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Regular Element for a Semigroup of Electroencephalography Signals during Epileptic Seizure

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Abstract: Electroencephalography (EEG) signal is one of the important scopes to diagnosis epilepsy, which is a recording of the electrical activity of the brain from the scalp. Algebraic structure of EEG signals during epileptic seizure can provide valuable insight and improve understanding of the mechanisms causing epileptic disorders. EEG signals during epileptic seizure can be viewed as a semigroup of square matrices under matrix multiplication. In this study, an element in that semigroup is shown to be regular.

Key words: Electroencephalography, semigroup, regular element

INTRODUCTION

Epileptic seizure is the outcome of sudden excessive electrical discharges in a collection of brain cells (neurons). Epilepsy describes the condition of a patient having recurring "spontaneous" seizures due to the sudden development of synchronous firing in the cerebral cortex caused by lasting cerebral abnormality (Engel Jr, 1993).

Electroencephalography (EEG) is the measurement of electrical activity produced by the firing of neurons in the brain. It functions by recording the instabilities in the potential difference of electrodes connected to the scalp of the patient, with these instabilities indicating the presence of neural activity. Furthermore, EEG signal is one of the actual roles and assistance for diagnosing epilepsy (Niedermeyer and da Silva, 2005).

The presence of the skull between the outer surface and the cortex tends to introduce far field effects and low-pass filters the signal. In consequence of the far field effects, scalp currents farther from the recording point may also be recorded. This tends to make the signals from different electrodes become correlated, not due to synchronization of the brain areas during a seizure but caused by the mixing effects presented by the skull.

The EEG system reads differences of voltage on the head relative to a given point. Therefore, if the activity of electrical is to be ascertained, then one shall need to place 3 electrodes, 1 on every hemisphere and another in the center, linked to both electrodes. This will give an absolute difference between activities of hemispheric brain.

The mathematical analysis of EEG signals helps medical professionals by providing an explanation of the brain activity being observed, hence increasing the understanding of the brain function of human. There are several techniques recommended in order to specify the EEG information. Among these, the fourier transform occurred as a very powerful tool capable of symbolizing the frequency components of EEG signals, even reaching diagnostic importance (Abarbanel *et al*., 1985). Nevertheless, fourier transform has some disadvantages that limit its applicability and thus, further technique for extracting “hidden”information from the EEG signals is needed.

Flat EEG: Zakaria and Ahmad (2007) have developed a novel technique for mapping high dimensional signal, namely EEG into a low dimensional plane. The whole procedures of this model consisted 3 parts. The first part deals with flattening the EEG data which mainly entails transformation of 3 dimensional spaces into 2 dimensional systems. This process involves position of sensors in the patient’s head with EEG signal. The second part involves processing EEG signals using Fuzzy c-Means. The last part involves finding the optimal number of clusters using cluster validity analysis.

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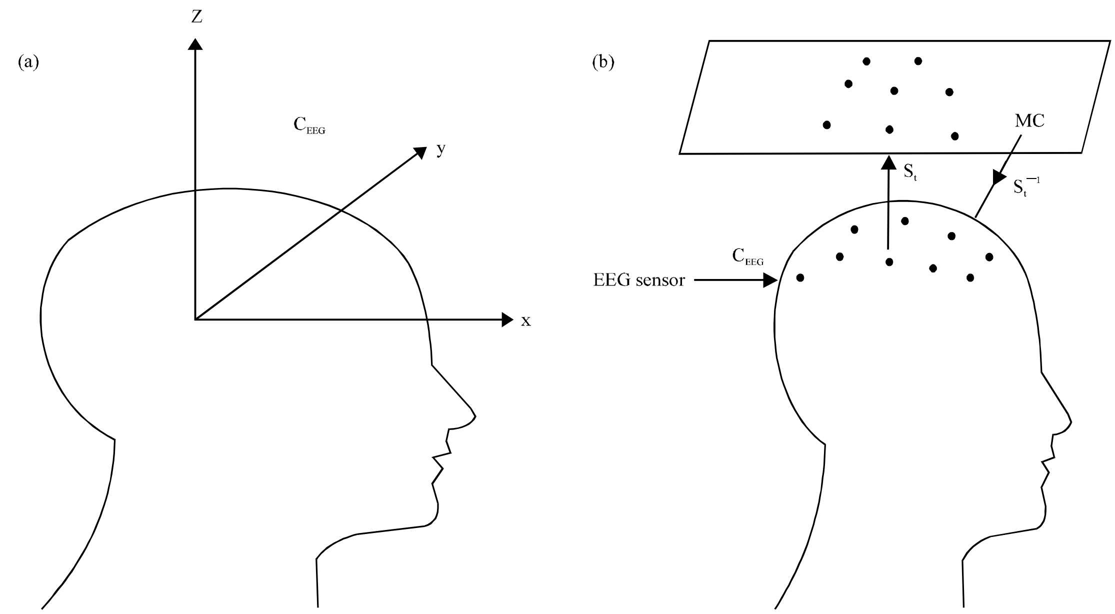

