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Generator Matrix of the Linear Codes and Gray Images over the Ring $F_2+vF_2+v^2F_2$

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Abstract: The study of linear codes and their Gray images over finite rings play an important role in the coding theory. Codes over rings have been studied extensively in the past decade. An important aspect of these rings is that they are all finite chain rings and also they are all principal ideal rings. In this study, focus was on linear codes and their Gray images over the ring $R = F_2+v F_2+v^2F_2$, which is a semi-local ring but not principal or finite chain. We defines a Gray map from the ring R to F_2, then obtains the generator matrix of the dual code $C^\perp$ and Gray image Ψ (C) from the generator matrix of the linear code C over the ring R.

Key words: Linear codes, generator matrix, gray image, dual code

INTRODUCTION

The study of linear codes and their Gray images over finite rings has obtained much useful results in coding theory (Carlet, 2000, Ling and Blackford, 2002). The two main classes of rings that have been studied are Galois rings and rings of the $F_{2^n}+uF_{2^n}$ and some variations of these (Dinh, 2009, 2010). Codes over F_3+uF_3 were studied and improvements to the bounds on ternary linear codes (Gulliver and Harada, 2001). In 2010, linear codes and cyclic codes over $F_2+uF_2+vF_2+uvF_2$ were studied where the ring $F_2+uF_2+vF_2+uvF_2$ is not a finite chain ring (Yildiz and Karadeniz, 2010, 2011). Linear codes and cyclic codes over the ring F_2+uF_2 were studied where the ring F_2+vF_2 is not a finite chain ring (Zhu *et al*., 2009, 2010). In order to popularize the conclusion of the coding theory over F_2+vF_2, we study the coding theory over the ring $F_2+vF_2+v^2F_2$ in this study.

BASIC CONCEPTS OF THE CODES OVER THE RING $F_2+v F_2+v^2F_2$

Let $R = \{a+bv+cv^2\}|a ,b, c\in F_2\}$, where $v^3 = v$. Note that R is a semi-local ring with characteristic 2 which is not finite chain. The ideals can be listed as:

$$\langle 0\rangle, \langle 1\rangle, \langle v\rangle = \langle v^2\rangle = \{0, v, v^2, v+v^2\}$$

$$\langle 1+v\rangle = \{0, 1+v, 1+v^2, v+v^2\}$$

$$\langle 1+v^2\rangle = \{0, 1+v^2\}$$

$$\langle v+v^2\rangle = \{0, v+v^2\}$$

$\langle v\rangle$ and $\langle 1+v\rangle$ are two maximal ideals of the ring R. The element 1 and $1+v+v^2$ are two units of R. The zero divisors in R are all in $\{0, v ,v^2, 1+v, 1+1+v^2, v+v^2\}$.

A linear code over the ring R of length n is an R-submodule of R^n. For any $x = (x_1, x_2,..., x_n)$, $y = (y_1, y_2,..., y_n)$, the inner product of x, y is defined as the following:

$$\langle x, y\rangle = x_1y_1 + x_2y_2 + ... + x_ny_n$$

Let C be a linear code of length n over R, then we can prove that $C^\perp = \{x|\langle x,y\rangle = 0, \forall y\in C\}$ is also a linear code over R of length n. We call $C^\perp$ to be the dual code of C.

STRUCTURE OF THE LINEAR CODE OVER THE RING $F_2+v F_2+v^2F_2$

Let $\tilde{C}$ and C are all linear codes over R of length n. If the code C can be transformed to $\tilde{C}$ through the transformation of coordinates (when necessary, v and 1+v can be interchanged), we call C permutation-equivalent to $\tilde{C}$. Similar to the literature (Ozen and Siap, 2006), the following theorem can be easily obtained.

