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A Parametric Modeling Method for Hyperboloidal-type Normal Circular-arc Gear

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Abstract: Hyperboloidal-type normal circular-arc gear is a new development of circular-arc tooth profile in the gear pair with crossed axis. In this study, a parametric modeling method for hyperboloidal-type normal circular-arc gear is developed. A series of mathematical representation for this gear are presented and a surface element, including the datum surface and the tooth surface, is utilized as the basic unit of gear modeling and the surface boundaries are solved by golden section search method. According to the generation theories of surfaces, the surface element is discretized to produce the data for generating the gear model. By using the programming tool VC++ 6.0 and Open Graphics Library (OpenGL), a 3D parametric modeling software package for this gear is developed. The results laying foundation for computer aided design and analysis of hyperboloidal-type normal circular-arc gear.

Key words: Parametric modeling, OpenGL, CAA, W-N gear, crossed-axis transmission

INTRODUCTION

Helical gear with circular-arc tooth profile has been applied at the parallel axis transmission and was named after the inventors, i.e., Wildhaber-Novikov gear (W-N gear) (Wildhaber, 1926; Novikov, 1956) and it is well known as the circular-arc gear in China due to circular arc tooth profile. Though its applications are limited to a certain extent by hardened involute gear, W-N gear still shows many advantages, such as high contact strength, low error sensitivity and compact structure. Many researchers took notice of these advantages and did their best to broaden the application of circular-arc tooth profile into the intersecting axis and cross axis transmissions. Kuo (2001) proposed a kind of bevel gear with circular-arc tooth profile and deduced the optimal parametric conditions of tooth surface. Aiming at W-N spiral bevel gear, Maiki and Watanabe (2005) presented a kind of machining operation on the NC machining center. Directed by the principle of moulding-surface conjugation, Chen *et al*. (2007) and Duan *et al*. (2012) proposed loxodromic-type normal circular-arc spiral bevel gear and programmed the tool path in the manufacture of tooth surface. As a novel application of circular arc tooth profile, Chen *et al*. (2006) proposed hyperboloidal-type normal circular-arc gear, in which the circular arc tooth profile is developed into the gear pair with crossed axis.

For a new type of gear transmission, its application prospect depends on the processing techniques and transmission performance. Usually, there are two ways to acquire those performances, i.e., the experimental method and the simulation method. With the rapid development of computer technology, Computer-aided Analysis (CAA) becomes an important and credible method in the design, manufacture and analysis of gear. As an essential step in these researches, three-dimensional models of gear pair should be firstly established. At present, the modeling methods of gear can be summarized into three categories: (1) Direct modeling. In many commercially available 3D softwares, the modeling function can help the users to represent any three-dimensional surface of objects as the basis for computer aided analysis. Tang and Nie (2010) applied the surface fitting function of CATIA to generate the hypoid gear model and simulated the machine process. (2) Quadratic programming on 3D software platforms. Many commercial 3D softwares provide the interface of quadratic development for the users, by which the modeling function of 3D softwares is expanded and the design efficiency can be improved because of avoiding repeated operations to similar products.

Aiming to spiral bevel gear, Yang *et al*. (2005) conducted the UG-based secondary development to reconstruct the tooth surface by extracting the data points from the numerical simulation. (3) Special software package. In order to improve the applicability of model and reduce the dependence on commercial 3D modeling softwares, many researchers committed themselves to developing some special software packages for special gears by means of computer graphics programming techniques. In order to conduct the dynamic simulation

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and the mechanics calculation of spiral bevel gear, Ji *et al*. (2009) applied VC^{++} 6.0 and OpenGL to develop out a modeling software system. Bibel *et al*. (1995) developed a procedure to generate a finite element model of spiral bevel gears in mesh and executed Finite Element Analysis on the platform of NASTRAN.

In this study, the mathematical equations of hyperboloidal-type normal circular-arc gear are presented and a surface element is extracted as the basic unit of gear modeling. The golden section search method is utilized to determine the surface boundaries and the surface element is discretized to produce the data for gear modeling. By utilizing VC^{++} 6.0 and OpenGL, a 3D parametric modeling software package for this gear is developed. The presented modeling software package lays foundation for computer aided design and analysis of hyperboloidal-type normal circular-arc gear.

MATHEMATICAL REPRESENTATION OF HYPERBOLOIDAL-TYPE NORMAL CIRCULAR-ARC GEAR

In hyperboloidal-type normal circular-arc gear pair, there are own special nomenclatures, such as the datum surface, the directrix, the gorge circle, etc. For brevity, only mathematical equations of structural surfaces, including datum surface, tooth surface and back cone surface, are presented and the parametric relationship in these equations can be referred to the literature (Chen *et al*., 2006).

