$ linear codes were discussed in Liang and Tang (2010). Two kinds of the MacWilliams identities of the F_p+uF_p-linear codes were researched in Li and Chen (2010). The complete weight and the Lee weight MacWilliams identities of the $F_2+uF_2+vF_2+uvF_2$-linear codes were discussed by Yildiz and Karadeniz (2010). Recently, the MacWilliams identities of the $F_p+uF_p+u^2F_p$ linear codes were considered by Xu and Mao (2013).

In this study, the definitions of the complete weight enumerator and the symmetrized weight enumerator of the linear codes over the ring $F_p+uF_p+\cdots+u^{k-1}F_p$ are given firstly. Secondly, the complete weight MacWilliams identity and the symmetrized weight MacWilliams identity over the ring $F_p+uF_p+\cdots+u^{k-1}F_p$ are obtained. Finally, an example will be given to illustrate the use of these two types of MacWilliams identities.

Basic concepts of $F_p+uF_p+\cdots+u^{k-1}F_p$-linear codes: Consider the ring $R = F_p+uF_p+\cdots+u^{k-1}F_p$, where $u^k = 0$ and p is prime. It is obvious that R is a finite chain ring with the ideals:

$$I_{u^k} = \{0\} \subseteq I_{u^{k-1}} = <u^{k-1}> \subseteq I_{u^{k-2}} = <u^{k-2}> \subseteq \cdots \subseteq I_u = <u> \subseteq R \qquad (1)$$

where, $I_{u^l} = <u^l> = u^lF_p + u^{l+1}F_p + \cdots + u^{k-1}F_p (l = 0,1,2,\cdots,k-1)$.

If the code C over the ring R is an R-submodule of R_n, then C is said to be linear:

$$\forall x = (x_1, x_2,\cdots, x_n),\ y = (y_1, y_2,\cdots, y_n) \in R_n$$

the inner product of x, y is defined by the following:

$$<x, y> = x_1y_1+x_2y_2+\cdots+x_ny_n$$

The dual code of C is defined to be the set $C^\perp = \{x | <x, y> = 0, \forall y \in C\}$.

Definition 1: The complete weight enumerator of the code C over R is defined by:

$$Cwe_C(X_0, X_1, \cdots, X_{(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)}) = \sum_{\bar{c}\in C} (X_0^{n_0(\bar{c})} X_1^{n_1(\bar{c})} \cdots X_{(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)}^{n_{(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)}(\bar{c})})$$

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where, $n_{g_i(\bar{c})}$ is the number of appearances of g_i in the codeword $\bar{c}$.

Definition 2: Classify all elements of R to k+1 subsets, as:

$$D_0 = I_{u^k} = \{0\}, D_1 = I_{u^{k-1}} \setminus I_{u^k}, D_2 = I_{u^{k-2}} \setminus I_{u^{k-1}}, \cdots, D_k = R \setminus I_u$$

Function I $(\cdot)$ is defined as:

$$I(a) = i$$

where, $a \in D_i (i = 0, 1, \cdots k)$.

Definition 3: The symmetrized weight enumerator of the R-linear code C is defined as:

$$Swe_C(X_0, X_1, \cdots, X_k) = Cwe_C(X_{I(0)}, X_{I(1)}, \cdots, X_{I((p-1)+u(p-1)+\cdots+u^{k-1}(p-1))})$$

COMPLETE WEIGHT MACWILLIAMS IDENTITY OF THE R-LINEAR CODES

Lemma 4: An abstract t will be introduced and the exponents of t will be elements of R such that $t^{u^{k-1}} = e^{2\pi i/p}, t^{a+b} = t^a t^b$, where $a, b \in R$, then $\sum_{g \in I} t^g = 0$ for all non-zero ideals I of R.

