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Research on the Method of Simply Supported Beam Modal Parameters Recognition by QY Iinclinometer

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Abstract: Modal parameters are of great significance in bridge structural health detection and damage identification. Nowadays structure, modal tests are mostly concentrated on acceleration response or velocity response and other responses are less to be researched. Simply supported beam is a representative structure and the research on it will help the study of other complex structures. The inclination angle data of the beam under environment excitation will be used for modal analysis in this article. The modal parameters of the simply supported beam can be got through the theoretical analysis and experimental methods separately and we find that the two results are roughly same. Therefore, by the inclination angle data of the simply supported beam we can detect modal parameters, so this article provides a new reference method for other structure modal parameters tests, expands the application range of inclinometer in engineering and provides a more reliable basis for the health monitoring and modal parameters identification of the complex bridge structures.

Key words: Simply-supported beam, natural frequency, natural mode of vibration, QY inclinometer, bridge health detection

INTRODUCTION

With the development of modern civilization and the rapid replacement of civil engineering, bridges play more and more important roles in people's lives. However, since the bridge structure is influenced by the load, the natural environment and other factors during its long-term use process, the structure will be damaged. So a real-time detection is necessary to prevent the bridge structure from damage and guarantee the bridge normal operational conditions. Meanwhile, the study of structural vibration is imperative for industrial production and economic development and the research facing practical engineering, based on vibration theory, knowledge, methods to solve the growing complex problems of the vibration in the project (Li and Li, 2002; Zhou and Shen, 1997; Hou *et al*., 2002; Hou *et al*., 2003).

Modal analysis and parameter identification is essential for structural health detection, fault diagnosis and dynamic characteristic analysis. So how to identify structural modal parameters from the original signal correctly and conveniently, is the basis for recognition of the structure behavior and is also the key problem of health detection. Structural vibration test is a traditional method of modal parameter identification which get the structure vibration response signal through a series of vibration equipment and signal acquisition and processing equipment and the test also require a large investment of manpower and expensive excitation equipment. For complex structure as large bridges, on the one hand, because of the limitation of field test conditions, the excitation equipment can not meet the requirement it is difficult to guarantee the quality of the measured data; on the other hand, excessive vibration excitation force is likely to cause damage to structures and it will affect the traffic since the test process need to stop the normal use of the bridge. Therefore, the traditional method is not applicable for the modal parameter identification of the bridge structure.

Ambient excitation-based method is modal analysis method which only need to obtain the response signal, is more in accordance with the using characteristics of long-span bridges, is widely used in practical project. Ambient excitation test is a kind of convenient, efficient and practical method of modal analysis for long-span bridge structure and has obvious advantages: No valuable excitation equipment, do not interrupt the normal use of structure, only need to obtain the response data, real-time monitoring, etc.

The simply-supported beam is common structure in modern engineering and the research in it is more representative. This article draws modal parameters

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identification of simply-supported beam with QY type inclinometer, using theoretical and experimental methods to obtain modal parameters (natural frequency, damping ratio and natural mode of vibration) of simply-supported beam. So we can conclude that it is possible to detect the modal parameters with QY inclinometer when the simply-supported beam under ambient excitation which provides references to the vibration test of complex structure. The working process of the detection system is: When vehicles passing a bridge, the bridge angle voltage signal detected by bridge angle detection unit will be transmitted to signal conditioning module by the inclinometer and through the filter, amplification and conversion of the module, the angle voltage signal will be transmitted to data acquisition and analysis module and the analog signal will convert to digital signal in this module. On the one hand, the signal will be send to storage unit for later analysis, on the other hand it will be uploaded to the analysis system as required. The analysis system can realize real time display of the multiplex angle waveform and converting the angle to bridge deflection curve according to the need.

BRIEF INTRODUCTION OF QY TYPE INCLINOMETER

Characteristics of use: This instrument has the advantage of good anti-interference ability, high precision, convenient and efficient, low requirements for environmental conditions. QY inclinometer is mainly used to measure the angle of bridge under static loading and through the equation, the bridge deflection value Y can be solved.

