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## Quantification of Precipitation Variation with Wavelet Analysis of Departure from Average and Cumulative Departure from Average in Major Grain Producing Area, Henan Province, China

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**Abstract:** Precipitation fluctuation research is essential to water security and grain production security. The monthly, annual and growing seasonal precipitation departure from average and cumulative precipitation departure from average were analyzed quantitatively via wavelet analysis to assess the characteristics of precipitation in multi-scales. The results indicated that precipitation changes and varies at three scales which are 4, 8 and 17 years. Wavelet analysis provides a special approach to evaluate the departure from average and cumulative departure from average.

**Key words:** Precipitation, departure from average, cumulative departure from average, morlet wavelet, major grain producing areas

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### INTRODUCTION

Precipitation is the crucial factor to assess water resources in a watershed or an area. Water resources variation is determined by precipitation change and variation. Water resources play an essential role in ecology security, society security, grain security and nation security because water resources are the basic strategic natural resources for the nature and society development (Allouche, 2011; Cook and Bakker, 2012).

Xinmi city which is one of the major grain producing areas is located in semi-arid and semi-humid areas in Henan province, China. Water resources are destroyed by the possible climate change and strong human activities including long term coal mining which result in water resources in shortage, water environment destroyed, rivers and springs disappeared, groundwater level in sharp decline. The challenges appear to the grain production, economy and society developments due to the water resources are destroyed. It is very important to study the precipitation fluctuation for grain producing and society security (Singh *et al.*, 2013; Da Silva, 2013).

Wavelet analysis whose key core is wavelet transform is a signal temporal scale analysis method (Arai *et al.*, 2006). The Multi Resolution Analysis (MRA) is available in wavelet transform to conduct time frequency analysis. Wavelet analysis is better than

traditional fourier analysis in frequency analysis mainly because wavelet analysis is capable to describe local features of sign in time and frequency domains and conduct multiple scales analysis in sign analysis (Kisi and Cimen, 2012; Nourani *et al.*, 2013).

Wavelet analysis was employed to study precipitation, atmospheric temperature, runoff and ocean disciplines. Multiple scales variation was obtained by wavelet analysis utilized to examine the variation of precipitation and atmospheric temperature in many areas. Precipitation time series contains multiple scales continuous periodical changes. Morlet wavelet analysis was employed to conduct local analysis in time and frequency domains to investigate variation and catastrophe features in multiple scales within Xinmi city for a 51 years (1960-2010) precipitation time series.

The annual average atmospheric temperature is 14.3°C. The statistics shows that the annual mean precipitation reach 656 mm and the maximum annual precipitation is 1181 mm (1964) and minimum annual precipitation is 397 mm (1986) based on the precipitation time series form 1960 to 2010.

### METHODOLOGY

Wavelet is a branch of mathematics which is capable to analyze the localization in the time and frequency

domains derived from stretching and displacing in the recent 20 years. The core content of wavelet analysis is wavelet multiplying the analytic function in order to decompose the original functions. Wavelet transform means that function named  $\psi(t)$  of mother wavelet is moved by a displacement named  $\tau$  and multiplies the analytic function named  $f(t)$  at multiple scales  $a$ . It can be described as Eq. 1:

$$C(a, t) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) \psi\left(\frac{t-\tau}{a}\right) dt, a > 0 \quad (1)$$

where,  $a$  refers to factors of scales;  $\tau$  means factor of translation;  $C$  is coefficient of wavelet;  $\psi(t)$  means square summable function which refers to  $\psi(t) \in L^2(\mathbb{R})$ .

The wavelet coefficients at different scales could be obtained by computing the inner product of the used wavelet and the function. Thus, according to the theories of wavelet transformation, the modules, real part and phases of wavelet coefficient are the important variances. Here, the modules indicate the intensity of the signal that characters the feature at different times. After the wavelet decomposition, the change features could be figured out at any time and frequencies for one time sequence. In this study, the Morlet wavelet is used to conduct wavelet analysis for the precipitation in Xinmi.