Parametric relationship: As illustrated in Fig. 1, the coordinate system $\{O_1;\ X_1Y_1Z_1\}$ is attached to pinion, thereinto, O_1 is the center of gorge circle of datum surface of pinion and Z_1 is the rotational axis of pinion. The coordinate system $\{O_2;\ X_2Y_2Z_2\}$ is attached to gear, thereinto, O_2 is the center of gorge circle of datum surface of gear and Z_2 is the rotational axis of gear. There are following relationships between coordinate systems $\{O_1;\ X_1Y_1Z_1\}$ and $\{O_2;\ X_2Y_2Z_2\}$ as:

$$x_2 = E\text{-}x_1\ y_2 = \text{-}z_1\ z_2 = \text{-}y_1 \quad (1)$$

where, E is the offset distance; x_1, y_1, z_1 and x_2, y_2, z_2 are coordinate components of $\{O_1;\ X_1Y_1Z_1\}$ and $\{O_2;\ X_2Y_2Z_2\}$, respectively.

For a pair of hyperboloidal-type normal circular-arc gear, the structural parameters of gear and pinion must satisfy the conditions:

$$R_1 = \frac{n_1 m_n}{2\sin\delta},\ R_2 = \frac{n_2 m_n}{2\cos\delta},\ \ E = R_1 + R_2,\ \ \tan\delta = 3\sqrt{\frac{n_1}{n_2}} \quad (2)$$

where, R_1 and R_2 are the gorge radii of pinion and gear, m_n is normal modulus; n_1 and n_2 denote the tooth number of pinion and gear, respectively; $\boldsymbol{\delta}$ is the directional angle of the meshing line $C^{(0)}$ and is the angle between the meshing line and Z_1-axis.

Datum surface: According to the definition of hyperboloidal-type normal circular-arc gear, the datum surfaces $\Psi_p^{(1)}$, $\Psi_p^{(2)}$ of pinion and gear is a pair of one-sheeted hyperboloids which are generated by the meshing line $C^{(0)}$ rotating about the axes of pinion and gear, respectively. According to the principle of moulding surface conjugation (Chen *et al*., 2007), the conjugate relationship of tooth surfaces can also be transformed into that of directrixes of tooth surfaces in hyperboloidal-type normal circular-arc gear pair. The meshing line $C^{(0)}$, obtained just by the conjugation of

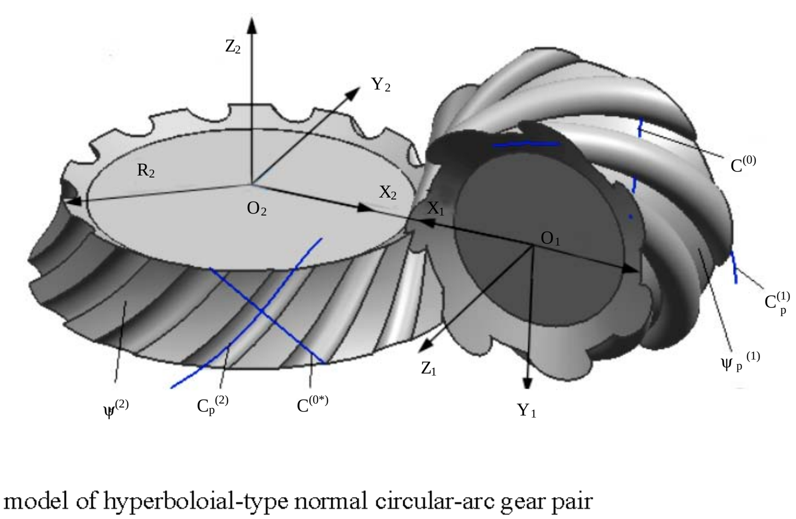

Fig. 1: Simulation model of hyperboloial-type normal circular-arc gear pair

directrixes of tooth surfaces, is a straight line and in the coordinate system $\{O_1; X_1Y_1Z_1\}$, the parametric equation of $C^{(0)}$ can be described as:

$$\begin{cases} x_1 = R_1 \\ y_1 = -u\sin\delta \\ z_1 = u\cos\delta \end{cases} \quad (3)$$

where, u is the length of $C^{(0)}$.

According to the generating procedure of one-sheeted hyperboloid, the parametric equation of datum surface $\Psi_p^{(1)}$ of pinion can be described in the coordinate system $\{O_1; X_1Y_1Z_1\}$ as:

$$\begin{cases} x_1 = R_1\cos\lambda_1 + u\sin\delta\sin\lambda_1 \\ y_1 = R_1\sin\lambda_1 - u\sin\delta\cos\lambda_1 \\ z_1 = u\cos\delta \end{cases} \quad (4)$$

where, λ_1 is the rotation angle parameter of $C^{(0)}$ around Z_1-axis.