Main technical parameters:

- Sensitivity: 60 mV/1'(Angle)
- The maximum range: 10'
- Drifting: 30 min ≤ 0.2 sec
- Power supply: +12 VDC
- Working environment: Temperature (0-50)°C humidity ≤ 85%
- Size: 210×120×100 mm
- Weight: 3.5 kg

Working mechanism: The structure of the inclinometer is shown in Fig. 1.

The capacitive sense and the passive servo technology has been used in the inclinometer, the basic principle of inclinometer is shown in Fig. 2. In the Figure, k is for spring stiffness, b is for the damping coefficient including air damping, the G1 = BL_1L_k, supposing the coil electric constant in order to meet the damping ratio and the BL_1 is electromechanical coupling coefficient, L_k is pendulum length as indicator, G_2 for the self-calibration coil electric constant, θ is the dip angle, K_c is the sensitivity of capacitance sensor, X is the displacement of capacitance sensor its motion equation is:

$$K_1\theta + b\theta + k\theta + G1i = -\frac{K_1}{l_0}X \tag{1}$$

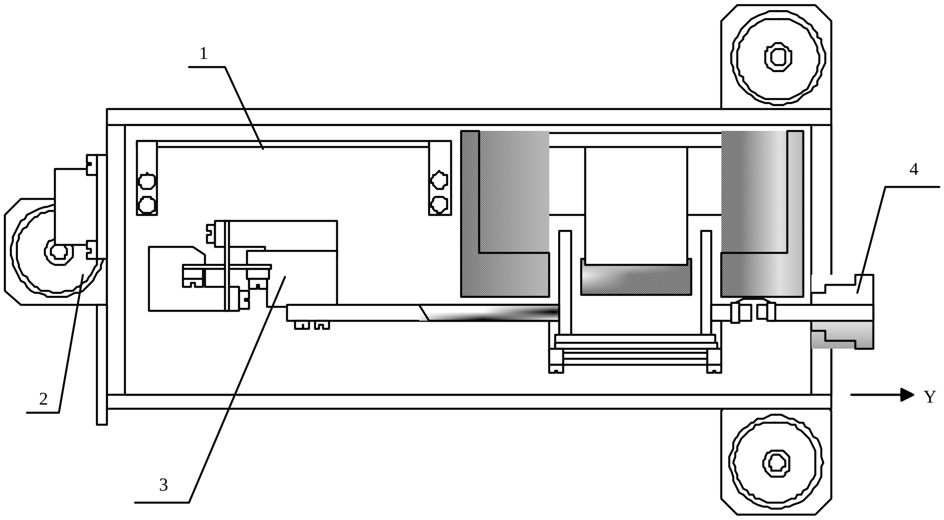

Fig. 1: Top view of the inclinometer, 1: Circuit board, 2: Foot, 3: Pendulum and 4: Lock pendulum button

K_1 is for the moment of inertia, $K_1 = L_0R_0m$ reduced pendulum length, R_0 is the distance from the center of rotation to the center of gravity of movement parts, m is the quality of the movement parts. When ignore the air damping, the solution of Eq. 1 as follows:

$$\theta = -\frac{s^2X}{n^2 l_0}\cdot\frac{1}{(\frac{s^2}{n^2}+\frac{2D}{n}s+1)} \tag{2}$$

S is operator, n is natural vibration circular frequency:

$$n^2 = \frac{k}{K_1}$$

D is the damping constant:

$$D = \frac{G_1^{\ 2}}{2K_1 nR}$$

R is the resistance of the coil circuit, the output voltage of inclinometer:

$$u_0 = K_C X = K_C L_K \theta = -\frac{K_C V_0 s^2 X}{n^2}\frac{1}{(\frac{s^2}{n^2}+\frac{2D}{n}s+1)} \tag{3}$$

$$\frac{u_0}{s^2X} = -\frac{K_C V_0}{n^2}\frac{1}{(\frac{s^2}{n^2}+\frac{2D}{n}+1)} \tag{4}$$