It is known that the Morlet is a complex sine modulated Gauss wave and has good localization feature within time and frequency domains (Paul *et al.*, 2013; Sang, 2013). Therefore, it is usually used to conduct the decomposition and time-frequency analysis of complicated signals. At the same time, the real part of Morlet wavelet is also one symmetrical wavelet function and the real part of wavelet coefficients could also contain the information of the strength and phase at different time location (Yi *et al.*, 2010; Rossi *et al.*, 2011). Without scale function, the Morlet wavelet function could be description by the following formula:

$$\psi(t) = e^{-\frac{t^2}{2}} e^{i\omega t}, \omega \geq 5 \quad (2)$$

The precipitation data in the Xinmi city is observed from 1960-2010. Monthly precipitation, annual precipitation and growing seasonal (May to September) precipitation were analyzed using wavelet analysis to examine precipitation variation from 1960 to 2010 at a total of 51 years which were 612 months. Firstly, monthly mean precipitation from January to December was calculated. Departure from average monthly precipitation series were obtained by monthly precipitation minus each mean monthly precipitation in order to avoid effect of natural

yearly cycle. The annual and growing seasonal precipitation time series were handled at the same way to the monthly precipitation.

Symmetric extension method was employed to extend the departure from average precipitation in order to reduce boundary effects (Grillakis *et al.*, 2011; Issac *et al.*, 2004) Precipitation departure from average time series  $2^n$  was employed to estimate wavelet coefficients for the extended precipitation departure from average. The minimum scale and maximum scale coefficients are 2 years and  $64(2^6)$  years for annual and growing seasonal precipitation series, respectively. The minimum and maximum scale coefficients are 2 months and  $512(2^9)$  months for monthly precipitation series.

These data would be further disposed to obtain the departure sequences and their Morlet wavelet coefficients are computed by their wavelet decomposition. In order to identify the main cycles of these departure sequences, the coefficient variance curves are plotted which could reflect the energy distribution with the analysis scales. The relative intensity of signal distribution of the precipitation time sequences could be indicated by the wavelet coefficient variances (Rathinasamy *et al.*, 2013; Kang and Lin, 2007).

The scales where the peak appeared are considered as the main scales, i.e., the main cycles. The wavelet coefficient variance is the integral of the wavelet coefficient module squares at the time domain and its computation formula is given by the Eq. 3.

$$\text{Var}(a) = \int_{-\infty}^{\infty} |C(a, \tau)|^2 dt \quad (3)$$

The wavelet analysis is conducted by MATLAB 7.0 and the following context presented the results about the wavelet analysis of the precipitation in Xinmi city.

## RESULTS AND DISCUSSION

**Monthly Precipitation departure features:** According to the monthly precipitation departure curve as displayed by Fig. 1 between the years of 1965 and 2011, the maximal monthly precipitation departure is about 246.7 mm in the April, 1964 while the minimal monthly precipitation departure is -146.4 mm in the July, 1987. What's more, the peak values and vale values of monthly precipitation appear mostly in the July or August and this owed to the concentrated precipitation in the whole years to the months of June to September for the Xinmi Area. The precipitation in the summer change abruptly and the precipitation in the rainy season occupied about 66.1% of the whole annual precipitation.

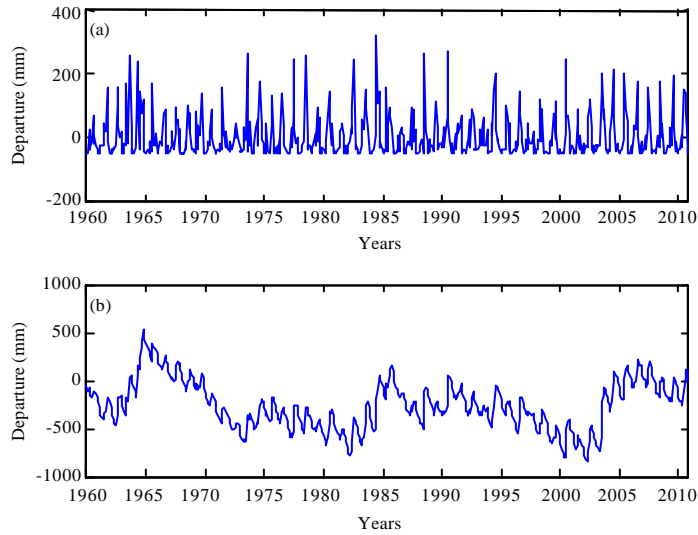


Fig. 1(a-b): Departure and cumulative departure curves departure form average in Xinmi Area, Henan, China, (a) Monthly precipitation and (b) Monthly cumulative precipitation

