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Effect of Various Fluid Densities on Vibration Characteristics in Variable Cross-section Pipes

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Abstract: Vibration characteristics of pipe conveying fluid with sudden enlargement-sudden contraction were studied in this study. The problem was modeled mathematically and simulated numerically. The governing equations of motion for a pipe conveying fluid were derived using beam theory and the mathematical model was solved using transfer matrix method in MATLAB-R2012 environment. The simulation was carried out by Mechanical APDL in ANSYS 14. Two types of fluids; namely, water and oil were used to demonstrate the effect of density on the natural frequencies and their corresponding mode shapes. The support type selected for this investigation is flexible support. The small and large diameters of the investigated pipe were 12.7 and 25.4 mm, made of copper. The external force was assumed to apply at mid length of the large diameter. A good agreement between the mathematical solution and the numerical simulation was achieved. In all cases, it was found that the natural frequencies with any fluid flow were lower than the case of pipe without fluid flow. The study can be extended for further cases of pipe geometries and for two phases fluid flows.

Key words: Induced vibration, pipe conveying fluid, flow structure interaction, pipe supports, pipe flow, beam theory

INTRODUCTION

The flow through a pipe with sudden enlargement and contraction occurs in many industrial applications and is characterized by increased pressure losses caused by flow separation close to the change in the cross sectional area. This increasing in pressure losses will increase the erosion rates and the heat in the regions where separated flow occurs (Mahdi, 2001).

The most important flow characteristics are the nature of flow (laminar or turbulent), velocity and pressure. Normally the nature of flow is circular and depended on the Reynolds Number (Re) and for most engineering applications if (Re>2300) the flow is laminar (Saleh, 1978).

Also, the fluid flowing through the pipe may impose pressures on the pipe walls which deflect the pipe, where at a high velocity flow through a thin wall pipe it can either buckle the pipe or cause it to fail. In certain applications involving very high velocity flows through flexible thin wall pipes combined with vibration such as (the feed lines to rockets and water turbines) the pipe may become susceptible to resonance and fatigue failure if its natural frequency falls below certain limits (Blevins, 1979).

Pittard (2003) studied the flow-induced vibration caused by fully developed pipe flow under turbulent conditions. This study focuses on the development of a numerical Fluid-Structure Interaction (FSI) model that will help define the relationship between pipe wall vibration and the physical characteristics of turbulent flow. The results show that a strong relationship between pipe vibration and flow rate exists.

A cantilevered pipe subjected to external transverse (or lateral) force is investigated by (Lilkova-Markova and Lolov, 2004). The major findings are the variations in frequency with flow velocity and displacements at different points and times.

Lolov and Lilkova-Markova (2006) studied the curved pipes conveying fluids. Methods of numerical solution of the dynamic stability of a pipe in its plane are developed. An example of a curved pipe is solved by these methods. A non-dimensional parameter of flow velocity and a non-dimensional circular frequency are obtained.

Experimental and numerical investigations of laminar flow in a pipe with a sudden contraction in the cross-sectional area were given by Durst and Loy (1985). Investigations were carried out to understand the increased pressure loss generated in this region. In addition, the detailed experimental velocity profile measurements permit comparison with numerical

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predictions. To yield reliable data, a laser-doppler anemometer together with a test section containing a liquid with the same refractive index as the test section wall materials, were employed. Numerical predictions of the flow employing finite difference computer code were also undertaken. A small separated flow region exists in the concave corner of the contraction.

A pipe conveying fluid with a sudden enlargement and with the effect of heat flux combined with vibration on these pipes was studied by Hammoudi (2007). Several end pipe supports (simply, flexible, fixed) were adopted. A mathematical model was developed using the transfer matrix method to show the effect of vibration and the effect of implementing different values of heat flux on pipes conveying fluid with a sudden enlargement. The most important result of this investigation is that the natural frequencies of the vibrated system decreased when the flowing fluid and thermal forces were taken in consideration. This reduction increased as the applied heat flux was increased. While increasing fluid velocity without applying heat flux did not affect the values of the natural frequencies.