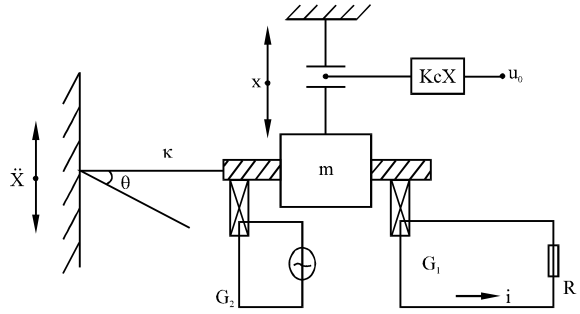

Fig. 2: Working principle of the inclinometer, 1: Circuit board, 2: Foot, 3: Pendulum and 4: Lock pendulum button

Supposing:

$$\frac{K_C V_0}{n^2} = 1,\ D = 0.707,\ n = 2\times\pi\times 2.7$$

STRUCTURAL PARAMETERS OF SIMPLY-SUPPORTED BEAM

Simply-supported beam with constant section is used in this experiment and its structural diagram is shown in Fig. 3. We make the following assumptions of the structure:

- Excluding the axial deformations of simply-supported beams
- The cross section and axial direction always maintain vertical when the simply-supported beam vibrated

Structural parameters of the simply-supported beam: length L = 2.7 m, width B = 0.2 m, height H = 0.01 m, density $\rho = 7.8\times10^3$ kg m^3, elastic modulus $E = 2.2\times10^{11}$ N m^2.

When experimenting, should reduce the influence of human factors in the excitation of the natural environment, experimental persons should keep quiet and no large mechanical operation around.

THEORETICAL ANALYSIS OF SIMPLY-SUPPORTED BEAM

Supposing q(x, t) is the uniform distribution force, according to Newton's second law we can obtain the partial differential equations (Long and Bao, 2004) of bending vibration of the simply-supported beam:

$$\rho(x)A(x)\frac{\partial^2 y(x,t)}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 y(x,t)}{\partial x^2}\right] = q(x,t) \tag{5}$$

In Eq. 5, ρ(x)`A(x)` EI(x) are density, area, flexural rigidity. If the simply-supported beam is homogeneous medium with constant cross section, so these three parameters are constants. Put $\bar{m} = \rho A$ as the mass of per

