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Vibrational Analysis of Laminated Composite Plates on Elastic Foundations

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Abstract: In this study, free vibrations of laminated composite plates on elastic foundations are studied. The plates have three different conditions of elastic boundary. The boundary conditions of plates are free, simply supported and fully clamped. Plates consist of layers and layers are arranged at different angles. Plates supported by the springs of five different points. The natural frequencies of plates are calculated by the finite element method. Finite element method is developed to solve problem. In this method, equations of boundary conditions are written for the analysis. In Matlab programming, a special software developed for the solution of equations. Free vibration frequencies of the different boundary conditions are displayed graphically and the results of graphic are discussed.

Key words: Vibration analysis, laminated composite plate, elastic foundations, finite element method

INTRODUCTION

Laminated composite plates on elastic foundation are widely used in engineering applications such as construction of the plane, spacecraft, ship, sports equipment and chemical industry. These plates are very attractive for researchers and research is continuing on. In severe conditions of environments, the buckling and vibration of these structures are occurred. It is important to know the mechanical behavior of the composite plates under the different load.

The number of parameters associated with the manufacturing and fabrication of composites and modeling of the foundation are many. The composite structures supported by elastic foundation display a considerable amount of uncertainties in their material and foundation properties. Therefore for accurate analysis of composite laminates supported on elastic foundation, it is necessary to estimate the values of the eigen solutions of the structures for a reliable design.

Plates are generally placed on an elastic foundation in many applications of engineering. The modeling and analysis of such structures under large deflections are important study area. For this reason, takes a lot of research on the subject. Many publications have appeared in literature on the free vibration analysis of laminated composite plates. The vibration analysis of elastic boundary of composite plates, very little in the literature are discussed.

Lal *et al.* (2008) analyzed nonlinear free vibration of laminated composite plates on elastic foundation. In analysis was used random system properties.

Setoodeh and Karami (2004) studied on thick composite plates. The analysis of static, free vibration and buckling was performed. The plates supported elastic and a 3-D layer-wise FEM was used for analysis.

Sobhy (2013) investigated buckling and free vibration of sandwich plates on elastic foundations. The plates were under various boundary conditions.

Malekzadeh *et al.* (2010) employed vibration analysis of composite plates. Composite plates were elastic foundation and simply supported.

Baltacioglu *et al.* (2011) researched static analysis of composite plates. The boundary conditions of plate was elastic installation. In analysis was used the method of discrete singular convolution.

Quintana and Nallim (2013) studied free vibrations of composite plates. Triangular and trapezoidal-type plates were elastic supported.

Wun and Lu (2011) presented vibration analysis of composite plates. Boundary conditions of plates were inner side columns and edges of the elastic support. The powerful pb-2 Ritz method was used for analyzing.

Ismail *et al.* (2013) investigated effect of the material properties on vibration of composite plate. Boundary conditions of plates were elastic. In analysis were used the Inverse method and Fourier series.

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Li *et al*. (2009) determined vibrations analysis of plates. An analytical method used for the analyzes and edges of plates were supported elastic.

Hao and Kam (2009) performed modal analysis of composite plates. Boundary conditions of composite plates were elastic supported at different places.

Vosoughi *et al*. (2013) analyzed forced vibration of composite plates on elastic installation. These plates were subjected to moving load.

Ashour (2006) studied vibration of angle-ply symmetric laminated composite plates. Elastically supported edges of the plate.

Karami *et al*. (2006) investigated vibrations of laminated composite plates. In analysis were used the differential quadrature method (DQM).

Civalek (2013) investigated a numerical model for geometrically nonlinear dynamic analysis of thicklaminated plates based on the first-order shear deformation theory.

Thai and Choi (2012) studied free vibration of functionally graded plates on elastic foundation. A refined shear deformation theory is developed for analysis.

Shen (2011) analyzed nonlinear vibrations of thin films. The films supported elastic in thermal environmental conditions.

In this study, free vibration analysis of composite Plates are investigated using symmetrical modes considering three different elastic boundary. Free vibration frequencies of the different boundary conditions are expressed as numerical values. This paper is contribution to vibration analysis of composite plates of elastic boundary.