To design safe and reliable piping systems free from excessive vibrations, the piping designer needs to know the frequencies of excitation forces in the piping and must be able to calculate the mechanical natural frequencies of the pipeline system.

The objective of the present research is to study the effect of different fluid densities in a pipe conveying fluid with sudden enlargement-sudden contraction. Also, evaluate the natural frequencies and their corresponding mode shapes due to external effect when the pipe is supported by flexible type of supports. The study has been conducted by developing mathematical model solved in MATLAB environment and a simulation model solved in ANSYS 14 commercial software. The case was studied at Reynolds value of 1500.

METHODOLOGY

The case of pipe conveying fluid was modeled mathematically and solved by Transfer Matrix Method (TMM). The construction of the mathematical model is implemented as below.

Governing equation of motion: Consider a straight pipe conveying uniform internal flow as shown in Fig. 1. The straight pipe, supported at both ends, has dimensions given by the Length (L), the cross-sectional outer Diameter (D) and the Thickness (th). It is assumed that the pipe is sufficiently slender, that is, (D/L)<<0.1. This diameter to length ratio makes it considered as a beam. Moreover, the fluid in the pipe is assumed to be incompressible. The flow regime in this study is assumed laminar, where the secondary flow effects are negligible.

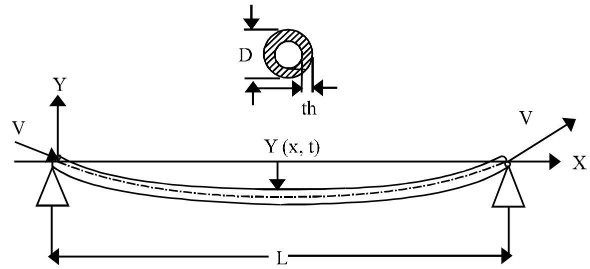

Fig. 1: Pipe conveying fluid

The equation of motion for free vibration of pipe conveying fluid derived and may be written as (Eq. 1):

$$EI\frac{\partial^4 y}{\partial x^4}+\left(m_f u_1 u_2+P_f A_p\right)\frac{\partial^2 y}{\partial x^2}+2m_f u_1\frac{\partial^2 y}{\partial x\partial t}+\left(m_f+m_p\right)\frac{\partial^2 y}{\partial t^2}=0 \quad (1)$$

So, for forced vibration, the equation of motion must be equated to the excitation harmonic force F (x,t) (Eq. 2):

$$EI\frac{\partial^4 y}{\partial x^4}+\left(m_f u_1 u_2+P_f A_p\right)\frac{\partial^2 y}{\partial x^2}+2m_f u_1\frac{\partial^2 y}{\partial x\partial t}+\left(m_f+m_p\right)\frac{\partial^2 y}{\partial t^2}=F(x,t) \quad (2)$$

where, F(x,t): is the external harmonic force:

Stiffness term:

$$EI\frac{\partial^4 y}{\partial x^4}$$

Curvature term:

$$\left(m_f u_1 u_2+P_f A_p\right)\frac{\partial^2 y}{\partial x^2}$$

Coriolis force term:

$$2m_f u_1\frac{\partial^2 y}{\partial x\partial t}$$

Inertia force term:

$$\left(m_f+m_p\right)\frac{\partial^2 y}{\partial t^2}$$

Now the equation of motion for forced vibration of pipe conveying fluid may be written as (Eq. 3):

$$E\cdot I\cdot y''''+\left(P_f\cdot A_p\cdot m_f\cdot u_1\cdot u_2\right)y''+2\cdot m_f\cdot u_1\cdot y'+\left(m_f+m_p\right)\ddot{y}=F(x,t) \quad (3)$$

If there is no fluid, (u, P, ρ, m_f) equal to zero and the equation of motion will reduce to (Eq. 4):

$$E \cdot I \cdot y'''' + (m_p)\ddot{y} = 0 \quad (4)$$

The following dimensionless variables are adopted which are already used by (Reddy and Wang, 2004).

