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## Optimal Sizing of Shunt Capacitors and its Placement in Unbalanced Radial Distribution Network for Power Loss Reduction Using Evolutionary Programming

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**Abstract:** Distribution systems are the end nodes that connect the consumers and the power utility. Power loss in distribution lines are high due to unbalance loads and unequal conductor sizes, causes a large voltage drop along the feeder lines. Hence the consumers at end nodes are affected by low voltage problem. Voltage at different nodes of the distribution network must be controlled by controlling the reactive power flow. Reactive power can be injected near to the consumer end by connecting shunt capacitors to reduce the reactive component of line current and hence voltage drops. Identification of optimal location and shunt capacitor size renders enhancement in voltage profile, power loss reduction etc. Evolutionary Programming (EP) approach is adopted to find out best possible size of the capacitors to be placed for minimizing the active power loss. Voltage Stability Index (VSI) is computed for all nodes to indicate the potential point for reactive power compensation. EP uses the fitness function information directly, instead of its derivatives which makes EP, a highly flexible and robust when compared with the conventional optimization methods. The proposed technique is implemented on modified IEEE 13-bus unbalanced radial distribution system by removing the voltage regulator from the standard IEEE 13 bus test system. The simulation results shows that the proposed method can reduce power losses which highlight the capability of the proposed technique for minimizing the power loss in distribution systems.

**Key words:** Unbalanced radial distribution system, capacitor placement and sizing, backward/forward sweep based radial distribution power flow technique, evolutionary programming, real power loss

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### INTRODUCTION

Shunt capacitors installed in radial distribution systems plays a vital role to reduce power loss, to increase available capacity of the feeder and for feeder voltage profile improvement. Harmony Search Algorithm (HSA) optimization technique which is heuristic in nature based on analogies with natural phenomena is integrated with harmonic power flow algorithm and Backward/Forward sweep for optimum shunt capacitor location and perfect capacitor sizing is proposed (Muthukumar and Jayalalitha, 2012). Hybrid Particle Swarm Optimization (HPSO) technique integrated with harmonic power flow technique for optimum capacitor allocation and proper sizing to bring down the total real power loss subjected to operating constraint and power quality constraints has been formulated by Eajal and El-Hawary (2010). Positioning of capacitor at proper locations using Harmony search algorithm in balanced radial distribution system has been suggested and validated on the IEEE 9 and IEEE 34 bus Radial Distribution System (Sirjani *et al.*, 2010).

Network reconfiguration and capacitor sizing problems are solved simultaneously using Harmony

Search Algorithm (HSA) has been proposed by Srinivasa Rao (2010) and the placement of shunt capacitor at proper location is identified based on voltage stability index values. A three phase load flow method was formulated with mathematical modeling to include the mutual coupling between the phases (Subrahmanyam, 2009). A technique for three phase load flow suitable for radial distribution networks based on network-topology has been discussed by Teng (2000).

Evolutionary Programming (EP) approach is proposed in this study to determine the proper sizing of shunt capacitor in Unbalanced Radial Distribution System (URDS) to compensate reactive power demand at appropriate location based on computation of voltage stability index value.

### MATHEMATICAL FORMULATION OF OPTIMIZATION PROBLEM

The choice of optimum capacitor size and appropriate location is framed as a problem of nonlinear integer optimization.

- **objective function:** The objective function is mathematically formulated such as the minimization of cost function related to real power loss along with investment cost of shunt capacitors as shown below (Eajal and El-Hawary, 2010):

$$F = k_p \text{Ploss} + \sum_{i=1}^{nc} K_{ci} Q_{ci} \quad (1)$$

Where:

- $K_p$ : The yearly cost (\$) of the RPL  $\text{kW}^{-1}$
- $K_{ci}$ : Yearly cost in (\$)  $\text{kVAR}^{-1}$  of the reactive power supplied at bus  $i$  by the capacitors
- $Q_{ci}$ : Injection of reactive power (kVAR) at bus  $i$  by the capacitor
- $nc$ : Number of shunt capacitors considered for installation
- $\text{Ploss}$ : The total amount of active power loss

Finally the sum of active power loss is attained by adding all the branch power losses as:

$$\text{Ploss} = \sum_{i=1}^{nb} \text{Ploss}_i \quad (2)$$

where,  $nb$  is the count of total number of branches.