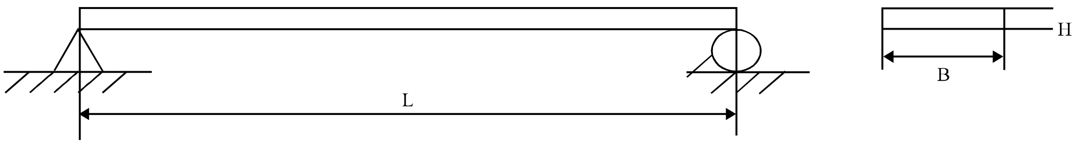

Fig. 3: Structural diagram of simply-supported beam

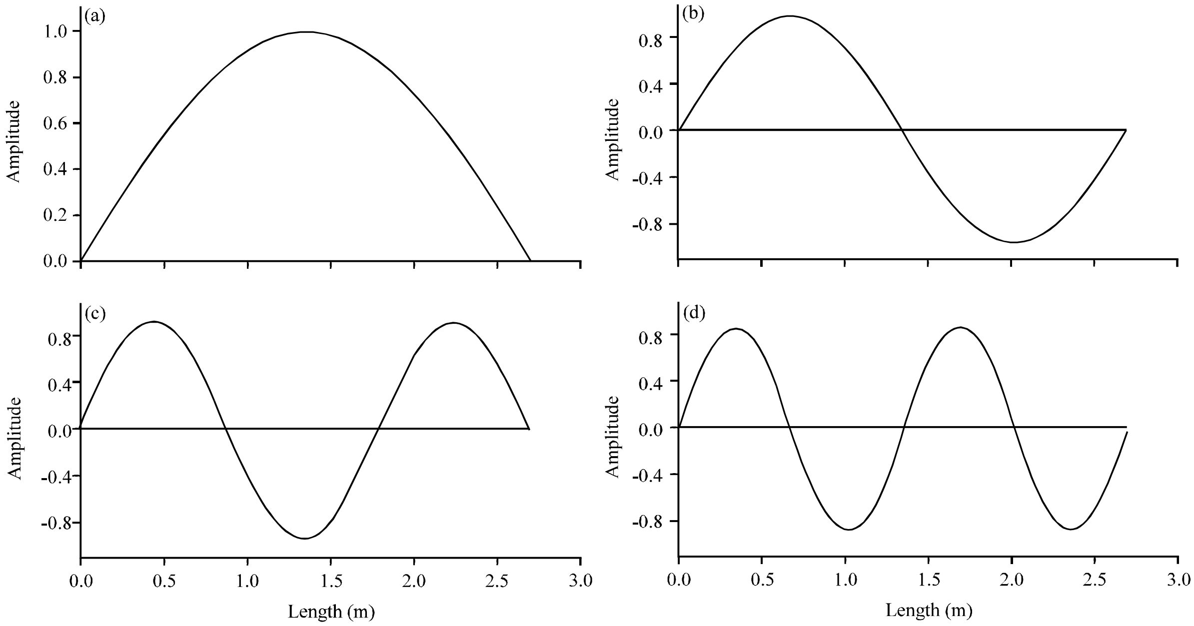

Fig. 4(a-d): Modes of vibration simply supported beam, (a) First mode of variation, (b) Second mode of variation, (c) Third mode of variation and (d) Forth mode of variation

Table 1: Amplitude of corresponding point of each natural mode of vibration

Natural mode of vibration	Amplitude											
Coordinate mode	0	0.38	0.58	0.96	1.16	1.54	1.74	1.93	2.12	2.32	2.51	2.7
Fist	0	0.43	0.62	0.90	0.98	0.97	0.90	0.78	0.62	0.43	0.22	0
Second	0	0.78	0.97	0.78	0.43	-0.43	-0.78	-0.97	-0.97	-0.78	-0.43	0
Third	0	0.98	0.90	-0.22	-0.78	-0.78	-0.22	0.43	0.90	0.98	0.62	0
Fourth	0	0.98	0.43	-0.98	-0.78	0.78	0.98	0.43	-0.43	-0.97	-0.78	0

unit length of the simply-supported beam, so q(x, t) = 0 in the experiment and equation 5 can be compiled as Eq. 6:

$$\bar{m}\frac{\partial^2 y(x,t)}{\partial t^2}+EI\frac{\partial^4 y(x,t)}{\partial x^4}=0 \tag{6}$$

Equation 6 is partial differential equation of free vibration of the simply-supported beam. And the solution of equation 6 as follows:

$$y\,(x,\,t) = Y(x,\,t)T(t) = Y(x)\sin\,(\omega_n t+\varphi) \tag{7}$$

y(x, t) is the main mode and put the (7) into (6) we can obtain:

$$Y\,(x,\,t) = (A\sin kx+B\cos kx+C\operatorname{sh} kx+D\operatorname{ch} kx).\sin\,(\omega_n t+\varphi) \tag{8}$$

According to simply-supported beam boundary conditions: at the both ends of the beam, there are four boundary conditions and two initial vibration conditions, so undetermined parameters A` B` C` D` ω_n` φ can be partly obtained. Thus we can get the natural frequency and the main vibration mode of simply-supported beam:

$$\omega_{nj}=Ck_j^2=\frac{j^2\pi^2}{l^2}\times\sqrt{\frac{EI}{\rho A}},\ (j=1,2,3,\cdots,\infty) \tag{9}$$

$$Y(x)=A_j\sin k_j x=A_j\sin\frac{j\pi}{l}x,\ (j=1,2,3\cdots\infty) \tag{10}$$

Putting the simply-supported beam's parameters into Eq. 9 and 10, we can obtain the natural frequency and natural vibration mode of the simply-supported beam. The first four natural frequencies: 9.4, 36.4, 88, 142.3 Hz. As can be concluded by the first four natural frequencies, the natural frequencies are in line with ω_1: ω_2: ω_3: ω_4: = 1^2: 2^2: 3^2: 4^2 (Liu and Jia, 2004). Based on equation 6, we could get data of each vibration mode by Fortran programming, as shown in Table 1, every natural vibration mode is shown in Fig. 4.

