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## Control of a Ball on Sphere System with Adaptive Neural Network Method for Regulation Purpose

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**Abstract:** Over the past decades, numerous control laboratory experimental setups, such as the inverted pendulum, ball and beam, ball and plate, ball and wheel and ball and sphere, have been developed for control education and research. Several attempts have been made to control a system which contains a ball on a body such as using feedback linearization and sliding mode control which couldn't regulate the angle position of ball in present of random disturbance. In this study, adaptive neural network position control of a class of Multi-Input Multi-Output (MIMO) nonlinear systems called "system of ball on a sphere" in present of random disturbance is considered. The system's dynamic is described and the equations are illustrated. Controlling simulation is in consideration of disturbance, to show exactness of the adaptive neural network controlling method. These simulation results show the propriety and accuracy of the controller's performance.

**Key words:** Adaptive neural network control, ball on a sphere, random disturbance, MIMO systems, nonlinear systems

### INTRODUCTION

Recently, several attempts have been made to analyze the dynamic and control a system which is contains a ball on a body and its stability. Although, the system of ball on a sphere is particularly a nonlinear system, it can be considered linear around the equilibrium point (Ho *et al.*, 2009). The system of ball on a sphere can be seen in Fig.1. Past researches have shown that artificial neural networks are effective soft-computing techniques to model and control a broad category of complex nonlinear systems, especially to those systems whose mathematical models are extremely difficult to obtain (Hu and Hwang, 2001). Artificial neural networks have traditionally been used for classification and prediction tasks (Hourfar and Salahshoor, 2009; El-Emam and Yousif, 2009). Attractive features of neural networks are that they can be trained easily by using past data records from systems under study. They are readily applicable to multivariable systems (Bazargan-Lari *et al.*, 2012; Coban, 2004; Mahi and Izabatene, 2011; Rashid *et al.*, 2012; Hourfar and Salahshoor, 2009; Aliasghary *et al.*, 2008). These facts suggest that neural networks, in conjunction with a suitable control strategy such as model-based control and neural control can be used to control non-linear systems

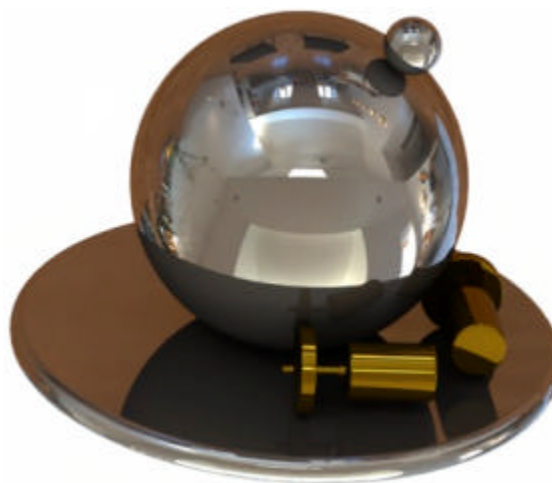


Fig. 1: A ball on a sphere system, system of ball on a sphere such an inherently nonlinear, unstable and under actuated system

as an adaptive sub system (Hourfar and Salahshoor, 2009; Qiu *et al.*, 2011; Halbaoui *et al.*, 2012). The solution is given for the regulation case.

In this study, In particular, the nonlinear nature of neural networks and the ability of neural networks to learn

from their environments in supervised as well as unsupervised ways, make them highly suited for solving difficult larger class of these nonlinear systems is proposed.

**DYNAMIC AND MODELING**

In the present study, a ball on a sphere system with arbitrary desires is controlled by the adaptive neural network controller. For this purpose, a model for the ball on a sphere system has been opted and then, its dynamical equations have been derived (Liu *et al.*, 2011; Ho *et al.*, 2009). Although, these dynamical equations are extremely non-linear and their parameters are interdependent in various directions, they have been considered linear around the equilibrium point, since in that point, the parameters are assumed independent in all directions. In this work, like a ball and wheel system, the system of ball on sphere is considered to be tow dimensional in all directions is shown in Fig. 2.

The generalized coordinates of system are  $\theta_x, \beta_x$  which respectively denote the ball and the spheres angles with respect to the x direction,  $\theta_y, \beta_y$  which denotes the ball and the spheres angles with respect to the Y direction. The parameters of system are  $I_B$  and  $I_b$  which are moments of inertia of the sphere and ball, respectively and  $m$  as the mass of ball. There is also  $R$  and  $r$  which already denote the sphere and balls' radiuses, respectively.