The real power loss is estimated by radial distribution power flow (RDPF) (Subrahmanyam, 2009) algorithm based on Backward/ Forward sweep technique. Size of shunt capacitors chosen for installation has impact on the cost of reactive power injection.

To avoid solution diverge from constraint limits, it is necessary to add quadratic penalty factors with the fitness function to get a modified fitness function  $F_c$  as,

$$F_c = \{f_c + \lambda_1 (\min(V) - V_{\min})^2 + \lambda_2 (\max(V) - V_{\max})^2 + \lambda_3 (Q - Q_T)^2\} \quad (3)$$

Where:

- $V$ : Bus voltage obtained from backward/forward-sweep based load flow after the installation of capacitors
- $\lambda_1, \lambda_2, \lambda_3$ : Penalty factors corresponding to minimum voltage, maximum voltage and total size of capacitor placed which can be adjusted during the optimization procedure

For reactive power compensation, shunt capacitors can be placed at buses except sub-station bus since it act as slack bus. The different sizes of capacitor that can be put at these locations are considered as the control variables.

- **Constraints:** Equality constraints are closely related with nonlinear load flow equation. The vector representation is as:

$$H(x, y) = 0 \quad (4)$$

Where:

- $x$ : State variable.
- $y$ : Control variable

Inequality constraints includes the rms values of bus voltage magnitudes and the total count of shunt capacitors considered for installation.

Throughout the optimization process, the rms value of the bus voltage magnitudes are maintained within specified operating limits. The rms value voltage magnitudes of each nodes should be between the lower and upper limits.

$$V_{\min} \leq |V_i| \leq V_{\max} \quad (5)$$

$$[|V|]_{i=1}^n = I[|V|_{i=1}^n] \quad (6)$$

where,  $n$  is the total count of buses.

Number of capacitors to be placed and the available standard capacitor sizes (discrete) are also considered as the constraints:

$$Q_{ci} \leq LQ_0, L = 1, \dots, nc \quad (7)$$

Where:

- $Q_0$ : Least capacitor size (commercial available)
- $Q_{ci}$ : The total injection of reactive power
- $nc$ : The total number of shunt capacitors to be considered for installation

The injection of reactive power from the shunt capacitors installed in distribution network should be kept lower than the reactive demand of distribution network:

$$\sum_{i=1}^{nc} Q_{ci} \leq Q_T \quad (8)$$

where,  $Q_T$  is the total reactive power demand.

### EVOLUTIONARY PROGRAMMING

Evolutionary Programming is one of the AI based search algorithms works on replicated procedure of natural selections, mutation and selection. EP uses population of potential solutions which can move over the search space to discover a global optimum solution.

It is initialized with random number generation which represents the parameters responsible for the optimization of the fitness value. EP involves statistics, calculation of fitness value, mutation and finally the selection process to get new population (Lee and Yang, 1998).

- $x_{k, \text{maximum}}$ : Maximum limit of jth element
- $x_{k, \text{minimum}}$ : Minimum limit of jth element
- $\beta$ : Mutation scale,  $0 < \beta \leq 1$
- $n$ : Total number of control variables

**Step 1: Initialization:** Evolutionary Programming works with a random set of initial data (capacitor sizes) which forms the parent population. The size of capacitor connected to any phase of a bus is limited by the upper bound (Qmax) of the systems total reactive power demand and the minimum size of commercially available capacitor is considered as the lower bound (Qmin). The control variable (size of capacitors) is initialized by randomly selecting “m” values ranging over [Qmin, Qmax], where “m” is the population size. The generated populations are considered as the parent population and corresponding fitness functions (Fc) are evaluated by running Backward/ Forward sweep based loadflow for every individuals of parent population

**Step 2: Fitness calculation and statistics:** The fitness function considered here is cost of real power loss which needs to be minimised. In this study, the fitness is calculated by using Backward/Forward sweep load flow algorithm. Subsequently, the maximum value of fitness, minimum value of fitness, sum and average values of fitnesses will be estimated and it will be utilized during the mutation process