Proof: Let:

$$g = u^l a_l + u^{l+1} a_{l+1} + \cdots + u^{k-1} a_{k+1} \in I_{u^l}$$

where l = 0, 1, 2,..., k-1, then:

$$\begin{aligned}
&\sum_{g \in I_{u^l}} t^g \\
&= \sum_{a_l \in F_p} \sum_{a_{l+1} \in F_p} \cdots \sum_{a_{k-1} \in F_p} t^{u^l a_l + u^{l+1} a_{l+1} + \cdots + u^{k-1} a_{k-1}} \\
&= \sum_{a_l \in F_p} \sum_{a_{l+1} \in F_p} \cdots \sum_{a_{k-2} \in F_p} t^{u^l a_l + u^{l+1} a_{l+1} + \cdots + u^{k-2} a_{k-2}} \Big(\sum_{a_{k-1} \in F_p} (t^{u^{k-1}})^{a_{k-1}} \Big) \\
&= \sum_{a_l \in F_p} \sum_{a_{l+1} \in F_p} \cdots \sum_{a_{k-2} \in F_p} t^{u^l a_l + u^{l+1} a_{l+1} + \cdots + u^{k-2} a_{k-2}} \Big(\sum_{a_{k-1}=0}^{p-1} (t^{u^{k-1}})^{a_{k-1}} \Big) \\
&= \sum_{a_l \in F_p} \sum_{a_{l+1} \in F_p} \cdots \sum_{a_{k-2} \in F_p} t^{u^l a_l + u^{l+1} a_{l+1} + \cdots + u^{k-2} a_{k-2}} \frac{1 - e^{2\pi i}}{1 - e^{2\pi i/p}} = 0
\end{aligned}$$

Theorem 5: Let C be the linear R-code of length n and let $C^\perp$ be its dual. With t as defined above, then:

$$\begin{aligned}
&Cwe_{C^\perp}(X_0, X_1, \cdots, X_{(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)}) \\
&= \frac{1}{|C|} cwe_C\Big(\sum_{l \in R} t^{0 \cdot l} X_l, \sum_{l \in R} t^{1 \cdot l} X_l, \ldots, \sum_{l \in R} t^{[(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)] l} X_l\Big).
\end{aligned}$$

Proof: For any $F(\bar{c}) = (c_1, c_2 \cdots, c_n) \in C$, let:

$$F(\bar{c}) = \sum_{\bar{x} \in R^n} t^{<\bar{c}, \bar{x}>} X_0^{n_0(\bar{x})} X_1^{n_1(\bar{x})} \cdots X_{(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)}^{n_{(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)}(\bar{x})}$$

then:

$$\begin{aligned}
&\sum_{\bar{c} \in C} F(\bar{c}) \\
&= \sum_{\bar{c} \in C} \sum_{\bar{x} \in R^n} t^{<\bar{c}, \bar{x}>} X_0^{n_0(\bar{x})} X_1^{n_1(\bar{x})} \cdots X_{(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)}^{n_{(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)}(\bar{x})} \\
&= \sum_{\bar{x} \in R^n} (X_0^{n_0(\bar{x})} X_1^{n_1(\bar{x})} \cdots X_{(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)}^{n_{(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)}(\bar{x})}) \sum_{\bar{c} \in C} t^{<\bar{c}, \bar{x}>}
\end{aligned}$$

Now, for fixed $\bar{x} \in R_n$, the function $f_{\bar{x}}$ will be considered from C to R. $f_{\bar{x}}$ is defined by $f_{\bar{x}}(\bar{c}) = <\bar{c}, \bar{x}>$. By the structure of the inner product, $f_{\bar{x}}$ can be proved to be an R-module homomorphism. Then, by the definition of the dual code, the following equivalent conditions hold true:

$$\ker(f_{\bar{x}}) = C \Leftrightarrow <\bar{c}, \bar{x}> = 0 (\forall \bar{c} \in C) \Leftrightarrow \bar{x} \in C^\perp$$

Then, for any $\bar{x} \in C^\perp$, the above equivalent conditions imply that $\sum_{\bar{x} \in C} t^{<\bar{c}, \bar{x}>} = |C|$.

Now, suppose that $\bar{x} \notin C^\perp$, this implies that $\ker(f_x) + C$. By the property of the homomorphism, $Im(f_{\bar{x}})$ can be verified to be a non-zero sub-module of R and hence a non-zero ideal of R. Then, $\sum_{\bar{x} \in C} t^{<\bar{c}, \bar{x}>} = 0$ can be obtained by Lemma 4 when $\bar{x} \notin C^\perp$. This means that:

$$\begin{aligned}
&\sum_{\bar{c} \in C} F(\bar{c}) \\
&= |C| \sum_{\bar{c} \in C^\perp} X_0^{n_0(\bar{x})} X_1^{n_1(\bar{x})} \cdots X_{(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)}^{n_{(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)}(\bar{x})} \\
&= |C| \, cwe_{C^\perp}\left(X_0, X_1, \cdots, X_{(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)}\right),
\end{aligned}$$

which is equivalent to:

$$cwe_{C^\perp}\left(X_0, X_1, \cdots, X_{(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)}\right) = \frac{1}{|C|} \sum_{\bar{c} \in C} F(\bar{c}) \quad (2)$$