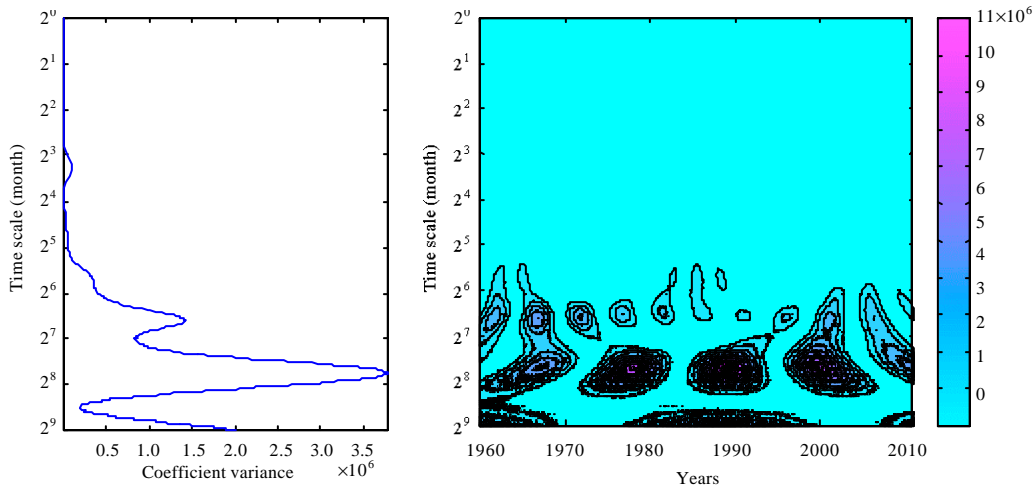


Fig. 2(a-b): Morlet wavelet analysis of the monthly precipitation cumulative departure in Xinmi, Henan, China

In order to discover the monthly precipitation trend for the long observed period with enough precipitation data, the cumulative departure curve is given in the Fig. 1. Obviously, the period change features with about 15 years could be figured out in the monthly cumulative precipitation departure fluctuation between the year 1960 and 2010. And the short fluctuated feature is about one year, i.e., 12 months and this could be presented as the annual change feature of monthly precipitation. However, the former long period change reflects the

change feature in many observed years which is mainly related to the periodical change of monthly precipitation.

Furthermore, the monthly precipitation cumulative departure is analyzed with Morlet wavelet transformation. Here, the wavelet coefficient is computed with the minimal scale 2 months and the maximal scale 512 months. According the wavelet coefficients at every observed time points and every scale, the wavelet coefficient energy contour is plotted as shown by Fig. 2. Thus, by the wavelet analysis with a larger number of scales, the whole

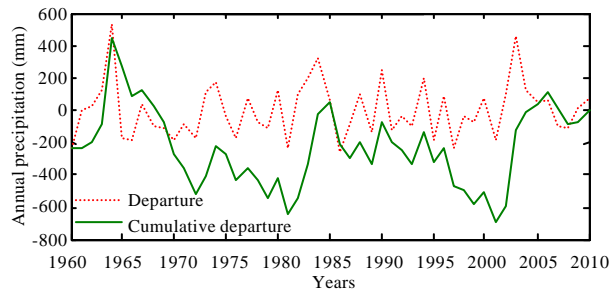


Fig. 3: Departure and cumulative departure curves of annual precipitation in Xinmi, Henan, China

feature could be observed in the contour, whose wavelet coefficient energy rally points disclosed the whole change feature in the times of the whole observed years. There are three typical wavelet coefficient energy rally points at the cross points between the years 1978, 1989, 2000 and the wavelet scale 217 months. This indicated the abrupt changes in the three adjacent observed years. The monthly precipitation in the July, 1978, is about 309 mm which is larger than that in other adjacent months. Likewise, the precipitation in the July, 1990, is about 320.3 mm while the precipitation in the July, 2000 is about 301.9 mm. For the whole peak in the wavelet coefficient energy contour, the whole monthly precipitation trend could be observed in the period with large wavelet transformation scales and this trend appears as slow monthly average precipitation changes.