**Step 3: Mutation:** This process is carried out based on the random numbers (populations) to produce siblings (child). The procedure is implemented by using Eq. 9 and 10. Component k of individual i is represented as (i, k):

$$C_{i,k} = P_{i,k} + N(\mu, \sigma^2) \tag{9}$$

$$\mu = 0$$

$$\sigma^2 = \beta * (x_{k, \text{maximum}} - x_{k, \text{minimum}}) * (f_i / f_{\text{maximum}}), k = 1, 2, 3 \dots, n \tag{10}$$

Where:

- $C_{i, k}$ : In child population
- $P_{i, k}$ : In parent population
- $N(\mu, \sigma^2)$ : Gaussian random variables with mean  $\mu=0$ , variance  $\sigma^2$
- $f_i$ : Fitness of  $i^{\text{th}}$  individual in parent population
- $f_{\text{maximum}}$ : Maximum fitness value for parent population

In Evolutionary Programming, a small fixed value of mutation factor can lead to premature convergence and thereby getting trapped into local optimum while searching with large value of mutation factor, prevents EP from converging. By using an adaptive mutation scale, this problem can be solved by choosing mutation scale as follows:

$$\beta(k+1) = \begin{cases} \beta(k) - \beta_{\text{step}}, & \text{if } f_{\text{minimum}}(k) \text{ unchanged} \\ \beta(k), & \text{if } f_{\text{minimum}}(k) \text{ decreased} \\ \beta_{\text{final}}, & \text{if } \beta(k) - \beta_{\text{step}} < \beta_{\text{final}} \\ \beta(0) = \beta_{\text{init}} \end{cases}$$

where, “k” is generation number.

$\beta_{\text{init}}, \beta_{\text{final}}, \beta_{\text{step}}$  are fixed values and its values would be  $\beta_{\text{init}} = 1, \beta_{\text{final}} = 0.005, \beta_{\text{step}}$  is between 0.001 to 0.01, depends on the maximum generation number (Ma and Lai, 1996). As the search process continues, the mutation factor will decrease. Mutation scale decrease faster, when fitness value is low. Such type of adaptive mutation factor prevents premature convergence.

**Step 4: Selection:** The offspring’s generated as the result of mutations are pooled along with their parents for selection process to find out the candidates that have the probability to be moved into the next new generation. Initially parent populations (size of capacitors) and its corresponding fitness values (cost of real power loss) will be estimated. After mutation, offsprings are created and merged with parent populations and arranged in the ascending order. The first half of the population ranked in the ascending order will be considered as the new population and choosen individuals are considered as the the new parent population for next generation process. The mutation and selection process is carried out repetitively until the convergence criterion is satisfied

**Step 5: convergence test:** Finally the convergence condition is achieved, when the variation between maximum and minimum fitness value is less than 0.0001 which is the resolution required. The flow chart as illustrated in Fig. 1 represents the overall EP procedure