Theorem 1: Any linear code over R of length n is permutation-equivalent to a code with generator matrix of the form:

$$G = \begin{pmatrix} I_{k_1} & A_1 & A_2 & A_{31}+vA_{32} & A_{41}+vA_{42} & A_5 \\ 0 & vI_{k_2} & B_1 & B_{21}+vB_{22} & vB_{31} & B_4 \\ 0 & 0 & (1+v)I_{k_3} & (1+v)C_1 & (1+v)C_2 & C_3 \\ 0 & 0 & 0 & (v+v^2)I_{k_4} & (v+v\ )D_1 & D \\ 0 & 0 & 0 & 0 & (1+v^2)I_{k_5} & E \end{pmatrix}_{k\times n} \quad (1)$$

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where, $I_{k_1}, I_{k_2}, I_{k_3}, I_{k_4}, I_{k_5}$ are all unit matrixes with order k_1, k_2, k_3, k_4, k_5, respectively. Let:

$$k = k_1 + k_2 + k_3 + k_4 + k_5$$

$$A_5 = A_{51} + vA_{52} + v^2 A_{53},\ B_4 = vB_{41} + v^2 B_{42}$$

$$C_1 = C_{11} + vC_{12} + v^2 C_{13},\ C_3 = (1+v)(C_{31} + vC_{32} + v^2 C_{33})$$

$$D = (v + v^2) D_2,\ \tilde{E} = (1 + v^2) E$$

Where:

$$A_1, A_2, A_{31}, A_{32}, A_{41}, A_{42}, A_{51}, A_{52}, A_{53}, B_1, B_{21}$$

$$B_{22}, B_{31}, B_{41}, B_{42}, C_{11}, C_{12}, C_{13}, C_2, C_{31}, C_{32}, C_{33}, D_1, D_2,$$

E are matrixes over the ring F_2.

Theorem 2: If C is an arbitrary linear code of $F_2 + v F_2 + v^2 F_2$, then the generator matrix of the dual code $C^{\perp}$ is:

$$H = \begin{pmatrix} K & J_2 & H_3 & E^T D_1^T + D_2^T & E^T & I_{n-k} \\ vJ_1 & v^2 H_2 & vF_4 & vD_1^T & vI_{k_5} & 0 \\ (1+v)H_1 & (1+v)F_2 & (1+v)F_3 & (1+v)I_{k_4} & 0 & 0 \\ (v+v^2)F_1 & (v+v^2)B_1^T & (v+v^2)I_{k_3} & 0 & 0 & 0 \\ (1+v^2)A_1^T & (1+v^2)I_{k_2} & 0 & 0 & 0 & 0 \end{pmatrix}_{(n-k_1)\times n} \quad (2)$$

where, $F_1 = B_1^T A_1^T + A_2^T$:

$$F_2 = (C_{11}^T + vC_{12}^T + v^2 C_{13}^T) B_1^T + B_{21}^T + vB_{22}^T$$

$$F_3 = C_{11}^T + vC_{12}^T + v^2 C_{13}^T$$

$$F_4 = D_1^T (C_{11}^T + vC_{12}^T + v^2 C_{13}^T) + C_2^T$$

$$\begin{aligned} H_1 = {} & (C_{11}^T + vC_{12}^T + v^2 C_{13}^T)(B_1^T A_1^T + A_2^T) \\ & + (B_{21}^T + vB_{22}^T) A_1^T + A_{31}^T + vA_{32}^T, \end{aligned}$$

$$\begin{aligned} H_2 = {} & D_1^T (C_{11}^T + vC_{12}^T + v^2 C_{13}^T) B_1^T + C_2^T B_1^T \\ & + D_1^T (B_{21}^T + vB_{22}^T) + B_{31}^T, \end{aligned}$$

$$\begin{aligned} H_3 = {} & E^T D_1^T (C_{11}^T + vC_{12}^T + v^2 C_{13}^T) + E^T C_2^T + D_2^T (C_{11}^T \\ & + vC_{12}^T + v^2 C_{13}^T) + C_{31}^T + vC_{32}^T + v^2 C_{33}^T \end{aligned}$$

$$\begin{aligned} J_2 = {} & E^T D_1^T (C_{11}^T + vC_{12}^T + v^2 C_{13}^T) B_1^T + E^T C_2^T B_1^T \\ & + E^T D_1^T (B_{21}^T + vB_{22}^T) + E^T B_{31}^T + (C_{31}^T + vC_{32}^T \\ & + v^2 C_{33}^T) B_1^T + vB_{41}^T + v^2 B_{42}^T + D_2^T (B_{21}^T + vB_{22}^T) \end{aligned}$$