Then, by using the Euler-Lagrange formulation, the systems equation will be derived:

$$L = K-U \tag{1}$$

$$\frac{\partial}{\partial t} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i, i=1,2,3,4 \tag{2}$$

$$Q_1 = 0 \tag{3}$$

$$Q_2 = 0 \tag{4}$$

$$Q_3 = T_x \tag{5}$$

$$Q_4 = T_y \tag{6}$$

$$\left( (R+r)m + I_b \frac{R+r}{r^2} \right) \ddot{\theta}_x + \left( -I_b \frac{R}{r^2} \right) \ddot{\beta}_x - mg \sin(\theta_x) = 0 \tag{7}$$

$$\left( -I_b \frac{R(R+r)}{r^2} \right) \ddot{\theta}_x + \left( I_B + I_b \frac{R^2}{r^2} \right) \ddot{\beta}_x = T_x \tag{8}$$

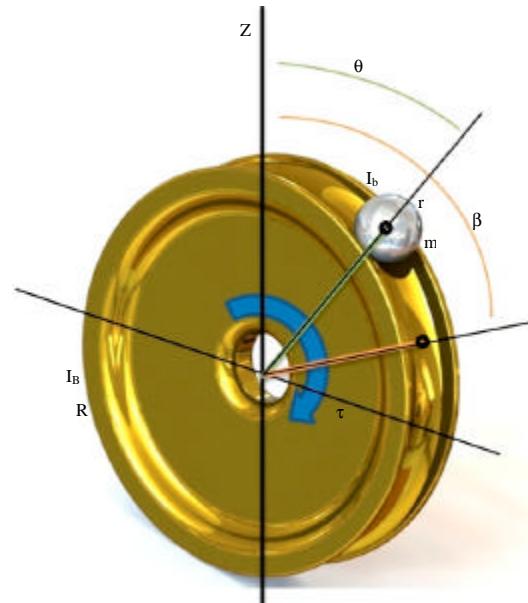


Fig. 2: 2-D schema of the system, system of ball on a sphere can considered truly as two independent ball and wheel systems around its equilibrium point

$$\left( (R+r)m + I_b \frac{R+r}{r^2} \right) \ddot{\theta}_y + \left( -I_b \frac{R}{r^2} \right) \ddot{\beta}_y - mg \sin(\theta_y) = 0 \tag{9}$$

$$\left( -I_b \frac{R(R+r)}{r^2} \right) \ddot{\theta}_y + \left( I_B + I_b \frac{R^2}{r^2} \right) \ddot{\beta}_y = T_y \tag{10}$$

$$Q = [\theta_x, \beta_x, \theta_y, \beta_y] \tag{11}$$

$$Mq+G = T \tag{12}$$

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \\ M_{41} & M_{42} & M_{43} & M_{44} \end{bmatrix} \tag{13}$$

$$M_{11} = (R+r)m + I_b \frac{R+r}{r^2}$$

$$M_{12} = -I_b \frac{R}{r^2}$$

$$M_{13} = 0$$

$$M_{14} = 0$$

**ADAPTIVE NEURAL NETWORK CONTROL**

In order to present an adaptive neural network control method for a (robotic) system, one may begin with an adaptive algorithm to make it robust with respect to some uncertainty and disturbances. To achieve this goal, a radial basis network that is a feed-forward neural network using the radial basis activation function is designed. In this study, the radial basis function of choice is the Gaussian radial basis function:

$$a_i(y) = \exp\left(-\frac{y - \mu_i^2}{2\sigma^2}\right)$$

where,  $\mu_i \in \mathbb{R}^n$  the center is vector and  $\sigma^2$  is the variance. Each Gaussian RBF network includes three layers: the input layer, the hidden layer and the output layer. The output of the network  $f(W, y)$  can be given by:

$$\hat{f}(W, y) = W^T \alpha(y)$$

where,  $\alpha(y) [\alpha_1(y) \alpha_2(y) \dots \alpha_n(y)]^T$  is generally positive for commonly used radial basis functions.