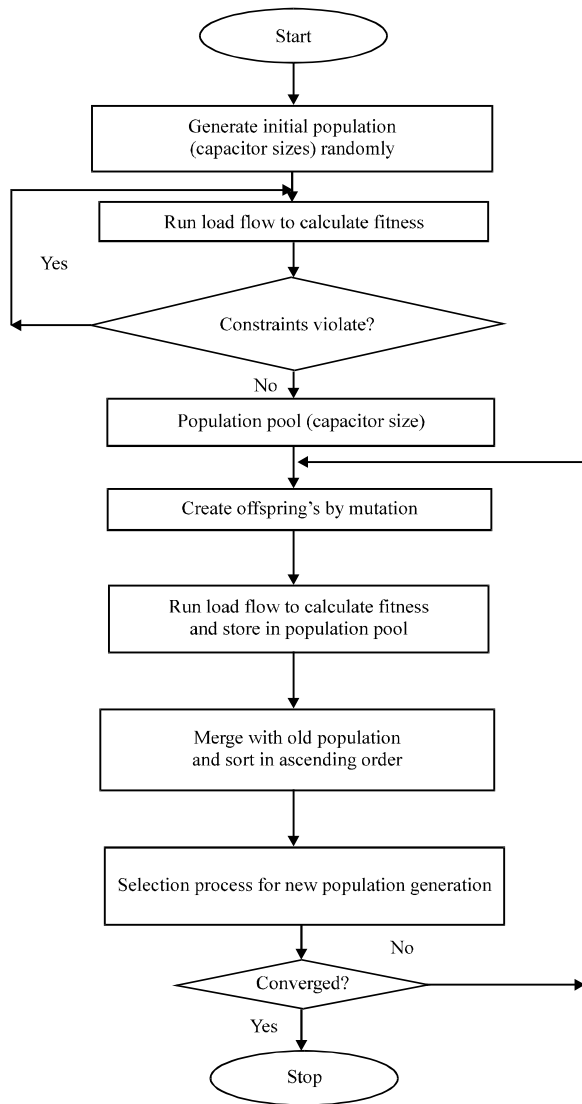


Fig. 1: Flowchart of proposed EP algorithm

**METHODS**

The optimization problem is formulated to identify the best possible capacitor ratings for an unbalanced RDS using Evolutionary Programming to achieve minimum value of real power losses with improved voltage profiles. The best location for placing shunt capacitor is identified by computing the voltage stability index values. Load flow based on backward/forward sweep technique is utilized (Subrahmanyam, 2009) to obtain active and reactive power flows, bus voltage magnitudes, phase angles, line currents, active and reactive power losses.

The suggested approach is examined on modified IEEE 13 bus unbalanced RDS network without voltage regulator (Kersting, 1991) to assess the performance of

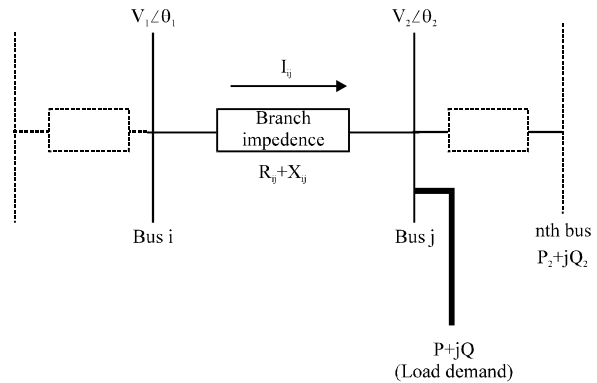


Fig. 2: One line diagram of two bus distribution network

the algorithms. If a distribution network with “P” number of buses and “Q” number of available size of capacitors, then the possible combination of solutions are  $(Q+1)^P$ . To minimize the computational burden and size of the solution vector, it is necessary to decide the most receptive nodes that require reactive power compensation which can be identified by computation of voltage stability index.

**VOLTAGE STABILITY INDEX**

In RDS for stable operation, the stability index value should be near to unity. Minimum value of VSI is the indication of buses in distribution systems that are prone to voltage instability and voltage collapse (Chakravorty and Das, 2001). Such nodes in radial distribution system are the exact places for placement of shunt capacitors.

Derivation of Voltage Stability Index (VSI).

Computing voltage stability index in all the buses as follows.

From Fig. 2:

$$I_{ij} = ((V_1 - V_2) / (R_j + jX_j)) \tag{11}$$

$$(P_2 - jQ_2) = (V_2 \times X I_{ij}) \tag{12}$$

Where:

- i, j: Transmitting and receiving end
- $P_2, Q_2$ : Total Real and Reactive Power load connected in bus j
- $V_1, V_2$ : Voltage magnitude at bus i and j

Using the above equations, Voltage stability index of bus “j” is computed as:

$$VSI(j) = V_1^4 - 4 V_1^2 (P_2 R_j + Q_2 X_j) - 4 (P_2 X_j - R_j Q_2)^2 \tag{13}$$

Table 1: Voltage stability index values before capacitive compensation

Bus No.	Phase	VSI
671	3	0.7032
675	3	0.7279
611	3	0.7316
684	3	0.7351
692	3	0.7386
680	3	0.7386
671	1	0.7495
632	3	0.7779
634	3	0.8053
675	1	0.8058
632	1	0.8082
652	1	0.8177
684	1	0.8203
692	1	0.8220
680	1	0.8220
632	2	0.8297
634	1	0.8391
633	3	0.8441
645	3	0.8444
646	3	0.8457
671	2	0.8486
633	1	0.8893
634	2	0.8913
646	2	0.9102
645	2	0.9136
675	2	0.9240
692	2	0.9269
680	2	0.9269
633	2	0.9307