MATERIALS AND METHODS

Main equations for the finite element method: A rectangular element, which is under the effect of bending vibrations, is shown at Fig. 1. There are three degrees of freedom at each node, at each corner. There are three degrees of freedom at each node, respectively, deflection of z direction and the two rotations, w, $\theta_x = \partial w/\partial y$ and $\theta_y = \partial w/\partial x$. In terms of then on-dimensional (ξ, η) coordinates, these become

$$\theta_x = \frac{1}{b}\frac{\partial w}{\partial \eta},\ \theta_y = -\frac{1}{a}\frac{\partial w}{\partial \xi}$$

Since the element has twelve degrees of freedom, the displacement function can be represented by a polynomial having twelve terms due to simplicity. It can be written as follows related to $\xi = \pm 1$ and $\eta = \pm 1$ coordinate, at the node points:

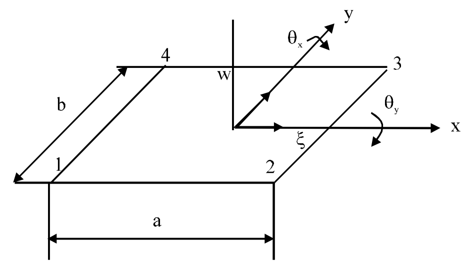

Fig. 1: Geometry of a rectangular element ($\xi = \frac{x}{a}, \eta = \frac{y}{b}$)

$$w = [N1(\xi, \eta) N_3(\xi, \eta) N_3(\xi, \eta) N_4(\xi, \eta)]\ \{w\}_e \quad (1)$$

where, $\{w\}_e$ is the displacement and rotations vector:

$$w = [\ N\ (\xi, \eta)]\{w\}e \quad (2)$$

At (2.1), defined the N (ξ, η) is:

$$N_j^T(\xi,\ \eta) = \begin{bmatrix} (1/8)(1+\xi_j\xi)(1+\eta_j\eta)(2+\xi_j\xi+\eta_j\eta-\xi^2-\eta^2) \\ (b/8)(1+\xi_j\xi)(\eta_j+\eta)(\eta^2-1) \\ (a/8)(\xi_j+\xi)(\xi^2-1)(1+\eta_j\eta) \end{bmatrix} \quad (3)$$

And (ξ_j, η_j)are the coordinates of node j (Petyt, 1990).

Mass matrix for plate element: The kinetic energy expressions for thin plate bending element is:

$$T = \frac{1}{2}\int_A \rho h\, \dot{w}^2 dA \quad (4)$$

where, is ρ density, h is thick plate and A is area of plate. Substituting Eq. 1 into 4 gives:

$$T_e = \frac{1}{2}\ \{w\}_e^T\ [M]_e\{w\}_e \quad (5)$$

Where:

$$[M]_e = \int_{Ae} \rho h[N]^T[N]dA$$
$$[M]_e = \rho hab \int_{-1}^{+1}\int_{-1}^{+1} [N(\xi,\ \eta)]^T\ [N(\xi,\ \eta)]d\xi d \quad (6)$$

Equation 6 is element mass matrix. If NJ (ξ, η) substitute from Eq. 3 and integrate Eq. 6, the result will be as follows (Petyt, 1990):

$$[M]_e = \frac{\rho hab}{6300}\begin{bmatrix} m_{11} & m_{21}^T \\ m_{21} & m_{22} \end{bmatrix} \quad (7)$$

Linear stiffness matrix for composite plate element: The strain energy can be expressed (Petyt, 1990):

$$U_L = \frac{1}{2}\int_A \frac{h^3}{12}\{\chi\}^T[D]\{\chi\}\,dA \qquad (8)$$

Where:

$$\{\chi\} = \begin{bmatrix} \frac{\partial^2 w}{\partial x^2} & \frac{\partial^2 w}{\partial y^2} & 2\frac{\partial^2 w}{\partial xy} \end{bmatrix} \qquad (9)$$

And:

$$D_{ij} = 1/3\sum_{k=1}^{N} (\bar{Q}_{ij})_k\ (z^3{}_k - z^3{}_{k-1}) \qquad (10)$$

where, $\bar{Q}_{ij}$ a matrix of reduced stiffness components for the kth layer whose surfaces are at distances z_{k-1}, z_k from the middle surface of the plate. $\bar{Q}_{ij}$ are the components transformed lamina stiffness matrix which are defined as follows:

$$\begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \qquad (11)$$

Terms of the $\bar{Q}_{ij}$ matrix are:

$$\begin{aligned}
\bar{Q}_{11} &= Q_{11}f^4 + 2(Q_{12} + 2Q_{66})f^2g^2 + Q_{22}g^4 \\
\bar{Q}_{12} &= (Q_{11}+Q_{22}\text{-}4Q_{66})f^2g^2 + Q_{12}(f^4 + g^4) \\
\bar{Q}_{22} &= Q_{11}g^4 + 2(Q_{12} + 2Q_{66})f^2g^2 + Q_{22}f^4 \\
\bar{Q}_{16} &= (Q_{11}\text{-}Q_{12}\text{-}2Q_{66})f^3g + (Q_{12}\text{-}Q_{22}+2Q_{66})fg^3 \\
\bar{Q}_{26} &= (Q_{11}\text{-}Q_{12}\text{-}2Q_{66})fg^3 + (Q_{12}\text{-}Q_{22}+2Q_{66})f^3g \qquad (12) \\
\bar{Q}_{16} &= (Q_{11}+Q_{22}\text{-}2Q_{12}\text{-}2Q_{66})f^2g^2 + Q_{66}(f^4 + g^4) \\
f &= \cos\theta,\quad g = \sin\theta \\
Q_{1_1} &= \frac{E_1}{1-912\ 2_1},\ Q_{1_1} = \frac{12\ E_1}{1-912\ 21},\ Q_{1_1} = \frac{21\ E_1}{1-912\ 21}, \\
Q22 &= \frac{E_2}{1-912\ 2_1},\ Q66 = G12
\end{aligned}$$

Substituting Eq. 8 into 1 gives:

$$U_e = \frac{1}{2}\{w\}_e^T[K]_e\{w\}_e \qquad (13)$$

$[K]_e$ can be written as follows:

$$[K]_e = \int_A [B]^T[D][B]\,dA \qquad (14)$$

Equation 14 is the element stiffness matrix. And:

$$[B] = \begin{bmatrix} \frac{1}{a^2}\frac{\partial^2}{\partial\xi^2} \\ \frac{1}{b^2}\frac{\partial^2}{\partial\eta^2} \\ \frac{2}{ab}\frac{\partial^2}{\partial\xi\partial\eta} \end{bmatrix} [N(\xi, \eta)] \qquad (15)$$

dA = dx dy, $\xi = \frac{x}{a}$, $\eta = \frac{y}{b}$, dA = abd $\xi d\eta$ equalby is (2.14) new expression:

$$[K]_e = ab\int_A [B]^T[D][B]\,d\xi d\eta \qquad (16)$$

Element stiffness matrix terms for easiness are separated square underside matrix and:

$$[K]_e = \begin{bmatrix} [K]_{11} & & & \text{simetrik} \\ [K]_{21} & [K]_{22} & & \\ [K]_{31} & [K]_{32} & [K]_{33} & \\ [K]_{41} & [K]_{42} & [K]_{43} & [K]_{44} \end{bmatrix} \qquad (17)$$

and stiffness matrix is symmetric (Morgul and Kucukrendeci, 2008).

Analysis of linear undumped free vibration: Free vibration analysis of the laminated composite plates is made by Eq. 18:

$$[M]\{\ddot{u}\}+[K]\{u\} = \{0\} \qquad (18)$$

where, [M] and [K] are system mass matrix and system stiffness matrix respectively. System matrices are composed of $[M]_e$ element mass matrix and $[K]_e$ element stiffness matrix:

$$([K]\text{-}\ \omega^2[M])\ \{\Phi\} = \{0\} \qquad (19)$$

In order to determine the frequencies, ω and modes {Φ}, of free vibration of a structure, it is necessary to solve the linear eigenvector problem (Morgul and Kucukrendeci, 2008).

In this study, the Eq. 18 [K] matrix by adding parameters of the spring ($[K^s]$) has been rewritten in the form of (Eq. 20-21). In addition, the computer program used for the analyzes re-arranged:

$$[M]\{\ddot{u}\}+([K]+[K^s])\{u\} = \{0\} \qquad (20)$$

$$[\ ([K]+[K^s])\text{-}\omega^2[M]]\ \{\Phi\} = \{0\} \qquad (21)$$

RESULTS AND DISCUSSION

Effect of different elastic boundary on free vibration: In this study, Morgul and Kucukrendeci (2008) is used as a model of the solution. The Eq. 22 is the natural frequency parameter and this parameter is used in the analyzes:

$$D_0 = \frac{E_1 h^3}{12\,(1-\mu_{12}\,\mu_{21})} \qquad \lambda = \frac{\rho h \omega^4 a^4}{D_0} \tag{22}$$

In this study, the equations of boundary conditions were written as special Eq. 21. Special software was developed to analyze of these equations in Matlab program. The linear undumped free vibrations of five layers symmetrically laminated rectangular plates were analyzed. The angle-plyis (θ, -θ, θ, -θ, θ) and as an example is (30^0, -30^0, 30^0, -30^0, 30^0) for plates. The angles of orientations of the layerare taken as 0^0, 15^0, 30^0, 45^0. The dimensions of the rectangular plate are a = 0.45 m, b = 0.30 m, a/b = 1, 5. Each layer thickness of 0.2 mm, the total cross-section of plate thickness of 1 mm. The model of three different boundary condition is designed for plates. The boundary conditions of plates shown in Fig. 2-4. In the Fig. 2 is fully clamped and supported with five spring. In the Fig. 3 is four corner simply supports and supported with five spring. In the Fig. 4 is four corner free and supported with five spring. Stiffness of spring is selected $k = 1500\ N\ m^{-1}$.