Define:

$$\begin{aligned} \dot{q}_r(t) &= \dot{q}_d(t) + q_v + \Lambda e(t) \\ r(t) &= \dot{q}_r(t) - \dot{q}(t) = \dot{e}(t) + \Lambda e(t) \end{aligned}$$

where,  $\Lambda$  is a positive definite matrix. Static neural networks are adequate to model  $D(q)$  and  $G(q)$  since that both of them are functions of  $q$  only. Assume that  $d_{kij}(q)$  and  $g_k(q)$  can be define as:

$$\begin{aligned} d_{kij}(q) &= \theta_{kij}^T \xi_{kij}(q) + \varepsilon_{dki}(q) \\ q_k &= \beta_k^T \eta_k(q) + \varepsilon_{gk}(q) \end{aligned}$$

where,  $\xi(q), \eta(q) \in \mathbb{R}$  are the Gaussian activation basis functions,  $\theta, \beta \in \mathbb{R}$  are the weight vectors of the neural networks and  $\varepsilon_{dki}(q), \varepsilon_{gk}(q) \in \mathbb{R}$  are the modeling errors of  $d_{kij}(q), g_k(q)$ , respectively, which are assumed to be bounded.

From above and according to Eq. 16, the control law can be found as:

$$\tau = \left[ \begin{matrix} \{\hat{\theta}\}^T \\ \{\Xi(q)\} \end{matrix} \right] \ddot{q}_r + \left[ \begin{matrix} \{\hat{\beta}\}^T \\ \{H(q)\} \end{matrix} \right] \dot{q}_r + K_r + K_s \text{sgn}(r)$$

The last term in the control law is added to repress the modeling errors of the neural networks. It can be proved for some values  $K$  and  $K_s$  if parameters update by:

$$\begin{aligned} M_{21} &= -Ib \frac{R(R+r)}{r^2} \\ M_{22} &= IB + Ib \frac{R^2}{r^2} \\ M_{23} &= 0 \\ M_{24} &= 0 \\ M_{31} &= 0 \\ M_{32} &= 0 \\ M_{33} &= (R+r)m + Ib \frac{R+r}{r^2} \\ M_{34} &= -Ib \frac{R}{r^2} \\ M_{41} &= 0 \\ M_{42} &= 0 \\ M_{43} &= -Ib \frac{R(R+r)}{r^2} \\ M_{44} &= IB + Ib \frac{R^2}{r^2} \\ G &= \begin{bmatrix} -mg \sin(q_1) \\ 0 \\ -mg \sin(q_3) \\ 0 \end{bmatrix} \\ T &= \begin{bmatrix} 0 \\ T_x \\ 0 \\ T_y \end{bmatrix} \end{aligned} \tag{14}$$

And finally as a result, the dynamic model of the system is stated in the form of matrices as shown in Eq. 16:

$$M(q) \ddot{q} + G(q) + \tau_f = T \tag{16}$$

where, that matrices  $M(q), G(q)$  and vector  $\tau_f$  represent the inertia, gravity term and disturbance, respectively.

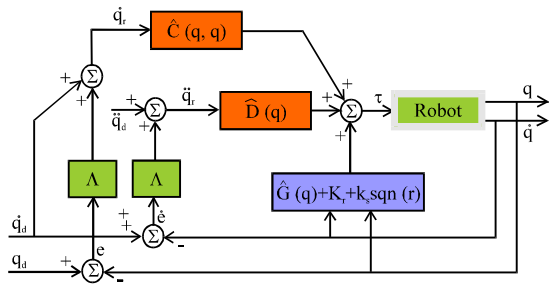


Fig. 3: Adaptive neural network algorithm, the control structure of proposed approach  $q$ ,  $\dot{q}$ ,  $\ddot{q}$  are the desired joint angle, the desired joint angular velocity and the desired joint angular acceleration, respectively

$$\begin{aligned} \hat{\theta}_k &= \Gamma_k \cdot \{\xi_k(\hat{q})\} \ddot{q}_k r_k \\ \hat{\alpha}_k &= Q_k \cdot \{\xi_k(z)\} \dot{q}_k r_k \\ \hat{\beta}_k &= N_k \eta_k(q) r_k \end{aligned}$$

Regulation error and its time derivative converge to zero when large enough time (Wei *et al.*, 2005).

The control law is shown in Fig. 3.

### RESULTS AND DISCUSSION

The desired values of ball angles in order to have a regulation control at its unstable equilibrium point is considered at  $\theta_x = 0$  and  $\theta_y = 0$ .

The physical properties of the ball and the sphere, which already described in the modeling section, are:

- $m = 0.06 \text{ kg}$
- $r = 0.0125 \text{ m}$
- $R = 0.15 \text{ m}$
- $I_b = 3.75 \times 10^{-6} \text{ kg m}^{-2}$
- $I_B = 0.99 \text{ kg m}^2$
- $g = 9.81 \text{ m sec}^{-2}$

The desired values for the initial condition are also should be specified. Initial position angle of the ball is considered around its equilibrium point at  $\theta_{x0} = 0.07$  and  $\theta_{y0} = 0.07$ . Also, the initial angular velocities of the ball are  $\dot{\theta}_{x0} = 0.02$ ,  $\dot{\theta}_{y0} = 0.05$ . The initial position and velocity of sphere in both X and Y directions are  $\beta_{x0} = 0$ ,  $\beta_{y0} = 0$  and  $\dot{\beta}_{x0} = 0$ ,  $\dot{\beta}_{y0} = 0$ , respectively.