MODAL IDENTIFICATION EXPERIMENT OF THE SIMPLY-SUPPORTED BEAM

Experimental laws: As the most common structure of constructional engineering, simply-supported beam is

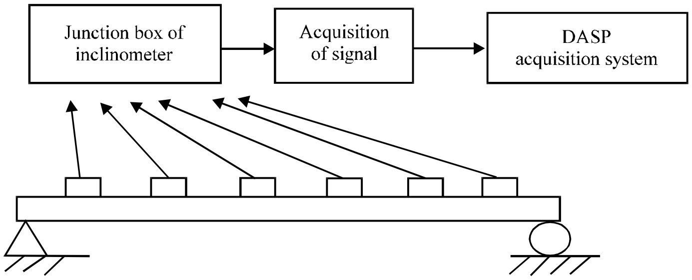

Fig. 5: Arrangement of experiment instrument

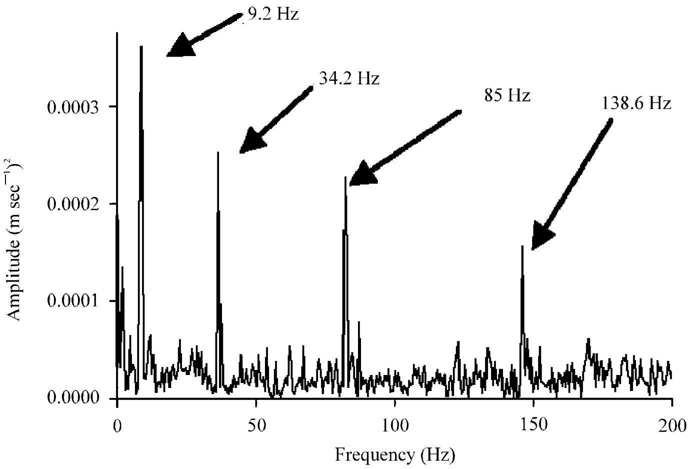

Fig. 6: Cross-power spectrum of 1and 2 reference point

relatively simple in vibration and easy to the test. But in theory, simply-supported beam is still infinite degree of freedom system, that is to say it has an unlimited number of natural frequencies and modal shapes. According to structural dynamics, the vibration of the beam is the superposition of infinite number of fundamental modes. However, when under the ambient excitation, including certain natural frequency, vibration of the beam will be generated by the natural frequency as leading and the vibration of other frequencies can be negligible.

Natural frequency identification of the simply-supported beam: The experimental apparatus arranged as shown in Fig. 5. In the trial, due to the interference of AC voltage signals, a multiple of 50 Hz interference frequency will be shown. Therefore using the band-stop filter in the dressing digital filtering that comes with the DASP system for digital filtering (INV, 2005). Band-stop filter is a filtering method in which most of the frequency components could get through and certain of frequency components will decline to very low. After the filtering, we will do modal parameter identification with cross-power spectrum modal identification method (Tao, 2007). The first four natural frequency values of identification are shown in Fig. 6.

Table 2: Comparison of the results of natural frequency

Mode	Theoretical value (Hz)	Inclinometer data (Hz)	Relative error (%)
Fist	9.4	9.2	2.1
Second	36.4	34.2	6.6
Third	88.0	85.0	3.4
Fourth	142.3	138.6	2.6

From the Fig. 6, we can get the first four natural frequency of the simply-supported beam: 9.2, 34.2, 85 and 138.6 Hz. As shown in Table 2. The natural frequency obtained by inclinometer data and the theoretical value are roughly same.

Natural mode vibration identification of simply-supported beam: The theory of inclinometer is different from acceleration vibration pickup, the collected data of inclinometer: At both ends of the simply-supported beam, the collected values of measuring points are large, whereas the corresponding modal displacements of the structure are small; the collected values in the middle of the beam are small but the corresponding modal displacements are large. We will also get the