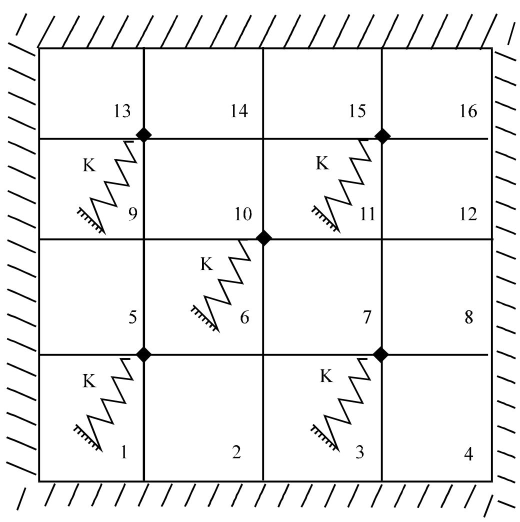

Fig. 2: Rectangular composite plate of symmetrically laminated is fully clamped and supported with five spring, (4×4) mesh element model (fully clamped)

In other studies, two or four-layer composite plates were analyzed. The free vibration analysis of angle-play, five-layered, elastic restricted of composite plates with finite element method are no other studies and is unique to this study. Finite element method used in this study is unique and there is no other studies.

The material of laminated composite plate is AS/3501 Graphite/epoxy. Its mechanical properties are as follows: density "$\rho = 1630\ kg\ m^{-3}$", modulus of elasticity

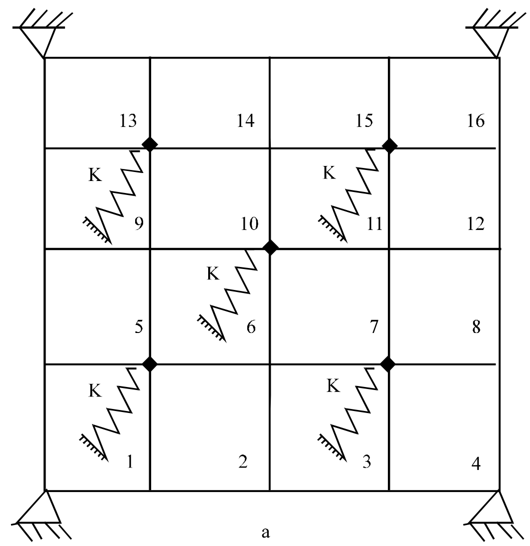

Fig. 3: Rectangular composite plate of symmetrically laminated is four corner simply supports and supported with five spring, (4×4) mesh element model (simply support)

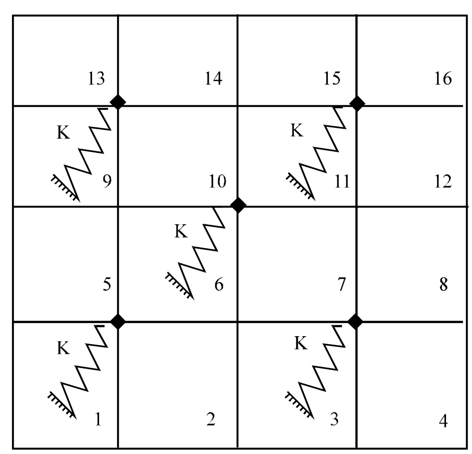

Fig. 4: Rectangular composite plate of symmetrically laminated is four corner free and supported with five spring and (4×4) mesh element model (free)

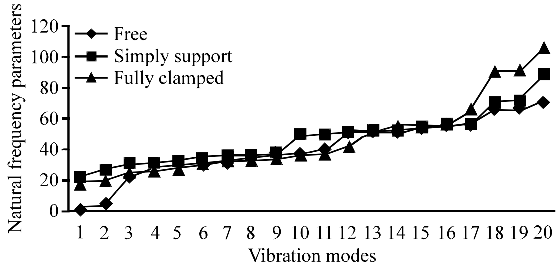

Fig. 5: AS/3501 Graphite/epoxy, Linear natural frequency parameters λ of symmetrically five-layer angle-ply (0^0) in different elastic boundary