Fig. 1(a-b): (a) EEG coordinate system and (b) EEG projection

The EEG coordinate system (Fig. 1a) is defined by Zakaria and Ahmad (2007) as follows:

$$C_{EEG} = \{((x, y, z), e_p): x, y, z, e_p \in \mathbb{R} \text{ and } x^2+y^2+z^2 = r^2\}$$

where, r is the radius of a patient head. Moreover, a function is defined from C_{EEG} to MC plane as the following:

$$S_t: C_{EEG} \rightarrow MC$$

(Fig. 1b) such that:

$$S_t\left((x, y, z), e_p\right) = \left(\frac{rx+iry}{r+z}, e_p\right) = \left(\frac{rx}{r+z}, \frac{rx}{r+z}\right)_{e_p(x, y, z)}$$

Together, C_{EEG} and MC were designed and proved as 2-manifolds (Fauziah, 2008). In this case, it must be well-known that S_t is an injective mapping of a conformal structure. Thus, S_t mapping can preserve information in a particular angle and orientation of the surface through the recorded EEG signals. In addition, they implemented this method followed by clustering on real time EEG data obtained from patients who suffer from seizure.

The signals were digitized at 256 samples per second using Nicolet One EEG software. The average potential difference was calculated from the 256 samples of raw data at every second. Similarly to the location of the electrodes, the EEG signal was also preserved through this new method. Then, every single second of the particular average, potential difference was stored in a file which contains the position of the electrode on MC plane.

MATERIALS AND METHODS

MC plane can be viewed as a set of (n×n) square matrices as following:

$$MC_n(\mathbb{R}) = \{[\beta_{ij}(z)_t]_{n\times n}: i, j \in Z^+, \beta_{ij}(z)_t \in \mathbb{R}\}$$

where, $\beta_{ij}(z)$ is a potential difference reading of EEG signals from a particular ij sensor at time t (Fig. 2). Furthermore, the set $Mc_n(\mathbb{R})$ can be transformed to the set of upper triangular matrices $Mc''_n(\mathbb{R})$ using QR-real Schur triangularization as following:

$$MC''_n(\mathbb{R}) = \{[\beta_{ij}(z)_t]_{n\times n}: \beta_{ij}(z)_t = 0, \forall 1 \le j < i \le n, i, j \in \mathbb{Z}^+, \beta_{ij}(z)_t \in \mathbb{R}\}$$

(Binjadhnan and Ahmad, 2010).

Theorem 1 (Binjadhnan, 2011): The set of upper triangular matrices (EEG signals) satisfies all the axioms of a semigroup under matrix multiplication. In other words, $MC''_n(\mathbb{R})$ is a semigroup under matrix multiplication.