**Annual precipitation departure change features:**

According to the observed annual precipitation data in Xinmi Area, China, the average annual precipitation is 656.4 mm in the years between 1960 and 2010 while the maximal annual precipitation 1180.8 mm in 1964 and the minimal annual precipitation 396.8 mm in 1986. The annual precipitation departure and its cumulative departure curves are described in Fig. 3. As indicated by Fig. 3, the maximal precipitation departure reached to 524 mm in 1964. Likewise, the annual precipitation departure also reached to its maximal values, i.e., 462 mm and its corresponding annual precipitation is about 1018 mm. The annual precipitation departure in 1986 is the least, i.e., -259 mm and the precipitation in 1984 and 1997 is approximate to this value. This indicated that the annual precipitation in these is far lower than the annual average precipitation and the drought may happen.

In fact, the annual precipitation cumulative departure curve for Xinmi Area in Fig. 3, could reflect the change trend of annual precipitation in the whole period of observed years. According to the similar departure and cumulative departure, they are consistent to each other.

If the annual precipitation is lower than the average annual precipitation for several sequent years, the corresponding cumulative departure would descend in the direct line way, vice versa. As disclosed in the Fig. 3, the annual precipitation in the period of 1960-1965 is larger than the annual average precipitation which leads to the rapid increase of the cumulative departure curve. In the following eight years, the annual precipitation in the Xinmi Area is decreased. During the period of 1973-2003, the annual precipitation is alternated between the abundant precipitation and the deficient precipitation until to the year 2003. The high intensity precipitation between the years 2003 and 2005 led to the linear increase of cumulative departure curve to the high position. In conclusion, the annual precipitation cumulative departure curve could reflect the trends and features of annual precipitation for the Xinmi Area.

The Morlet wavelet analysis is also conducted to disclose the possible trend and feature for the annual precipitation departure with multiple scales and the wavelet coefficient energy contour could be plotted as shown by Fig. 4. Obviously, there are many wavelet coefficient energy rally points in the contour which are located in the area with 4-16 scales. This indicated that the the annual precipitation is change abruptly in the whole Xinmi Area. In the range of wavelet scale 8 year, several energy rally points are located in the alternative way and the period change features could be found in the annual observed precipitation and the corresponding change period is about eight years. Meanwhile, in the range of scale 16, one wavelet energy rally point and two adjacent wavelet energy tundish areas could be also found. With the wavelet analysis theories, there is one great annual precipitation in the observed years for the Xinmi Area, i.e., 1982-1985.

**Growing seasonal precipitation departure feature:** By averaging the observed annual growing seasonal precipitation from 1960-2010 in Ximi Area, the annual

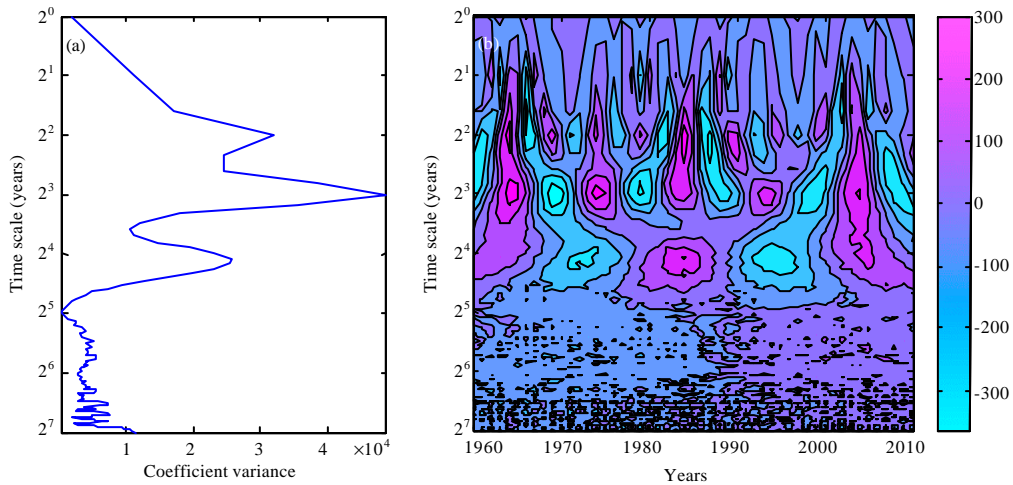


Fig. 4(a-b): Wavelet analysis of annual precipitation departure in Xinmi, Henan, China