On the other hand, let $\delta(x, y)$ denote the Kronecker Delta function:

$$\delta(x, y) = \begin{cases} 1 & x = y \\ 0 & x \neq y \end{cases}$$

So:

$$\begin{aligned}
&F(\bar{c}) \\
&= \sum_{\bar{x} \in R^n} t^{<\bar{c}, \bar{x}>} X_0^{n_0(\bar{x})} X_1^{n_1(\bar{x})} \cdots X_{(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)}^{n_{(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)}(\bar{x})} \\
&= \sum_{(x_1, x_2, \ldots, x_n) \in R^n} \left(\prod_{j=1}^{n} (t^{c_j x_j} X_0^{\delta(x_j, 0)} X_1^{\delta(x_j, 1)} \cdots X_{(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)}^{\delta(x_j, (p-1)+u(p-1)+\cdots+u^{k-1}(p-1))} \right) \\
&= \prod_{j=1}^{n} \left(t^{c_j \cdot 0} X_0 + t^{c_j \cdot 1} X_1 + \cdots + t^{c_j \cdot [(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)]} X_{(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)} \right) \\
&= \prod_{j \in R} \Big(\sum_{l \in R} t^{j \cdot l} X_l \Big)^{n_j(\bar{c})}.
\end{aligned}$$

By the definition of the complete weight enumerator, the following identity can be obtained:

$$\sum_{\bar{c} \in C} F(\bar{c}) = cwe_C\Big(\sum_{i=1}^{p^k} t^{g_1 \cdot g_i} X_i, \sum_{i=1}^{p^k} t^{g_2 \cdot g_i} X_i, \ldots, \sum_{i=1}^{p^k} t^{g_{p^k} \cdot g_i} X_i\Big). \quad (3)$$

Combining 2 with 3, the theorem 5 can be proved.

SYMMETRIZED WEIGHT MACWILLIAMS IDENTITY OF THE R-LINEAR CODES

Lemma 6: With the same notations as the definition 2, then, when p>2,the following proposition hold true:

- $|D_0| = 1$, $|D_1| = p-1$, $|D_2| = p(p-1)$, $|D_3| = p^2(p-1)$,···, $|D_k| = p^{k-1}(p-1)$
- $\sum_{g\in D_0} t^g = 1, \sum_{g\in D_1} t^g = -1, \sum_{g\in D_l} t^g = 0 (l = 2,3,\ldots,k-1)$

Proof 1:

- It can be easy to be proved by the definition of Di(i = 0, 1, 2,···k-1)
- It is easy to know that $\sum_{g\in D_0} t^g = \sum_{g\in D_0} t^0 = 1$. By the lemma 4, then:

$$\sum_{g\in D_1} t^g = \sum_{g\in I_{k-1}} t^g - \sum_{g\in D_0} t^g = 0 - 1 = -1$$

When $g\in D_l(l = 2, 3\cdots, k-1)$, let:

$$g = u^{k-l}a_{k-l} + u^{k-l+1}a_{k-l+1} + \cdots + u^{k-1}a_{k-1}$$

where, $a_{k-l}\in F_P\backslash\{0\}$, $ah\in F_P$ (h = k-l+1, k-l+2,..., k-1 then:

$$\begin{aligned}\sum_{g\in D_l} t^g &= \sum_{a_{k-l}\in F_p\backslash\{0\}} \sum_{a_{k-l+1}\in F_p} \cdots \sum_{a_{k-2}\in F_p} \sum_{a_{k-1}\in F_p} t^{u^{k-l}a_{k-l}+u^{k-l+1}a_{k-l+1}+\cdots+u^{k-1}a_{k-1}} \\ &= \sum_{a_{k-l}\in F_p\backslash\{0\}} \sum_{a_{k-l+1}\in F_p} \cdots \sum_{a_{k-2}\in F_p} t^{u^{k-l}a_{k-l}+u^{k-l+1}a_{k-l+1}+\cdots+u^{k-2}a_{k-2}} \left(\sum_{a_{k-1}\in F_p} t^{u^{k-1}a_{k-1}}\right) \\ &= \sum_{a_{k-l}\in F_p\backslash\{0\}} \sum_{a_{k-l+1}\in F_p} \cdots \sum_{a_{k-2}\in F_p} t^{u^{k-l}a_{k-l}+u^{k-l+1}a_{k-l+1}+\cdots+u^{k-2}a_{k-2}} \left(\sum_{a_{k-1}=0}^{p-1} (t^{u^{k-1}})^{a_{k-1}}\right) \\ &= \sum_{a_{k-l}\in F_p\backslash\{0\}} \sum_{a_{k-l+1}\in F_p} \cdots \sum_{a_{k-2}\in F_p} t^{u^{k-l}a_{k-l}+u^{k-l+1}a_{k-l+1}+\cdots+u^{k-2}a_{k-2}} \frac{1-e^{2\pi i}}{1-e^{2\pi i/p}} = 0\end{aligned}$$

Lemma 7: With the same notations as the definition 2, the following proposition hold true:

- If $g\in D_0$, then $\sum_{g_i\in D_s} t^{g\cdot g_i} = |D_s| \ (s = 0,1,\ldots,k)$
- If $g\in D_1$, then $\Sigma_{g_i\in D_s} t^{g\cdot g} = |D_s|$ (s = 0, 1, 2, ..., k-1) $\Sigma_{g_i\in DK}$ $t^{g\cdot gi} = -p^{k-1}$
- If $g\in D_l(l = 2, 3,\cdots k)$, then $\sum_{g_i\in D_s} t^{g\cdot g_i} = |D_s| \ (s = 0,1\cdots,k-l)$, $\sum_{g_i\in D_{k-l+1}} t^{g\cdot g_i} = -p^{k-1}$, $\sum_{g_i\in D_s} t^{g\cdot g_i} = 0 (s = k-l+2, k-l+3,\cdots,k)$

Proof: The above proposition (3) will be chosen to prove,others including the proposition (1) and (2) are similar to be proved.

If $g\in D_l(l = 2, 3\cdots, k)$, let:

$$g = u^{k-l}a_{k-l} + u^{k-l+1}a_{k-l+1} + \cdots + u^{k-1}a_{k-l}$$

where, $a_{k-l}\in F_P\backslash\{0\}a_h\in F_p$ (h = k-l+1, k-l+2,···, k-1)

When $g_h\in D_0$ then:

$$\sum_{g_h\in D_0} t^{g\cdot 0} = \sum_{g_h\in D_0} t^0 = \sum_{g_h\in D_0} 1 = 1$$

When $g_h\in D_1$ then:

$$\sum_{g_h\in D_1} t^{g\cdot g_h} = \sum_{b_{k-1}\in F_p\backslash\{0\}} t^{(u^l a_l + u^{l+1}a_{l+1}+\cdots+u^{k-1}a_{k-1})\cdot u^{k-1}b_{k-1}} = \sum_{b_{k-1}\in F_p\backslash\{0\}} t^0 = p-1$$

And so on:

$$\sum_{g_h\in D_s} t^{g\cdot g_h} = \sum_{g_h\in D_s} t^0 = \sum_{g_h\in D_s} 1 = |D_s| \ (s = 0,1,\cdots,k-l)$$

When $g_h\in D_{k-l+1}$, let:

$$g_h = u^{l-1}b_{l-1} + u^l b_l + u^{l+1}b_{l+1} + \cdots + u^{k-1}b^{k-1}$$

where, $b_{l-1}\in F_p\backslash\{0\}$, $b_d\in F_p$(d = l, l+1,···,k-1) then:

$$\begin{aligned}&\sum_{g_h\in D_{k-l+1}} t^{g\cdot g_h} \\ &= \sum_{b_{l-1}\in F_p\backslash\{0\}} \sum_{b_l\in F_p} \cdots \sum_{b_{k-1}\in F_p} t^{(u^{k-l}a_{k-l}+u^{k-l+1}a_{k-l+1}+\cdots+u^{k-1}a_{k-1}).(u^{l-1}b_{l-1}+u^l b_l+u^{l+1}b_{l+1}+\cdots+u^{k-1}b_{k-1})} \\ &= \sum_{b_{l-1}\in F_p\backslash\{0\}} \sum_{b_l\in F_p} \cdots \sum_{b_{k-1}\in F_p} t^{u^{k-1}a_{k-l}b_{l-1}} = p^{k-1} \sum_{b_{l-1}\in F_p\backslash\{0\}} t^{u^{k-1}a_{k-l}b_{l-1}} \\ &= p^{k-1} \sum_{b\in F_p\backslash\{0\}} t^{u^{k-1}b} = p^{k-1}\sum_{g\in D_1} t^g = -p^{k-1}\end{aligned}$$