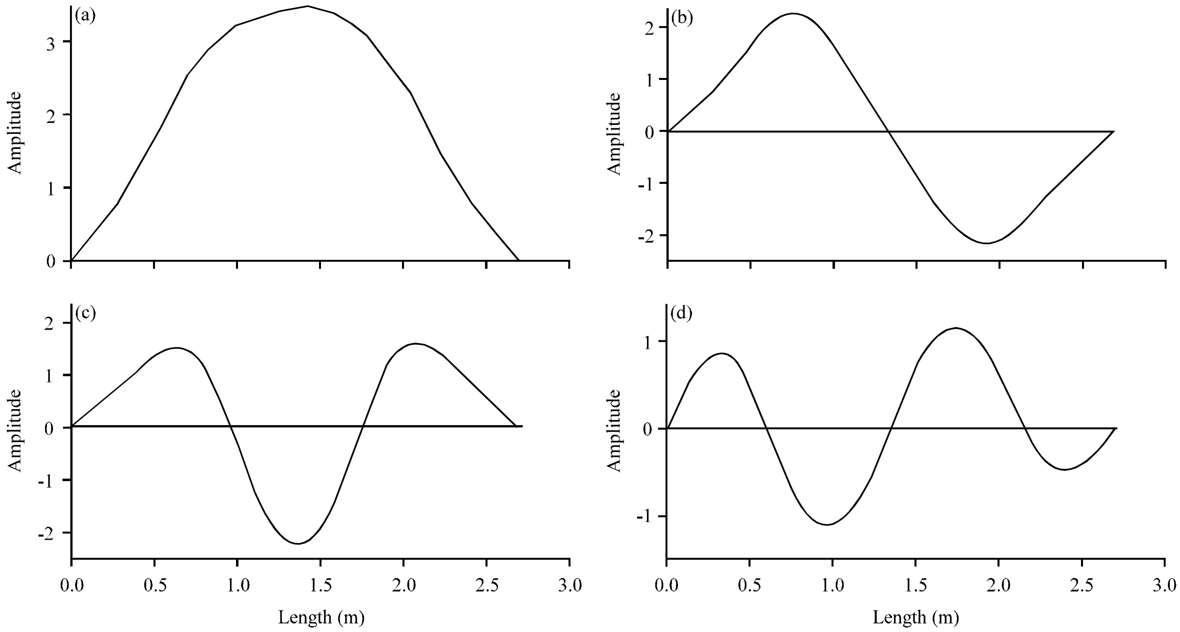

Fig. 7(a-d): Modes of vibration simply supported beam, (a) First mode of variation, (b) Second mode of variation (c) Third mode of variation and (d) Forth mode of variation

Table 3: Data of the first mode of vibration

Measuring point coordinate	Original data ($\times 10^{-3}$)	Reciprocal ($\times 10^{3}$)	First measured point of reference
0.00	0.0	0.0	0.0
0.38	11.5	0.8	1.0
0.77	3.8	2.7	3.0
1.16	3.4	2.9	3.4
1.54	3.2	3.1	3.6
1.93	4.0	2.5	2.9
2.32	11.9	8.4	0.9
2.70	0.0	0.0	0.0

corresponding results after the collected data is transformed by cross-power spectrum, so it is essential to process the results. Take the data of first vibration mode as example, using the reciprocal method for processing the numerical results of the cross-power spectrum which is shown in the Table 3. We can get the first modal vibration type of the structure with the data in Table 3 which is fitted by the least square fitting method (Dang and Wu, 2010), so does other modal vibration type, as it is shown in Fig. 7.

From the analysis of the results of the inclinometer data, we can draw a conclusion: With this identification method each modal vibration type of simply-support beam is consistent with the theoretical data result and it is scientific to test structural mode using inclinometer.

CONCLUSION

In this article, through the modal experimental research by using inclinometer, we draw the following conclusions:

- In the simply-supported experiment, we can get the natural frequency obtained by the inclinometer data is roughly the same with theoretical value and in line with the proportion relations. The natural mode vibration of simply-supported beam is also consistent with theoretical requirement. So it is feasible to identify bridge mode with inclinometer
- Nowadays, QY inclinometer is mainly used in testing bridge deflection. Analysis and study of the beam shows that QY inclinometer is not only used in bridge deflection test but also used in testing modal parameters of structure, improving utilization of experimental instrument and having perfect practical engineering significance, expanding the method and the scope of bridge structural health monitoring
- Using the inclinometer test does not require any stationary reference point, reducing the dependence on environmental conditions. So it can be used to test the dynamic parameters of the river-crossing bridge, sea-crossing bridge and high bridge etc
- For active bridge it will greatly shorten the time of interrupt bridge and for the transportation busy railway bridge, highway bridge it has better engineering value

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