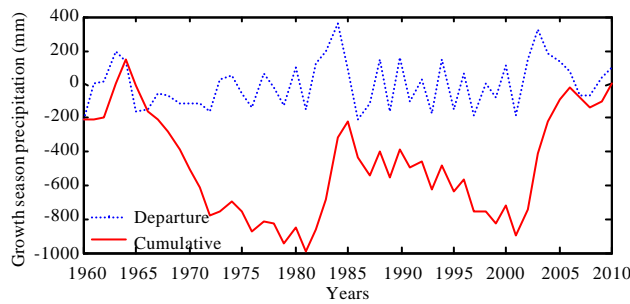


Fig. 5: Departure and cumulative departure curves of annual growing seasonal precipitation for Xinmi, Henan, China

average growing seasonal precipitation is about 491 mm and the 75% of the annual whole precipitation. Thus, the departure and cumulative departure curve of annual growing seasonal precipitation could be plotted in Fig. 5. Special annual growing seasonal precipitations are obviously higher than other annual growing seasonal precipitation while other annual precipitations are lower than the average precipitation. Furthermore, the growing seasonal precipitation in the years 1973-2000, are fluctuated with the corresponding annual precipitation. The precipitation fluctuation in growing seasonal periods could further disclose the precipitation change in the abundant and deficient years for Ximi Area. This change is consistent to that of annual precipitation. Furthermore, the wavelet analysis results of growing seasonal precipitation departure in Ximi Area, is also consistent to that of annual precipitation. This result discloses that the growing seasonal precipitation has strong relation to the annual precipitation as the former is one part of the latter.

**Typical fluctuation curve of precipitation departure:** By analyzing the wavelet coefficient variance of the monthly, annual and growing seasonal precipitation in Ximi Area, there are several obvious maximum points. These points are corresponding to the main scales where the wavelet coefficient energy rally points appear in the wavelet coefficient energy contour. The wavelet coefficient fluctuations could figure out the main cycles of the corresponding precipitation changes.

For the observed monthly precipitation in the years 1960-2011, Ximi, China, the wavelet analysis is conducted to obtain the wavelet coefficient variance curves of its cumulative departure, as shown in Fig. 3. There are two maximal scales in the variance curves, i.e., 97 months and 217 months. For the wavelet transformation special scales 97 and 217, the monthly precipitation in the consecutive 60 years from 1960-2010 are observed where two main cycles are obtained, i.e., the periods of monthly precipitation are 97 months and 217 months, respectively. These two

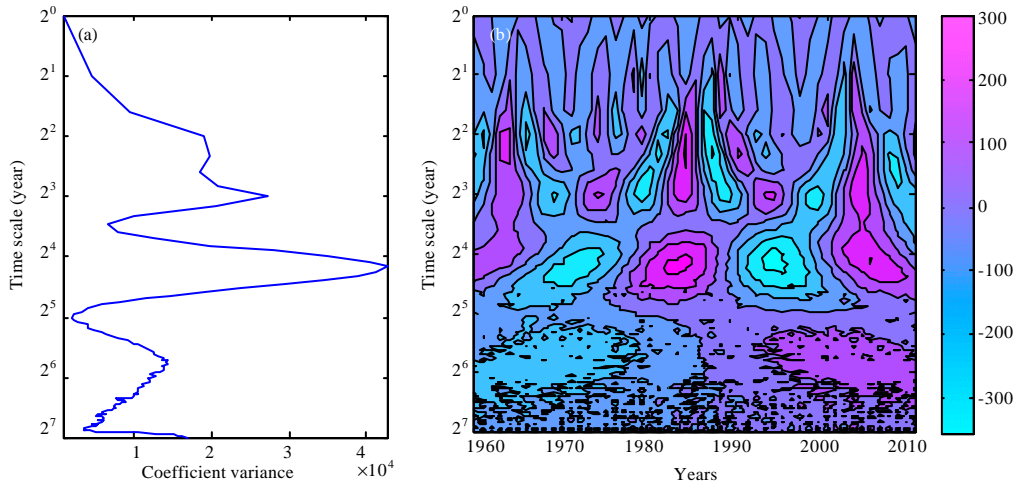


Fig. 6(a-b): Wavelet coefficient analysis results of growing seasonal precipitation in Xinmi, Henan, China