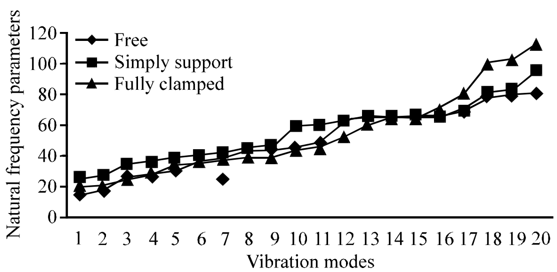

Fig. 6: AS/3501 Graphite/epoxy, Linear natural frequency parameters λ of symmetrically five-layer angle-ply (15^0) in different elastic boundary

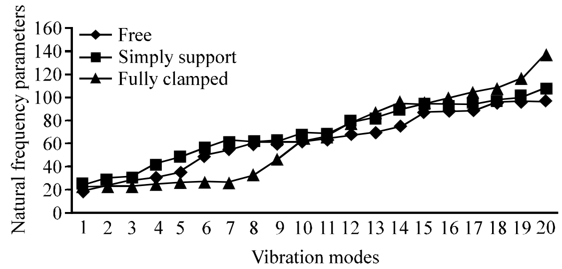

Fig. 7: AS/3501 Graphite/epoxy, Linear natural frequency parameters λ of symmetrically five-layer angle-ply (30^0) in different elastic boundary

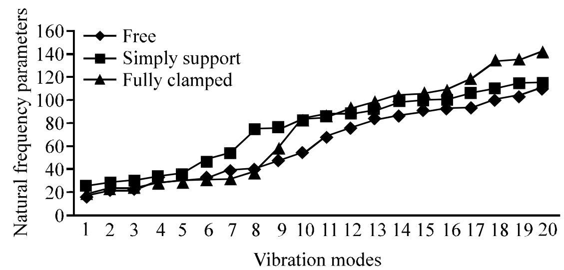

Fig. 8: AS/3501 Graphite/epoxy, Linear natural frequency parameters λ of symmetrically five-layer angle-ply (45^0) in different elastic boundary

"E_1 = 138 Gpa, and E_2= 9.0 Gpa", shear module "G_{12} = 6.9 Gpa" and poisons ratios "ν_{12}= 0.3 and ν_{21}= 0.019". Plate type and material properties retrieved from (Schwartz, 1992).

Composite plates of AS/3501 Graphite/epoxy of five-layer symmetrically analyzed for different elastic boundary. Linear natural frequency parameters of the un-damped vibration of the plates are calculated. The linear frequency for the symmetric mode 20 of these plates are displayed in the Fig. 5-8.

Figure 5-8, frequency parameters for the first 20 modes are shown free, simply support, fully clamped of boundary conditions of composite plate. Angles of plate layer (angle-play) are (0^0, -0^0, 0^0, -0^0, 0^0), (15^0,-15^0, 15^0, -15^0, 15^0), (30^0,-30^0, 30^0, -30^0, 30^0), (45^0, -45^0, 45^0,-45^0, 45^0).

In Figure 5, in all the boundary conditions, between 4-9 and 13-16 modes are approximate the frequency values. In the first ten mode, fully clamped and simply support conditions frequencies are consistent. After the seventeenth mode differs values. Between 3 and 16 modes in all the boundary conditions the characteristic of vibration can be said to same. In Figure 6, all the boundary conditions between 1-16 modes the frequency values are approximate. After seventeenth mode values differs. In Fig. 7, in all modes conditions of simple support and free frequency values overlap. In fully clamped condition, frequency values of the first three and 10-16 modes are approximate the other terms. After sixteenth mode has higher values. In Figure 8, in all the boundary conditions, the frequency values of the first five modes and between 12-16 modes are approximate. In other modes differs values.

CONCLUSIONS

Linear frequency parameters calculated according to the plate on elastic foundations. The effect of boundary conditions on the plate vibrations was observed (Fig. 5-8). In Fig. 5-8 frequency values close to each other. Frequency values of Fig. 7-8 greater than Fig. 5-6. Composite plates can be formed with different angle of arrangement of layers. These plates can be found dynamic properties. Types of plates can be selected according to the characteristics of user location. As a result, in usage area of plates with different boundary conditions, is preferred over one another. This are important to overcome the difficulties of construction, reduce costs durability.

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