For each element of  $D(q)$  and  $G(q)$ , a 100-node for hidden layer of neural network is used. The values of  $i$  and  $\sigma^2$  are considered 0.0 and 10.0, respectively. To show the potential of the adaptive neural network

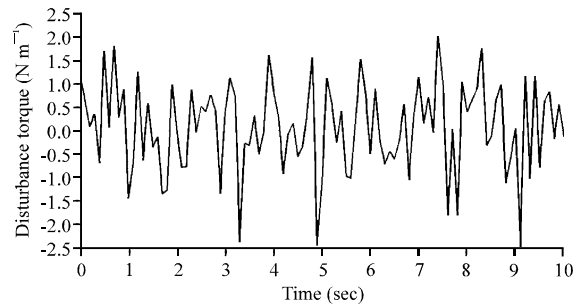


Fig. 4: Disturbance, the controlling simulation has been done due to the disturbance

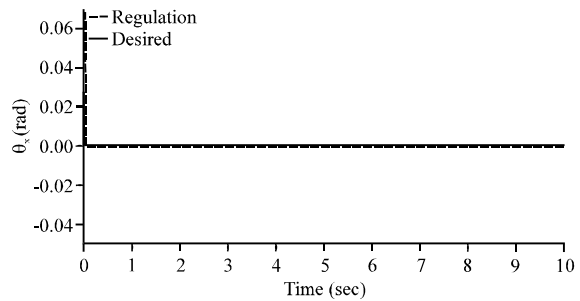


Fig. 5: Time evolution of  $\theta$  in X direction in present of disturbance after 10 sec ball angle in X direction becomes stable in 0.1 sec

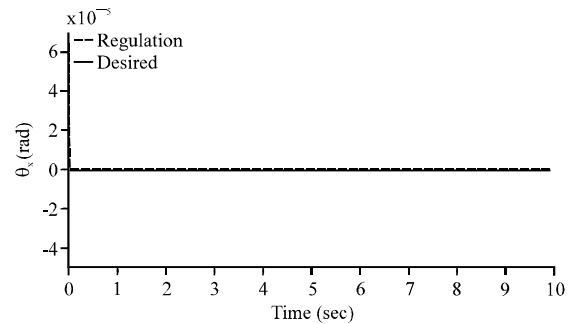


Fig. 6: Regulated  $\theta$  in Y direction in 10 sec. It is stable in presented disturbance after 0.1 sec

controlling method, a complex random term in which denotes the disturbance of the system has been applied in X-direction. The disturbance is shown in Fig. 4.

The controlling simulation has been done due to the disturbance given in Fig. 4. These simulation results are summarized in Fig. 5-12. In Fig. 5 it can be observed ball angle in X direction become regulated in 0.1 seconds in present of disturbance and the error tends to zero after 0.1 sec. It can be also observed the same in Y direction in Fig. 6 which shows regulation after 0.1 sec. The time

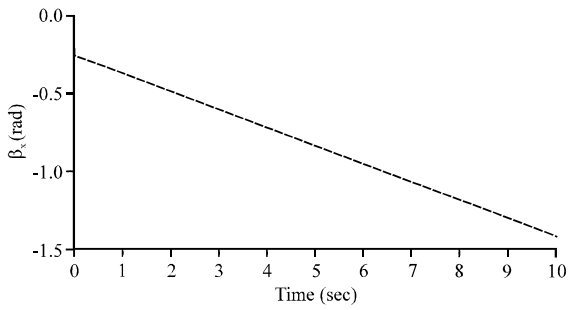


Fig. 7: Time evolution of  $\beta$  in X direction in present of disturbance after 10 sec

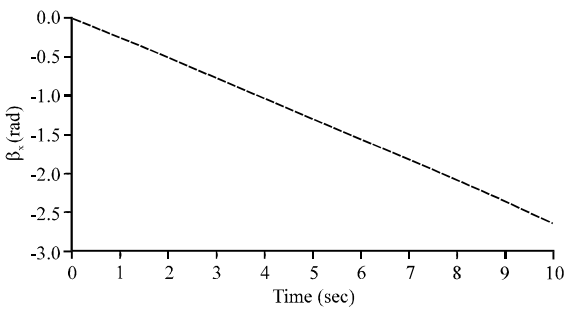


Fig. 8:  $\beta$  in Y direction is plotted vs. time in 10 sec, the angle of ball related to Y direction