$$\begin{aligned} J_1 = {} & D_1^T (C_{11}^T + vC_{12}^T + v^2 C_{13}^T)(B_1^T A_1^T + A_2^T) \\ & + D_1^T (B_{21}^T + vB_{22}^T) A_1^T + D_1^T (A_{31}^T + vA_{32}^T) \\ & + C_2^T A_2^T + B_{31}^T A_1^T + A_{41}^T + vA_{42}^T \end{aligned}$$

$$\begin{aligned} K = {} & E^T J_1 + D_2^T [(C_{11}^T + vC_{12}^T + v^2 C_{13}^T) A_2^T + A_{31}^T + vA_{32}^T \} \\ & + (C_{31}^T + vC_{32}^T + v^2 C_{33}^T)(A_2^T + B_1^T A_1^T) + E^T C_2^T B_1^T A_1^T \\ & + D_2^T (B_{21}^T + vB_{22}^T) A_1^T + A_{51}^T + vA_{52}^T + v^2 A_{53}^T \end{aligned}$$

Proof: Let C' be a linear code of R with generator matrix 1. Because $GH^T = 0$, we have $C' \subseteq C^{\perp}$. So, in order to prove the theorem, we need to prove that $C^{\perp} \subseteq C'$.

For any c = $(c_1, c_2, \ldots c_n)$, by putting some linear combination of the first n-k rows in 2 to c, we can transform c to:

$$\begin{aligned} c' = {} & (c_1, \cdots, c_{k_1}, c_{k_1+1}, \cdots, c_{k_1+k_2}, c_{k_1+k_2+1}, \cdots, c_{k_1+k_2+k_3}, \\ & c_{k_1+k_2+k_3+1}, \cdots, c_{k-k_5}, c_{k-k_5+1}, \cdots, c_k, 0, \cdots, 0) \end{aligned}$$

where, the last n-k positions of c' are all 0. Because c' is orthogonal to the last k_5 rows in Eq. 1, so $c_{k-k_5+1}, \cdots, c_k$ must all 0 or $v+v^2$. Then, by putting some linear combination of the middle k_5 rows in Eq. 2 to c', we can transform c' to:

$$\begin{aligned} c'' = {} & (c_1, \cdots, c_{k_1}, c_{k_1+1}, \cdots, c_{k_1+k_2}, c_{k_1+k_2+1}, \cdots, \\ & c_{k_1+k_2+k_3}, c_{k_1+k_2+k_3+1}, \cdots, c_{k-k_5}, 0, \cdots, 0) \end{aligned}$$

where, the last $n-k+k_5$ positions of c" are all 0. Because c" is orthogonal to the middle k_4 rows in Eq. 1, so $c_{k_1+k_2+k_3+1}, \cdots, c_{k-k_5}$ must all 0 or $1+v^2$ or $1+v$. Then, by putting some linear combination of the middle k_4 rows in Eq. 2 to c", we can transform c" to:

$$c''' = (c_1, \cdots, c_{k_1}, c_{k_1+1}, \cdots, c_{k_1+k_2}, c_{k_1+k_2+1}, \cdots, c_{k_1+k_2+k_3}, 0, \cdots, 0)$$

where, the last $n-k_1-k_2-k_3$ positions of c''' are all 0. Because c''' is orthogonal to the middle k_3 rows in Eq. (1), so $c_{k_1+k_2+1}, \cdots, c_{k_1+k_2+k_3}$ must all 0 or $v+v^2$. Then, by putting some linear combination of the middle k_3 rows in Eq. (2) to c''', we can transform c''' to:

$$c^{(4)} = (c_1, \cdots, c_{k_1}, c_{k_1+1}, \cdots, c_{k_1+k_2}, 0, \cdots, 0)$$

where, the last $n-k_1-k_2$ positions of $c^{(4)}$ are all 0. Because $c^{(4)}$ is orthogonal to the middle k_2 rows in Eq. 1, so $c_{k_1+1}, \cdots, c_{k_1+k_2}$ must all 0 or $1+v^2$. Then, by putting some linear combination of the last k_2 rows in Eq. 2 to $c^{(4)}$, we can transform $c^{(4)}$ to $c^{(5)} = (c_1, \cdots, c_{k_1}, 0, \cdots, 0)$, where, the last $n-k_1$ positions of $c^{(5)}$ are all 0. Also, $c^{(5)}$ is orthogonal to the first k_1 rows in Eq. 1, so $c_1, \cdots, c_{k_1}$ must all 0. Then, we have $0 \in C'$, so $c = (c_1, c_2, \cdots, c_n) \in C'$. Therefore, we have proved the theorem.