$$\bar{X} = \frac{x}{L_m}, \bar{Y} = \frac{y}{L_m}, \bar{U}\left(\frac{m_f}{E \cdot I}\right)^{1/2} \cdot u_f \cdot L_m$$

$$\beta = \left(\frac{m_f}{m_f + m_p}\right)^{1/2}, \gamma = \left(\frac{L_m^2}{E \cdot I}\right) \cdot P \cdot A_p$$

$$\tau = \left(\frac{E \cdot I}{m_f + m_p}\right)^{1/2} \left(\frac{t}{L_m^2}\right)$$

Then, the equation for forced vibration becomes (Eq. 6):

$$\bar{Y}'''' + (\gamma + \bar{U}_1 \cdot \bar{U}_2)\bar{Y}'' + 2 \cdot \bar{U} \cdot \frac{\beta}{\tau} \cdot \dot{\bar{Y}}' + \frac{1}{\tau^2}\ddot{\bar{Y}} = F(\bar{X}, \tau) \quad (5)$$

where, F ($\bar{X}$, τ) is the non-dimensional external force applied normal to the pipe axis in (y-direction).

Investigation of the flow stream: Since, the fluid discharge to atmosphere; therefore, the out let pressure of the pipe (P_3) = 1 atm and the inlet pressure to the pipe (P_1) can be found from the energy Eq. 6 as follows:

$$\frac{P_1}{\rho \cdot g} + \frac{u_1^2}{2 \cdot g} + Z_1 = \frac{P_3}{\rho \cdot g} + \frac{u_3^2}{2 \cdot g} + Z_3 + \text{Losses} \quad (6)$$

For horizontal pipe ($z_1 = z_3 = 0$) substitute in above equation gives (Eq. 7):

$$P_1 = P_3 + \left(\frac{u_3^2 - u_1^2}{2} + \text{Losses}\right) * \rho \quad (7)$$

Where:

Losses = $P_{L1}+P_{Le}+P_{L2}+P_{Lc}+P_{L3}$

P_{L1} = Losses for the first part of pipe (before enlargement)

P_{L2} = Losses for the second part of pipe (after enlargement)

P_{L3} = Losses for the third part of pipe (after contraction)

P_{Le} = Losses at enlargement:

$$[\frac{1}{2} C_e \cdot \rho \cdot u_1^2]$$

P_{Lc} = Losses at contraction:

$$[\frac{1}{2} C_c \cdot \rho \cdot u_3^2]$$

C_e = C_c = Constant = 0.5 (Douglas *et al*., 1981)

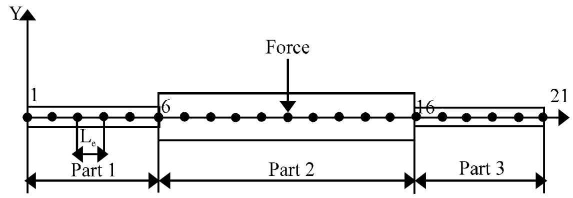

Fig. 2: Pipe with discrete elements and masses

Transfer matrix method: The adopted solution method of the mathematical model is the transfer matrix method, known as T.M.M. In this method the vibrating system is arranged in a line and the behavior at any point in the system is only influenced by the behavior at neighboring points as shown in Fig. 2.

To fully describe the situation at each node, four quantities must be known; the deflection, Y, slope, Φ, moment, M and shear forces, Q. In the present study, two fluid physical properties have been added, which are velocity, U and pressure, P. Accordingly, six variables are affecting the node status and can be arranged in a state vector as:

$$Z = \{Y, \Phi, M, Q, U, P\}$$

The matrices below show the adopted field and point matrices in this investigation.