Table 2: Standard capacitor sizes with associated costs/KVAR (KiloVolt Ampere Reactive)

Capacitor size (Qc)	K, \$ (KVAR)
150	0.500
300	0.350
450	0.253
600	0.220
750	0.276
900	0.183
1050	0.228
1200	0.170
1350	0.207
1500	0.201

For radial distribution system to be in stable conditions,  $VSI(j) = 0$ , for  $j = 2, 3...n$ .

In this proposed method, branch currents and bus voltages, active and reactive power flow of all the lines are estimated using the backward/forward sweep based load flow technique and from Eq. 13, the VSI of each bus is computed and tabulated in Table 1. From the tabulated results, for the proposed test system the bus number 671, 3rd phase is chosen for the perfect location for placing shunt capacitor which is having lower voltage stability index of 0.7032. To generate parent population in Evolutionary Programming, the capacitor values are chosen randomly from 150 to 1500 KVAR as listed from Table 2 (Eajal and El-Hawary, 2010).

### RESULTS AND DISCUSSION

The proposed Evolutionary Programming approach is utilized to identify the optimal shunt capacitor sizing in

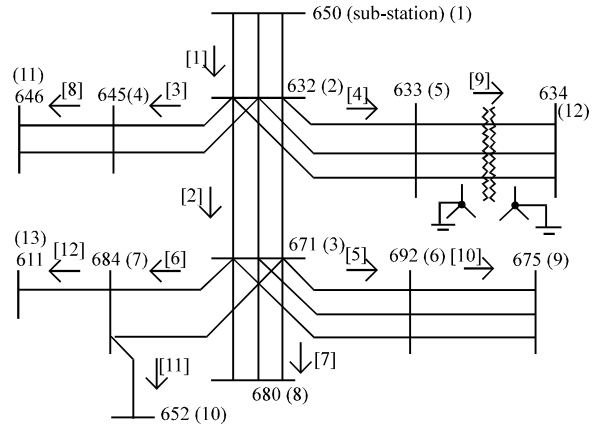


Fig. 3: Modified IEEE 13-bus unbalanced radial distribution system (URDS)

modified IEEE 13 bus unbalanced Radial distribution system is deployed in MATLAB computing environment on Intel Duo Core of 1.60 GHz and RAM of 1 GB. The modified IEEE 13 bus-URDS consists of three, two and single phase lines and loads. The total active power loads on the system is 3466 kW and the total reactive power load is 2102 kVAR, respectively. The system loads considered as distributed loads and spot loads. Bus 1 is considered as substation and act as slack bus. The remaining buses (2-13) except the slack bus are modeled as constant PQ buses. 10 MVA is taken as the base value and the line to line base voltage is the feeder nominal voltage 4.16 kV. Throughout the optimization process, the limits imposed on the rms values of bus voltages are selected between  $V_{min} = 0.9$  pu and  $V_{max} = 1.1$  pu. The cost of real power losses are taken as 168 U.S.\$  $\text{kW}^{-1}$  year<sup>-1</sup> (Eajal and El-Hawary, 2010). The pictographic view of the modified IEEE 13 bus unbalanced radial distribution system as shown in Fig. 3. A detailed description of IEEE 13 bus distribution network can be found by Kersting (1991).

Installation of shunt capacitor in bus number 671 at phase “3” with the size of 450 KVAR (case 2) results the total active power loss reduction from 116.4267 to 99.76 kW and net savings achieved is 2686.15 \$ year<sup>-1</sup>. The total cost of RPL before the capacitive compensation is 19559.68 \$ year<sup>-1</sup> (case 1) and after capacitive compensation, it is reduced to 16873.53 \$ year<sup>-1</sup> (case 2). Comparatively, the reactive power compensation by shunt capacitor installed at optimal place has reduced the overall cost.