GRAY IMAGE OF THE LINEAR CODES OVER THE RING F_2+v F_2+v^2F_2

For any $\bar{x} \in R$, then $\bar{x} = a + vb + v^2c$ $(a,b,c \in F_2)$.

Define Ψ: $R \to F_2^3$ by: $\Psi(\bar{x})$. then Ψ is a ring homomorphism. The Lee weight of $\bar{x}$ are defined by $W_L(\bar{x}) = W(\psi(x))$. For any $\bar{x}, \bar{y} \in F_2 + vF_2 + v^2F_2$ we have :

$$W_L(\bar{x} - \bar{y}) = d_L(\bar{x}, \bar{y}) = d(\psi(\bar{x}), \psi(\bar{y}))$$
$$= W(\psi(\bar{x}) - \psi(\bar{y}))$$

The Gray map Ψ can be extended to R^n. For any $x = (x_1, x_2, \cdots, x_n) \in R^n$ let $x_i = a_i + vb_i + v^2c_i \in R$ then, for any x, we have:

$$\psi(x) = (a_1, a_2, \cdots, a_n, c_1, c_2, \cdots, c_n,$$
$$a_1 + b_1 + c_1, \cdots, a_n + b_n + c_n).$$

It is obviously that Ψ is a bijective from R^n to F_2^{3n}.

By the definition of the Gray map Ψ, we can obtained the following lemma easily.

Lemma 3: The Gray map Ψ is a distance preserving map from R^n to F_2^{3n}.

Theorem 4: Let C be a linear code of length n over the ring R with generator matrix of the form Eq. 1, $\Psi(C)$ is the Gray image of C. Then, $\Psi(C)$ is permutation-equivalent to a linear binary code of length 3n with generator matrix of the form:

$$M = \begin{pmatrix}
I_{k_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
A_1^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
A_2^T & I_{k_2} & B_1^T & 0 & 0 & 0 & 0 & 0 & 0 \\
A_{31}^T & C_{11}^T & B_{21}^T & 0 & 0 & 0 & 0 & 0 & 0 \\
A_{41}^T & C_2^T & 0 & I_{k_5} & 0 & 0 & 0 & 0 & 0 \\
A_{51}^T & C_{31}^T & 0 & E^T & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & I_{k_1} & 0 & 0 & 0 & 0 \\
0 & 0 & I_{k_3} & 0 & A_1^T & I_{k_3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & A_2^T & 0 & I_{k_1} & 0 & 0 \\
A_{32}^T & C_{12}^T + C_{13}^T & B_{22}^T & 0 & A_{31}^T + A_{32}^T & B_{22}^T & C_{11}^T + C_{12}^T + C_{13}^T & I_{k_4} & A_{32}^T \\
A_{42}^T & 0 & B_{31}^T & I_{k_5} & A_{41}^T + A_{42}^T & B_{31}^T & C_2^T & D_1^T & A_{42}^T \\
A_{52}^T + A_{53}^T & C_{32}^T + C_{33}^T & B_{41}^T + B_{42}^T & E^T & A_{51}^T + A_{52}^T + A_{53}^T & B_{41}^T & C_{31}^T + C_{32}^T + C_{33}^T & D_2^T & A_{52}^T \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_{k_1} \\
0 & 0 & 0 & 0 & 0 & I_{k_2} & 0 & 0 & A_1^T \\
0 & 0 & 0 & 0 & 0 & B_1^T & 0 & 0 & A_2^T \\
0 & 0 & 0 & 0 & 0 & B_{21}^T + B_{22}^T & 0 & 0 & A_{31}^T + A_{32}^T \\
0 & 0 & 0 & 0 & 0 & B_{31}^T & 0 & 0 & A_{41}^T + A_{42}^T \\
0 & 0 & 0 & 0 & 0 & B_{41}^T + B_{42}^T & 0 & 0 & A_{51}^T + A_{52}^T + A_{53}^T
\end{pmatrix}^T$$

where, A_1, A_2, A_{31}, A_{32}, A_{41}, A_{42}, A_{51}, A_{52}, A_{53}, B_1, B_{21}, B_{22}, B_{31}, B_{241} , B_{42}, C_{11}, C_{12}, C_{13}, C_2, C_{31}, C_{32}, C_{33}, D_1, D_2, E are matrixes over the ring F_2.