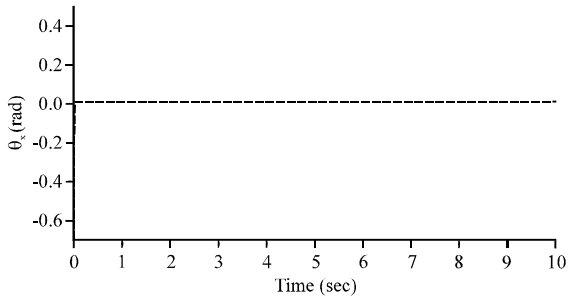


Fig. 9: Error of angel  $\theta$  is X direction the error tends to 0 after 0.1 sec

evolution of beta in X and Y directions in present of disturbance after 10 seconds is shown in the Fig. 7 and 8, respectively. In the Fig. 9, the error of angel theta in X direction can be observed. The error of angle theta in Y direction is shown in Fig. 10. The error tends to zero after 0.1 sec. The torque required is applied to the sphere in X direction to control the position of the ball in present of disturbance is shown in Fig. 11. Also, simulation result of the Y-axis control input response of the motor is shown in Fig. 12.

According to the results, it can be realized that the ball is regulated in both X and Y directions in less than 0.1 sec, much faster than previous works. Also, from

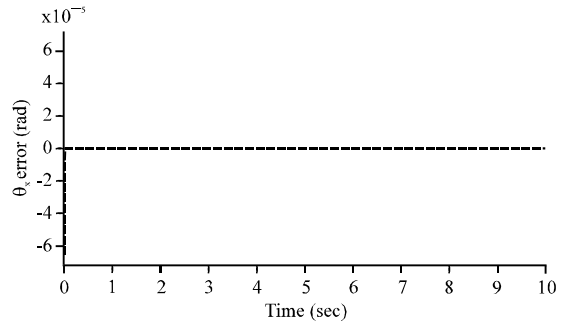


Fig. 10:  $\theta$  error in Y-direction, Error of angel  $\theta$  in Y direction reaches its desired value. The error tends to zero after 0.1 sec

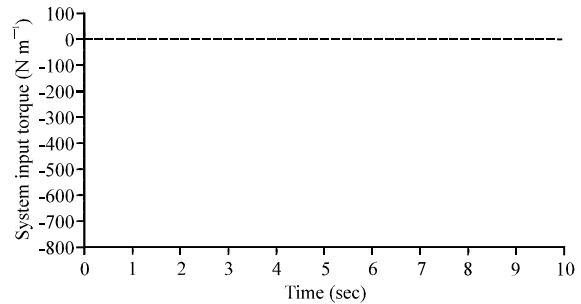


Fig. 11: Torque in X-direction, torque is applied to the sphere in the X direction to control the position of the ball in present of disturbance by means of changing  $\beta$  in X direction

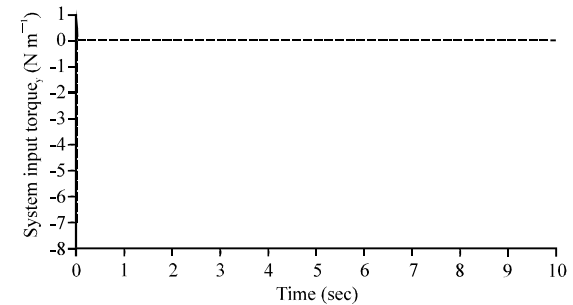


Fig. 12: Torque in Y-direction, torque is applied to the sphere in the Y-direction to control the position of the ball in present of disturbance by means of changing  $\beta$  in Y direction

figures it is obvious that the errors tends to zero quickly with smoother maximum over shoots thanks to adaptive neural network control approach.

**CONCLUSION**

The purpose of this article was to control a system of “ball on a sphere”. At first, a dynamical model that had

been constructed for this special system had been utilized. By choosing a proper controller called adaptive neural network method, as can be shown in the simulation results and figures, an appropriate regulation control has been achieved. As can be observed from results, adaptive neural controller was able to lead the system to the desired position in consideration of undesirable disturbances. The great accuracy of the diagrams represents the propriety and accuracy of used controller which can continue doing its task well in the presence of disturbance.

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