- **Field matrix:** The field matrix [F] for a pipe element may be written in matrix notation in dimensionless form, as (Eq. 8):

$$[\bar{Z}]_i^L = [F_i] \cdot [\bar{Z}]_{i-1}^R \quad (8)$$

Or, in detailed matrix format:

$$\begin{bmatrix} -\bar{Y} \\ \bar{\Phi} \\ \bar{M} \\ \bar{Q} \\ \bar{U} \\ \bar{P} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & F_{12} & F_{13} & F_{14} & 0 & 0 & F_{17} \\ 0 & 1 & F_{23} & F_{24} & 0 & 0 & F_{27} \\ 0 & 0 & 1 & F_{34} & 0 & 0 & F_{37} \\ 0 & 0 & 0 & 1 & 0 & 0 & F_{47} \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & F_{67} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\bar{Y} \\ \bar{\Phi} \\ \bar{M} \\ \bar{Q} \\ \bar{U} \\ \bar{P} \\ 1 \end{bmatrix}_{i-1}^R$$

Where:

$$F_{12} = \frac{L_i}{L_m}, F_{13} = \frac{L_i^2}{2(EI)_i \delta \cdot L_m}, F_{14} = \frac{1}{\varphi \cdot L_m}\left(\frac{L_i^3}{6(EI)_i} - \chi \frac{L_i}{(GA_p)_i}\right),$$

$$F_{17} = -\frac{\bar{W}_i}{\varphi \cdot L_m}\left(\frac{L_i^3}{48(EI)_i} - \frac{\chi \cdot L_i}{(GA_p)_i}\right), F_{23} = \frac{Li}{(EI)_i \delta}, F_{24} = \frac{L_i^2}{2(EI)_i \varphi},$$

$$F_{27} = -\frac{\bar{W}_i L_i^2}{8(EI)_i \varphi}, F_{34} = \frac{L_i \delta}{\varphi}, F_{37} = \frac{\bar{W}_i L_i \delta}{2\varphi}, F_{47} = -\bar{W}_i, F_{67} = -\frac{2 f_i \rho_f u^2 L_i}{D_i \cdot P_1}$$

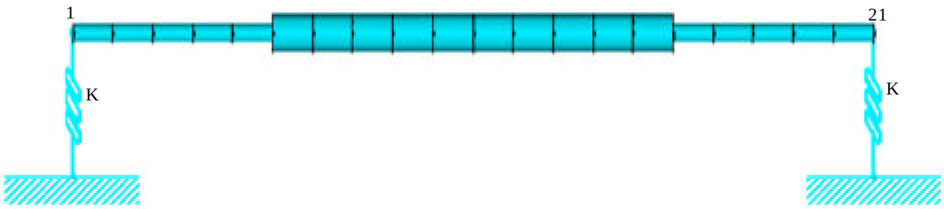

Fig. 3: Schematic diagram of pipe system with flexible support

- **Point matrix at the supported node:**

$$\begin{bmatrix} -\overline{Y} \\ \overline{\Phi} \\ \overline{M} \\ \overline{Q} \\ \overline{U} \\ \overline{P} \\ 1 \end{bmatrix}_i^R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \left[\overline{\omega}^2 - \dfrac{K \cdot L_m^3}{(EI)_m}\right] & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\overline{Y} \\ \overline{\Phi} \\ \overline{M} \\ \overline{Q} \\ \overline{U} \\ \overline{P} \\ 1 \end{bmatrix}_i^L$$

Boundary conditions: The boundary conditions in the transfer matrix method give the description of the state vector parameters at the supported ends of the pipe, which is selected as flexible support for this analysis. For flexible supported the moment and the shear forces are equal to zero and all other parameters have a value more than zero as shown in Fig. 3.

RESULTS AND DISCUSSION

Effect of fluid density on natural frequencies: To study the effect of fluid density on the natural frequencies, two types of fluids were selected which are water and oil. Their physical properties are listed in Table 1. Where, the deflections at mid length of copper pipe conveying fluid with Reynolds number (1500) and same supports (flexible) with various excitation frequencies for different kinds of fluid density are presented in (Fig. 4a-c). The values of three lowest natural frequencies from the peaks of these figures are given in Table 2. It can be noticed from the results in (Fig. 4b) that the values of the 1st, 2nd and 3rd natural frequencies for the case of vibrated pipe system conveying water at Reynolds numbers (1500) are less than the values of the 1st, 2nd and 3rdnatural frequencies for the case of vibrated pipe system without fluid (Fig. 4a). This can be related to the effect of the fluid mass which is added to the mass of the system and it is inversely proportional to the natural frequencies. Also, the values of 1st, 2nd and 3rd natural frequencies (Fig. 4c) of vibrated pipe system conveying oil are higher than those of pipe conveying water due to the lower density.