Proof: By the theorem 1 and the definition of the Gray map Ψ, $\Psi(C)$ can be generated by linear combination of the Gray image of the row vector of the following matrix $\tilde{G}$:

$$\tilde{G} = \begin{pmatrix}
I_{k_1} & A_1 & A_2 & A_{31} + vA_{32} & A_{41} + vA_{42} & A_{51} + vA_{52} + v^2A_{53} \\
vI_{k_1} & vA_1 & vA_2 & vA_{31} + v^2A_{32} & vA_{41} + v^2A_{42} & v(A_{51} + A_{53}) + v^2A_{52} \\
v^2I_{k_1} & v^2A_1 & v^2A_2 & v^2A_{31} + vA_{32} & v^2A_{41} + vA_{42} & v^2(A_{51} + A_{53}) + vA_{52} \\
0 & vI_{k_2} & B_1 & B_{21} + vB_{22} & vB_{31} & B_4 \\
0 & v^2I_{k_2} & vB_1 & vB_{21} + v^2B_{22} & v^2B_{31} & vB_4 \\
0 & 0 & (1+v)I_{k_3} & (1+v)C_1 & (1+v)C_2 & C_3 \\
0 & 0 & (1+v^2)I_{k_3} & (1+v^2)C_1 & (1+v^2)C_2 & (1+v)C_3 \\
0 & 0 & 0 & (v+v^2)I_{k_4} & (v+v^2)D_1 & D \\
0 & 0 & 0 & 0 & (1+v^2)I_{k_5} & \tilde{E}
\end{pmatrix}$$

Because:

$$\psi(I_{k_1}, A_1, A_2, A_{31} + vA_{32}, A_{41} + vA_{42}, A_{51} + vA_{52} + v^2A_{53})$$
$$= (I_{k_1}, A_1, A_2, A_{31}, A_{41}, A_{51}, 0, 0, 0, 0, 0, A_{53}, I_{k_1}, A_1, A_2,$$
$$A_{31} + A_{32}, A_{41} + A_{42}, A_{51} + A_{52} + A_{53})$$

$$\psi(v^2I_{k_1}, v^2A_1, v^2A_2, v^2A_{31} + vA_{32}, v^2A_{41} + vA_{42}, v^2A_{51} + vA_{52} + v^2A_{53})$$
$$= (0, 0, 0, 0, 0, 0, I_{k_1}, A_1, A_2, A_{31}, A_{41}, A_{51} + A_{53}, I_{k_1}, A_1, A_2,$$
$$A_{31} + A_{32}, A_{41} + A_{42}, A_{51} + A_{52} + A_{53})$$

$$\psi(vI_{k_1}, vA_1, vA_2, vA_{31} + v^2A_{32}, vA_{41} + v^2A_{42}, vA_{51} + v^2A_{52} + vA_{53})$$
$$= (0, 0, 0, 0, 0, 0, 0, 0, 0, A_{32}, A_{42}, A_{52}, I_{k_1}, A_1, A_2, A_{31} + A_{32},$$
$$A_{41} + A_{42}, A_{51} + A_{52} + A_{53})$$

$$\psi(0, vI_{k_2}, B_1, B_{21} + vB_{22}, vB_{31}, B_4) = (0, 0, B_1, B_{21}, 0, 0,$$
$$0, 0, 0, 0, 0, 0, 0, B_{42}, 0, I_{k_2}, B_1, B_{21} + B_{22}, B_{31}, B_{41} + B_{42})$$

$$\psi(0, v^2I_{k_2}, vB_1, vB_{21} + v^2B_{22}, v^2B_{31}, vB_4) = (0, 0, 0, 0, 0, 0, 0,$$
$$I_{k_2}, 0, B_{22}, B_{31}, B_{41}, 0, I_{k_2}, B_1, B_{21} + B_{22}, B_{31}, B_{41} + B_{42})$$