When $g_h\in SD_{k-1+2}$, let:

$$g_h = u^{l-2}b_{l-2} + u^{l-1}b_{l-1} + \cdots + u^{k-1}b_{k-1}$$

where, $b_{l-2}\in F_p\backslash\{0\}$, $b_d\in F_p$ (d = l-1, l,···,k-1), then:

$$\begin{aligned}&\sum_{g_h\in D_{k-l+2}} t^{g\cdot g_h} \\ &= \sum_{b_{l-2}\in F_p\backslash\{0\}} \sum_{b_{l-1}\in F_p} \cdots \sum_{b_{k-1}\in F_p} t^{(u^{k-l}a_{k-l}+u^{k-l+1}a_{k-l+1}+\cdots+u^{k-1}a_{k-1}).(u^{l-2}b_{l-2}+u^{l-1}b_{l-1}+\cdots+u^{k-1}b_{k-1},)} \\ &= \sum_{b_{l-2}\in F_p\backslash\{0\}} \sum_{b_{l-1}\in F_p} \cdots \sum_{b_{k-1}\in F_p} t^{u^{k-2}a_{k-l}b_{l-2}+u^{k-1}a_{k-l}b_{l-1}} \\ &= p^{k-l+3} \sum_{b_{l-2}\in F_p\backslash\{0\}} t^{u^{k-2}a_{k-l}b_{l-2}} \sum_{b_{l-1}\in F_p} t^{u^{k-1}a_{k-l}b_{l-1}} \\ &= 0\end{aligned}$$

Evidenced by the same token:

$$\sum_{g_i\in D_s} t^{g\cdot g_i} = 0 (s = k-l+2, k-l+3,\cdots,k)$$

Theorem 8: Let C be the R-linear code of length n, then:

$$swe_{C^\perp}(X_0, X_1, \ldots, X_k) = \frac{1}{|C|} swe_C(Y_0, Y_1, \ldots, Y_k)$$

Where:

$$
\begin{aligned}
Y_0 &= X_0+(p-1)X_1+p(p-1)X_2+p^2(p-1)X_3+\cdots+p^{k-1}(p-1)X_k \\
Y_1 &= X_0+(p-1)X_1+p(p-1)X_2+p^2(p-1)X_3+\cdots+p^{k-2}(p-1)X_{k-1}-p^{k-1}X_k \\
Y_2 &= X_0+(p-1)X_1+p(p-1)X_2+p^2(p-1)X_3+\cdots+p^{k-3}(p-1)X_{k-2}-p^{k-2}X_{k-2} \\
Y_3 &= X_0+(p-1)X_1+p(p-1)X_2+p^2(p-1)X_3+\cdots+p^{k-4}(p-1)X_{k-3}-p^{k-3}X_{k-2} \\
&\ldots\ldots \\
Y_{k-1} &= X_0+(p-1)X_1-pX_2 \\
Y_k &= X_0-X_1
\end{aligned}
$$

Proof: By the definition of the symmetrized weight enumerator and the theorem 3.3, then:

$$
\begin{aligned}
&swe_{C^\perp}(X_0, X_1, \ldots, X_7) \\
&= cwe_{C^\perp}(X_{I(0)}, X_{I(1)}, \cdots, X_{I((p-1)+u(p-1)+\cdots+u^{k-1}(p-1))}) \\
&= \frac{1}{|C|} cwe_C\Big(\sum_{l\in R} t^{0\cdot 1} X_{I(1)}, \sum_{l\in R} t^{1\cdot 1} X_{I(1)}, \ldots, \sum_{l\in R} t^{[(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)]\cdot 1} X_{I(1)}\Big). \\
&= \frac{1}{|C|} cwe_C\Big(\sum_{s=0}^{k} \sum_{g_i\in D_s} t^{0\cdot g_i} X_s, \sum_{s=0}^{k} \sum_{g_i\in D_s} t^{1\cdot g_i} X_s, \ldots, \sum_{s=0}^{k} \sum_{g_i\in D_s} t^{[(p-1)+u(p-1)+\cdots+u^{k-1}(p-1)]\cdot g_i} X_s\Big)
\end{aligned}
$$