Table 1: Main physical properties for the fluid phases (Al-Yaari and Abu-Sharkh, 2011)

Property	Water phase	Oil phase
Density (ρ kg m^{-3})	998.2	780
Dynamic viscosity (μ Pa sec)	0.001003	0.00157

Table 2: Comparison of three lowest natural frequencies values of (Sudden enlargement-sudden contraction) copper pipe conveying different fluid with different densities

Flexible support	ω_{n1}	ω_{n2}	ω_{n3}
Without fluid	195	585	1634
With water	161	502	1430
With oil	167	517	1468

Table 3: Results of comparison between the transfer matrix method by using MATLAB-R2012 program and finite element method by using ANSYS-14 software for flexible support copper pipe without fluid

Analysis type	ω_{n1}	ω_{n1}	ω_{n1}
T.M.M	195	585	1634
F.E.M	195.41	589.56	1634.53
Error (%)	0.209	0.773	0.032

Effects of density on the mode shape: Figure 5a-c and 6a-c represent 1st, 2nd and 3rd mode shapes, respectively for flexible support copper pipe and for different fluid density with Reynolds number equal 1500. It can be observed that there is changing in the phase of the mode shape at the natural frequency for some of these figures like (Fig. 6a, c) where the mode shapes for flexible support pipe conveying oil is positive, while the mode shapes for flexible support pipe conveying water is negative see (Fig. 5a, c) that's because the effect of changing system properties at the different density. Also, in these figures, new phenomenon could be noticed, whereas a change in the sequence of maximum amplitude values at changing of fluid density for the same supports type.

Comparison between the mathematical and simulation results: The results obtained from transfer matrix method by adopting MATLAB-R2012 program were compared with prediction results obtained from the finite element method using ANSYS-14 software. The comparisons are presented in Table 3-5. The comparisons between the results for these two programs show a good agreement.