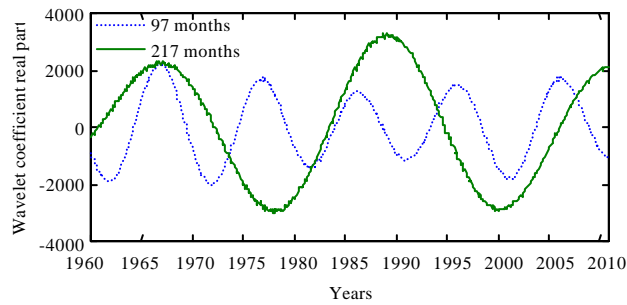


Fig. 7: Wavelet coefficient fluctuations of monthly cumulative precipitation departure under the typical main scales in the years 1960-2011, Xinmi, Henan, China

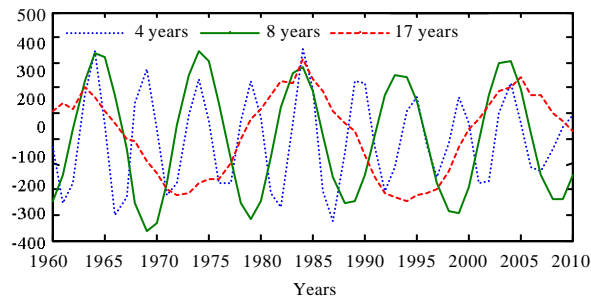


Fig. 8: Wavelet Coefficient real part curve of the annual precipitation departure at the typical main scales in Xinmi, Henan, China

cycles are equivalent to 8 years and 18 years. They could be figured out in the wavelet coefficient real part curve of monthly precipitation at the two special scales, as presented in Fig. 7.

In the followings, the Fig. 6 presents the wavelet coefficient variance curve of the observed annual precipitation and there are three maximal points, i.e.,

4 year, 8 years and 17 years. The main scales 8 and 17 years are consistent to the main scales 8 years and 18 years of monthly precipitation as disclosed in Fig. 2 above. The Fig. 8 displays the wavelet coefficient real part curves at the typical scales 4, 8 and 17 years, in Ximi Area from 1960-2010. Likewise, three main scales in the wavelet coefficient variance curve of growing seasonal

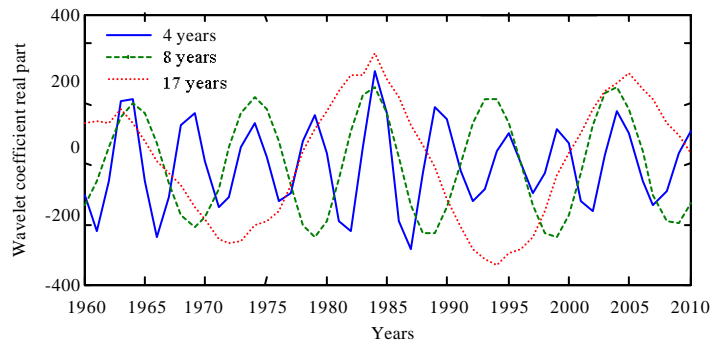


Fig. 9: Wavelet Coefficient real part curve of the growing seasonal precipitation departure at the typical main scales in Xinmi, Henan, China

precipitation in Xinmi Area could also be figured out, i.e., 4, 8 and 17 years. Their corresponding growing seasonal wavelet coefficient real part could be plotted in Fig. 9, where three typical curve are described. As one conclusion, the cumulative departure of monthly precipitation and the departures for the annual and growing seasonal precipitation are analyzed respectively. Their wavelet coefficient analysis results display the two typical common fluctuation periods in the years from 1960-2010, i.e., 8 and 17 years which reflects the trend and feature for the precipitation in Xinmi, Area, China.

### CONCLUSIONS

Multiple scale features could be figured out in the monthly precipitation, annual precipitation, growing seasonal precipitation and the departure and cumulative departure and their corresponding wavelet coefficient analysis are complemented to each other. What's more, they have common period features, i.e., 4, 8 and 17 years, respectively.

The wavelet analysis based Morlet wavelet is conducted to disclose the multiple scale feature of precipitation sequence, which could be considered as one effective method to figure out the climate features.

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