Solutions obtained from the proposed method are tabulated in Table 3 and 4. which indicates the improvement in voltage profiles after the installation of shunt capacitor at appropriate node. The best location

Table 3: Bus voltage profile (magnitude and phase angles) before capacitor installation

Bus No.	Phase 1		Phase 2		Phase 3	
	Magnitude	Angle	Magnitude	Angle	Magnitude	Angle
650	1.0000	0.0000	1.0000	-120.0000	1.0000	120.0000
632	0.9736	-1.0000	0.9841	-120.6734	0.9605	119.2430
671	0.9522	-2.1490	0.9812	-120.7288	0.9270	118.6696
645	0.0000	0.0000	0.9783	-120.7263	0.9601	119.3061
633	0.9718	-1.0413	0.9830	-120.7021	0.9589	119.2391
692	0.9522	-2.1490	0.9812	-120.7288	0.9270	118.6696
684	0.9520	-2.1615	0.0000	000.0000	0.9259	118.6534
680	0.9522	-2.1490	0.9812	-120.7288	0.9270	118.6696
675	0.9476	-2.2432	0.9819	-120.7790	0.9249	118.7299
652	0.9509	-2.1471	0.0000	000.0000	0.0000	000.0000
646	0.0000	0.0000	0.9771	-120.7368	0.9594	119.3628
634	0.9574	-1.4658	0.9719	-120.9950	0.9475	118.9311
611	0.0000	0.0000	0.0000	000.0000	0.9248	118.6284

Table 4: Bus voltage profile (magnitude and phase angles) after capacitor installation

Bus No.	Phase 1		Phase 2		Phase 3	
	Magnitude	Angle	Magnitude	Angle	Magnitude	Angle
650	1.0000	0.0000	1.0000	-120.0000	1.0000	120.0000
632	0.9730	-0.7319	0.9806	-120.8333	0.9716	119.0565
671	0.9508	-1.6013	0.9741	-121.0508	0.9491	118.2970
645	0.0000	0.0000	0.9747	-120.8864	0.9712	119.1185
633	0.9712	-0.7728	0.9794	-120.8623	0.9700	119.0531
692	0.9508	-1.6013	0.9741	-121.0508	0.9491	118.2970
684	0.9506	-1.6141	0.0000	000.0000	0.9480	118.2812
680	0.9508	-1.6013	0.9741	-121.0508	0.9491	118.2812
675	0.9469	-1.6736	0.9740	-121.0813	0.9482	118.2301
652	0.9496	-1.5996	0.0000	000.0000	0.0000	000.0000
646	0.0000	0.0000	0.9735	-120.8969	0.9704	119.1743
634	0.9568	-1.1979	0.9682	-121.1574	0.9587	118.7522
611	0.0000	0.0000	0.0000	000.0000	0.9469	118.2568

Table 5: Simulation result before and after capacitor placement

Description	CASE -1 (Before capacitor placement)	CASE 2 (After capacitor placement)
Minimum bus voltage (in p.u)	0.9248	0.9469
Maximum bus voltage (in p.u)	0.9841	0.9806
Active power losses (kW)	116.4267	99.76
Reactive power injection (kVAR)	-	Bus 671, Phase 3, Q <sub>c</sub> = 450 kVAR
Cost of total active power loss (\$)	19559.68	16873.53
Loss reduction (%)	-	13.73
Net savings ( \$/year)	-	2686.15

and suitable size of shunt capacitor is identified to achieve power loss reduction and summary of results are tabulated in Table 5.

### CONCLUSION

The proposed Evolutionary Programming approach is used to identify the optimum capacitor sizing in modified IEEE 13 bus URDS. This results in reduction of total active power loss with voltage profile improvement. The backward/forward sweep based load flow technique is used to estimate the total real power loss of the test system and bus voltage profiles. The suggested EP

technique to find the optimum size of shunt capacitors has been examined on the modified IEEE 13 bus unbalanced radial distribution system and simulation results are presented. The simulations result shows that EP is well suited for multi objective reactive power planning, particularly in non-continuous, nonsmooth situations. This approach can be extended to IEEE standard bus systems to test the accuracy of the proposed algorithm.

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