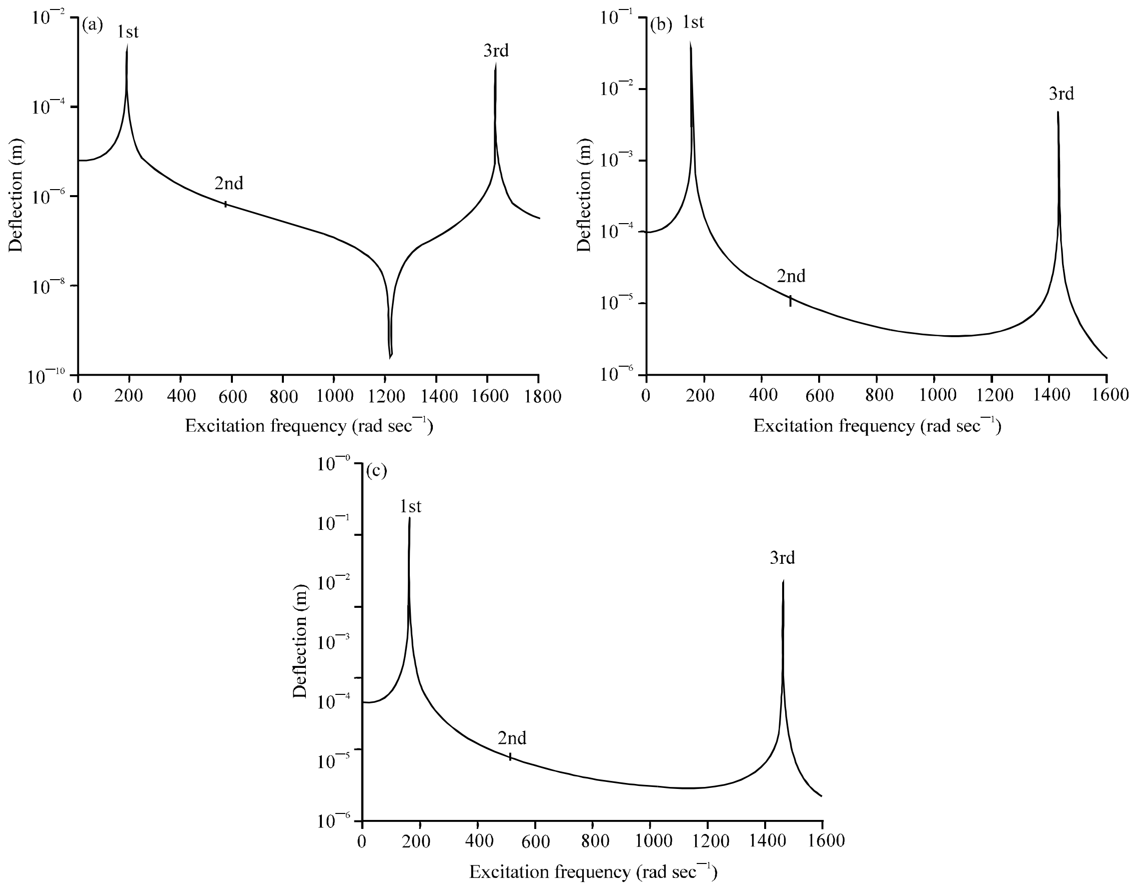

Fig. 4(a-c): Deflection at mid length of flexible supports copper pipe (a) Without fluid, (b) Conveying water and (c) Conveying oil with various excitation frequencies represents three lowest natural frequency, using MATLAB-R2012 program

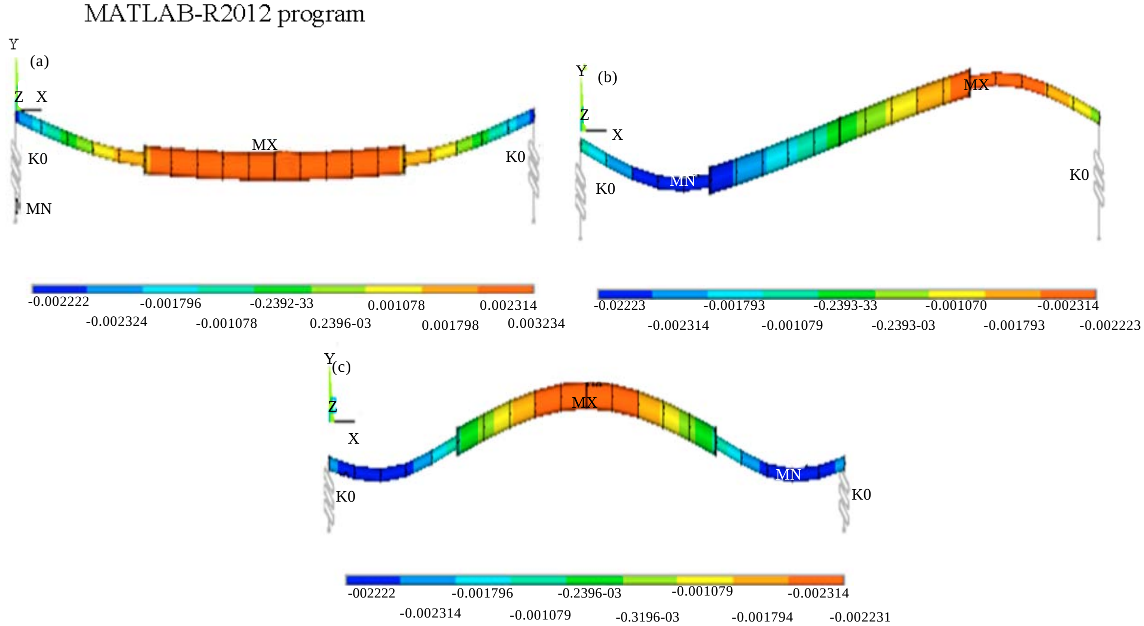

Fig. 5(a-c): (a)1st, (b) 2nd and (c) 3rd mode shapes for flexible support pipe conveying water using ANSYS-14 software