$$\psi(0, 0, (1+v^2)I_{k_3}, (1+v^2)C_1, (1+v^2)C_2, (1+v)C_3)$$
$$= (0, 0, I_{k_3}, C_{11}, C_2, C_{31}, 0, 0, I_{k_3}, C_{11}, C_2, C_{31}, 0, 0, 0, 0, 0, 0)$$

$$\psi(0, 0, 0, (v+v^2)I_{k_4}, (v+v^2)D_1, D)$$
$$= (0, 0, 0, 0, 0, 0, 0, 0, 0, I_{k_4}, D_1, D_2, 0, 0, 0, 0, 0, 0)$$

$$\psi(0, 0, 0, 0, (1+v^2)I_{k_5}, E)$$
$$= (0, 0, 0, 0, I_{k_5}, E, 0, 0, 0, 0, I_{k_5}, E, 0, 0, 0, 0, 0, 0)$$

$$\psi(0, 0, (1+v)I_{k_3}, (1+v)C_1, (1+v)C_2, C_3) = (0, 0, I_{k_3},$$
$$C_{11}, C_2, C_{31}, 0, 0, 0, C_{12} + C_{13}, 0, C_{32} + C_{33}, 0, 0, 0, 0, 0, 0)$$

Then, $\Psi(C)$ can be generated by the following matrix G":

$$G'=\begin{pmatrix}
I_{k_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
A_1^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
A_2^T & 0 & 0 & B_1^T & 0 & I_{k_1} & I_{k_1} & 0 & 0 \\
A_{31}^T & 0 & 0 & B_{21}^T & 0 & C_{11}^T & C_{11}^T & 0 & 0 \\
A_{41}^T & 0 & 0 & 0 & 0 & C_2^T & C_2^T & 0 & I_{k_5} \\
A_{51}^T & 0 & 0 & 0 & 0 & C_{31}^T & C_{31}^T & 0 & E^T \\
0 & 0 & I_{k_1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & A_1^T & 0 & I_{k_2} & 0 & 0 & 0 & 0 \\
0 & 0 & A_2^T & 0 & 0 & I_{k_1} & 0 & 0 & 0 \\
0 & A_{32}^T & A_{31}^T & 0 & B_{22}^T & C_{11}^T & C_{12}^T+C_{13}^T & I_{k_4} & 0 \\
0 & A_{42}^T & A_{41}^T & 0 & B_{31}^T & C_2^T & 0 & D_1^T & I_{k_5} \\
A_{53}^T & A_{52}^T & A_{51}^T+A_{53}^T & B_{42}^T & B_{41}^T & C_{31}^T & C_{32}^T+C_{33}^T & D_2^T & E^T \\
I_{k_1} & I_{k_1} & I_{k_1} & 0 & 0 & 0 & 0 & 0 & 0 \\
A_1^T & A_1^T & A_1^T & I_{k_2} & I_{k_2} & 0 & 0 & 0 & 0 \\
A_2^T & A_2^T & A_2^T & B_1^T & B_1^T & 0 & 0 & 0 & 0 \\
A_{31}^T+A_{32}^T & A_{31}^T+A_{32}^T & A_{31}^T+A_{32}^T & B_{21}^T+B_{22}^T & B_{21}^T+B_{22}^T & 0 & 0 & 0 & 0 \\
A_{41}^T+A_{42}^T & A_{41}^T+A_{42}^T & A_{41}^T+A_{42}^T & B_{31}^T & B_{31}^T & 0 & 0 & 0 & 0 \\
A_{51}^T+A_{52}^T+A_{53}^T & A_{51}^T+A_{52}^T+A_{53}^T & A_{51}^T+A_{52}^T+A_{53}^T & B_{41}^T+B_{42}^T & B_{41}^T+B_{42}^T & 0 & 0 & 0 & 0
\end{pmatrix}^T$$

After applying column transformation to G', we can prove the theorem.

CONCLUSION

In this study, we studied linear codes over the ring R. Another direction for research in this topic is of course the cyclic and constacyclic codes over the ring R.

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