Definition 1 (Binjadhnan, 2011): The elementary EEG signals are a square matrix of EEG signals reading at time t in terms of one of the following types:

- Diagonal matrix (special case sub-identity matrix)
- Unipotent matrix
- Permutation

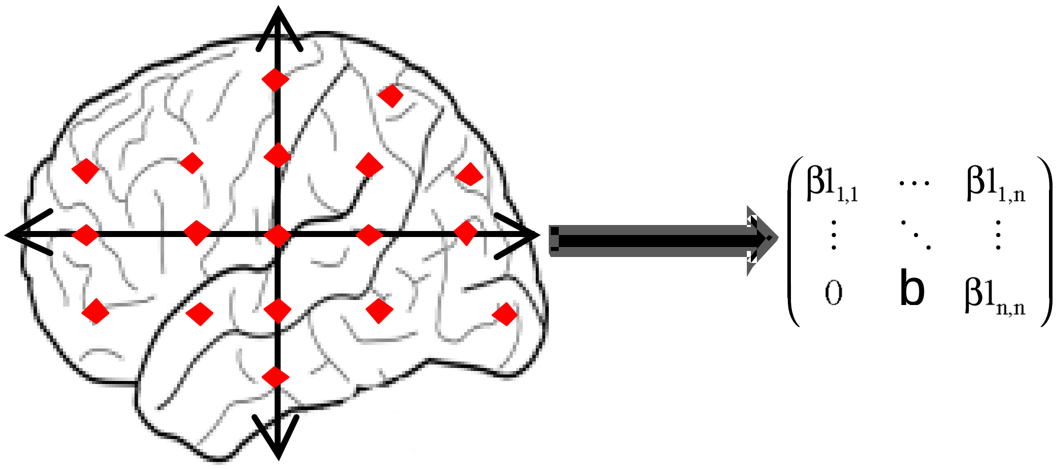

Fig. 2: MC plane in terms of upper triangular matrix

Definition 2 (Clifford and Preston, 1967): A semigroup congruence ~ is an equivalence relation that is compatible with the semigroup operation. That is a subset $\sim \subseteq S \times S$, that is an equivalence relation and x~y and u~v implies xu~yv for every x, y, u, v in S. A semigroup congruence ~ induces congruence classes. $[\alpha]_{\sim} = \{x \in S | x \sim a\}$.

Let A_t, B_t 2 matrices in $MC''_n(\mathbb{R})$ and we define a relation Ω on a semigroups $MC''_n(\mathbb{R})$ by $A_t \Omega B_t$ if and only if $A_t = \lambda B_t$ for some non-zero field element λ. A relation Ω on the $MC''_n(\mathbb{R})$ is called left compatible if $\forall A_t, B_t, C_t \in MC''_n(\mathbb{R})$, $A_t \Omega B_t \Rightarrow C_t A_t \Omega C_t B_t$ and right compatible if $\forall A_t, B_t, C_t \in MC''_n(\mathbb{R})$, $A_t \Omega B_t \Rightarrow A_t C_t \Omega B_t C_t$ and it is called compatible if $\forall A_t, B_t, A'_t, B'_t \in MC'_n(\mathbb{R})$, $A_t \Omega A'_t$ and $B_t \Omega B'_t \Rightarrow A_t B_t \Omega A'_t B'_t$. A left (right) compatible equivalence relation is called a left (right) congruence. A compatible equivalence relation is called congruence.

Definition 3 (Clifford and Preston, 1967): A row operation on an upper triangular matrix is said to be invertible if we can add a multiple of 1 row to a row above or scaling a row by non-zero field element.

Remark 1: Column operations are defined analogously.

Let A_t, $B_t \in MC''_n(\mathbb{R})$. It is easy to see that $B_t A_t$ can be obtained from A_t by a certain sequence of row operation determined by the matrix A_t. Conversely, every row operation can be represented as left- multiplication by a certain triangular matrix. There is an analogous relationship between right-multiplication and column operations.

A direct consequence of these explanations is the following characterization of Green's relations $\mathcal{L}$, $\Re$ and $\mathcal{J}$ on the semigroups $MC''_n(\mathbb{R})$.

In 1951, James Alexander Green introduced 5 equivalence relations that characterize the elements of a semigroup in terms of the principal ideals. These relations are useful for understanding the nature of divisibility in semigroup. The prime decomposition theorem (Krohn and Rhodes, 1965). Moreover, Green relations are particularly significant in the study of regular semigroup.