By the lemma 7, then:

$$
swe_{C^\perp}(X_0, X_1, \ldots, X_k) = \frac{1}{|C|} swe_C(Y_0, Y_1, \ldots, Y_k),
$$

$$
Y_j = \sum_{s=0}^{k} \sum_{g_i\in D_s} t^{g\cdot g_i} X_s \; (g \in D_j, j = 0,1,2,\ldots k)
$$

Thus the theorem 8 can be proved.

Corollary 9: Let C be the linear code of length n over F_p+uF_p, then:

$$
swe_{C^\perp}(X_0, X_1, X_2) = \frac{1}{|C|} swe_C(Y_0, Y_1, Y_2)
$$

Where:

$Y_0 = X_0+(p-1)X_1+(p^2-p)X_2$, $Y_1 = X_0+(p-1)X_1-pX_2$, $Y_2 = X_0-X_1$

Proof: By letting k = 2 into the theorem 8, the corollary can be easily verified. This is consistent with the results in literature (Li and Chen, 2010).

Corollary 10: Let C be the linear code of length n over $F_p+uF_p+u^2F_p$, then:

$$
swe_{C^\perp}(X_0, X_1, X_2, X_3) = \frac{1}{|C|} swe_C(Y_0, Y_1, Y_2, Y_3)
$$

Where:

$$
\begin{aligned}
Y_0 &= X_0+(P-1)X_1+(p^2-p)X_2+(p^3-p^2)X_3 \\
Y_1 &= X_0+(P-1)X_1+(p^2-p)X_2-p^2X_3 \\
Y_2 &= X_0+(P-1)X_1-pX_2 \\
Y_3 &= X_0-X_1
\end{aligned}
$$

Proof: By letting k = 3 into the theorem 8, the corollary can be easily verified. This is consistent with the results in literature (Xu and Mao, 2013).

Example: In order to show the application of theorem 5 and the theorem 8, an example will be given in this section.

Example 1: The MacWilliams identities of a linear code over the ring F_2+uF_2.

Let C = {(0, 0), (u, u)} be the linear code of length 2 over the ring F_2+uF_2, then the complete weight MacWilliams identity of the code C is:

$$
Cwe_C(X_0, X_1, X_u, X_{1+u}) = X_0^2 + X_u^2.
$$

So, the symmetrized weight MacWilliams identity of the code C is:

$$
Swe_C(X_0, X_1, X_2) = X_0^2 + X_1^2
$$

By the theorem 5, the complete weight MacWilliams identity of the code $C^\perp$ is:

$$
\begin{aligned}
&Cwe_{C^\perp}(X_0, X_1, X_u, X_{1+u}) \\
&= \frac{1}{2}\{(X_0+X_1+X_u+X_{1+u})^2 + (X_0-X_1+X_u-X_{1+u})^2\} \\
&= X_0^2+X_1^2+X_u^2+X_{1+u}^2+2X_0X_u+2X_1X_{1+u}
\end{aligned}
$$

By the complete weight MacWilliams identity of the dual code $C^\perp$, the elements of the dual code $C^\perp$ can be obtained easily, namely:

$C^\perp$ = {(0, 0), (1, 1), (u, u), (1+u, 1+u), (0, u), (u, 0), (1, 1+u), (1+u, 1)}

It also suggests that the dual code of $C^\perp$ can be obtained by the complete weight MacWilliams identity.

By the theorem 8, the symmetrized weight MacWilliams identity of the code $C^\perp$ is:

$$
\begin{aligned}
&Swe_{C^\perp}(X_0, X_1, X_2) \\
&= \frac{1}{|C|} Swe_C(X_0+X_1+2X_2, X_0+X_1-2X_2, X_0-X_1) \\
&= \frac{1}{2}\{(X_0+X_1+2X_2)^2 + (X_0+X_1-2X_2)^2\} \\
&= X_0^2+X_1^2+2X_0X_1+4X_2^2
\end{aligned}
$$

CONCLUSION

In this study, two kinds of MacWilliams identities of the linear codes over the ring R were studied. Another direction for research in this topic is of course the constacyclic codes and the dual codes over the ring R.

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