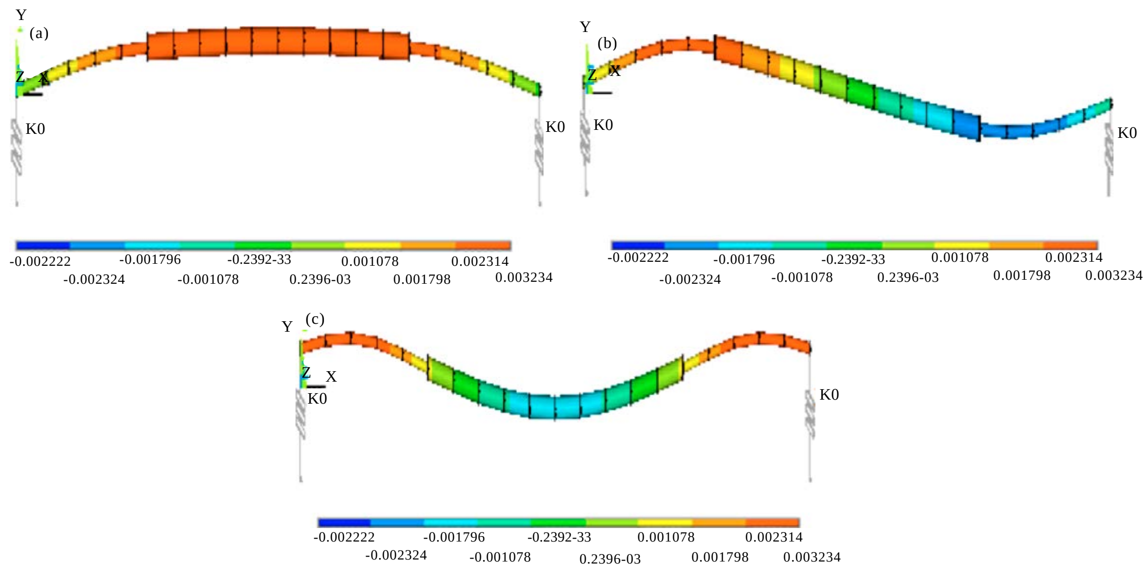

Fig. 6(a-c): (a) 1st, (b) 2nd and (c) 3rd mode shapes for flexible support pipe conveying oil using ANSYS-14 software

Table 4: Results of comparison between the transfer matrix method by using MATLAB-R2012 program and finite element method by using ANSYS-14 software for flexible support copper pipe conveying water

Analysis type	ω_{n1}	ω_{n1}	ω_{n1}
T.M.M	161	502	1430
F.E.M	162.11	505.42	1431.94
Error (%)	0.684	0.676	0.135

Table 5: Results of comparison between the transfer matrix method by using MATLAB-R2012 program and finite element method by using ANSYS-14 software for flexible support copper pipe conveying oil

Analysis type	ω_{n1}	ω_{n1}	ω_{n1}
T.M.M	167	517	1468
F.E.M	167.76	521.00	1469.26
Error (%)	0.453	0.767	0.085

CONCLUSION

For the (Sudden enlargement-sudden contraction) pipe conveying fluid with different density and exposed to vibration, it can be concluded:

- The natural frequencies for pipe system conveying fluid is less than the natural frequencies for pipe system without fluid
- Increasing the density at the same values of Reynolds number leads to decrease the values of the natural frequencies
- The change of fluid density effect on maximum amplitude values for the same supports type
- The results of the transfer matrix method by using (MATLB-R2012) program and finite element method by using (ANSYS-14) software show a good agreement with percentage for a maximum difference of (0.767%) and a minimum difference of (0.032%)

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