Definition 4 (Howie, 1995): An element s of a semigroup S is called a regular element of S, if there is an element t of S such that s = sts and S is said to be a regular semigroup if every element of S is regular. Note that the regular elements of semigroup of upper triangular matrices are characterized as those matrices whose rank is the equal to the number of their non-zero diagonal entries.

If A_t is an element of a semigroup $MC''_n(\mathbb{R})$, the smallest left ideal contain A_t is A_t is $MC''_n(\mathbb{R})A_t \cup \{A_t\}$, which we may conveniently write as $MC'_n \backslash (\mathbb{R})A_t$ and called the principal left ideal generated by A_t. An equivalence relation $\mathcal{L}$ on $MC''_n(\mathbb{R})$ is defined by the rule that $A_t \mathcal{L} B_t$ if and only if A_t and B_t generate the same principal left ideal, on the other word $MC''_n \backslash (\mathbb{R})A_t = MC''_n \backslash (\mathbb{R})B_t$. Similarly, we define $\Re$ by the rule that $A_t \Re B_t$ if and only if A_t and B_t generate the same principle right ideal, on the other word, $A_t MC''_n \backslash (\mathbb{R}) = B_t MC''_n \backslash (\mathbb{R})$. Furthermore, we define $\mathcal{J}$ by the rule that $A_t \mathcal{J} B_t$ if and only if $MC''_n \backslash (\mathbb{R}) A_t MC''_n \backslash (\mathbb{R}) = MC''_n \backslash (\mathbb{R}) B_t MC''_n \backslash (\mathbb{R})$.

Proposition 1 (Kambites, 2007): Let S(n, F) be a semigroup of all n×n upper triangular matrices with entries drawn from field f. Let A_1, $A_2 \in S(n, f)$ then:

- A_1, A_2 are $\mathcal{L}$ related exactly if each can be obtained from the other by row operation
- A_1, A_2 are $\mathfrak{R}$ related exactly if each can be obtained from the other by column operation
- A_1, A_2 are $\mathcal{J}$ related exactly if each can be obtained from the other by row and column operation

REGULAR ELEMENT FOR A SEMIGROUP OF EEG SIGNALS DURING SEIZURE

This section will show that an element of a semigroup of EEG signals $MC''_n(\mathbb{R})$ during epileptic seizure is regular.

Theorem 2: Assume that A_t is an upper triangular matrix of EEG signals during epileptic seizure ($A_t \in MC''_n(\mathbb{R})$). Then, the following are equivalent:

(i) A_t is regular
(ii) Every row (column) in A_t is a linear combination of rows (columns) in A_t with non-zero diagonal entries
(iii) A_t is $\mathcal{J}$ related to subidentity

Proof:

(i) ⇒ (ii)

Since the regular elements of $MC''_n(\mathbb{R})$ are characterized as those matrices whose rank is equal to the number of their non-zero diagonal entries (by definition 5) and since the rank of matrices is the number of linearly independent rows or columns, therefore, A_t together with the observation that sets of rows (columns) with non-zero diagonal entries in that case are necessarily linearly independent. Thus, every row (column) in A_t is a linear combination of rows (columns) in A_t with non-zero diagonal entries:

(ii) ⇒ (iii)

Let (ii) holds. It is easy to see that one can use row operations to make zero all entries in rows with diagonal zeros, column operations to make zero all entries in columns with diagonal zeros, row operations to make all the non-zero diagonal entries 1 and then the column operations to remove the remaining non-diagonal entries, so that, by last proposition, A_t is $\mathcal{J}$ related to a subidentity in $MC''_n(\mathbb{R})$. Hence (iii) holds:

(iii) ⇒ (i)

Since sub-identities are idempotent and hence regular and regularity is a property of $\mathcal{J}$ classes, so (iii) implies (i).

CONCLUSION

This study showed an element of a semigroup of upper triangular matrices of EEG signals during epileptic seizure is regular. This regularity associated with Green's relations.

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