In order to represent the datum surface of gear, the meshing line $C^{(0)}$ should be firstly transformed into the coordinate system $\{O_2; X_2Y_2Z_2\}$. Then, in the coordinate system $\{O_2; X_2Y_2Z_2\}$, the parametric equation of datum surface of gear can be described as:

$$\begin{cases} x_2 = R_2\cos\lambda_2 + u\cos\delta\sin\lambda_2 \\ y_2 = R_2\sin\lambda_2 - u\cos\delta\cos\lambda_2 \\ z_2 = u\sin\delta \end{cases} \quad (5)$$

Tooth surface: The tooth surface of hyperboloidal-type normal circular-arc gear is the tubular surface in geometry and it is generated by the sweeping of normal circular arc tooth profile, whose center is at the directrix of tooth surface, along the directrix with single degree of freedom. As shown in Fig. 1, the directrixes $C_p^{(1)}$, $C_p^{(2)}$ of pinion and gear are a pair of spirals attached on the datum surfaces $\Psi_p^{(1)}$, $\Psi_p^{(2)}$ and both of them are orthogonal with the meshing line $C^{(0)}$.

When $u = R_1\lambda_1\sin\delta$, Eq. 4 will describe the directrix $C_p^{(1)}$ of tooth surface in the pinion as follows:

$$\begin{cases} x_1 = R_1\left(\cos\lambda_1 + \lambda_1\sin^2\delta\sin\lambda_1\right) \\ y_1 = R_1\left(\sin\lambda_1 - \lambda_1\sin^2\delta\cos\lambda_1\right) \\ z_1 = R_1\lambda_1\sin\delta\cos\delta \end{cases} \quad (6)$$

Similarly, when $u = R_2\lambda_2\cos\delta$, Eq. 5 will describe the directrix $C_p^{(2)}$ of tooth surface in the gear as follows:

$$\begin{cases} x_2 = R_2\left(\cos\lambda_2 + \lambda_2\cos^2\delta\sin\lambda_2\right) \\ y_2 = R_2\left(\sin\lambda_2 - \lambda_2\cos^2\delta\cos\lambda_2\right) \\ z_2 = R_2\lambda_2\sin\delta\cos\delta \end{cases} \quad (7)$$

Note that, only when $\lambda_2 = I\lambda_1$ (I is the transmission ratio), the obtained curves just are conjugate and then can be used as the directrixes of tooth surfaces of pinion and gear.

According to the definition of tubular surface, the tooth surface of pinion can be represented in the coordinate system $\{O_1; X_1Y_1Z_1\}$ with the parametric equation:

$$\begin{cases} x_1 = R_1A_1 + rB_1 \\ y_1 = R_1C_1 - rD_1 \\ z_1 = R_1E_1 + rF_1 \end{cases} \quad (8)$$

Here:

$$A_1 = \cos\lambda_1 + \lambda_1\sin^2\delta\sin\lambda_1$$

$$B_1 = \sin\delta\sin\lambda_1\cos\theta_1 + \frac{\cos\delta\cos\lambda_1 + \lambda_1\sin^2\delta\cos\delta\sin\lambda_1}{\sqrt{\cos^2\delta + \left(\lambda_1\sin^2\delta\right)^2}}\sin\theta_1$$

$$C_1 = \sin\lambda_1 - \lambda_1\sin^2\delta\cos\lambda_1$$

Here:

$$D_1 = \sin\delta\cos\lambda_1\cos\theta_1 - \frac{\cos\delta\sin\lambda_1 - \lambda_1\sin^2\delta\cos\delta\cos\lambda_1}{\sqrt{\cos^2\delta + \left(\lambda_1\sin^2\delta\right)^2}}\sin\theta_1$$

$$E_1 = \lambda_1\sin\delta\cos\delta$$

$$F_1 = \cos\delta\cos\theta_1 - \frac{\lambda_1\sin^3\delta}{\sqrt{\cos^2\delta + \left(\lambda_1\sin^2\delta\right)^2}}\sin\theta_1$$

where, r is the radius of normal tooth profile, θ_1 is the polar angle of tooth profile in the normal plane of the directrix.

Similarly, the tooth surface of gear can also be described in the coordinate system $\{O_2; X_2Y_2Z_2\}$ as:

$$\begin{cases} x_2 = R_2A_2 + rB_2 \\ y_2 = R_2C_2 - rD_2 \\ z_2 = R_2E_2 + rF_2 \end{cases} \quad (9)$$

Here:

$$A_2 = \cos\lambda_2 + \lambda_2\cos^2\delta\sin\lambda_2$$

$$B_2 = \cos\delta\sin\lambda_2\cos\theta_2 + \frac{\sin\delta\cos\lambda_2 + \lambda_2\cos^2\delta\sin\delta\sin\lambda_2}{\sqrt{\sin^2\delta + \left(\lambda_2\cos^2\delta\right)^2}}\sin\theta_2$$

$$C_2 = \sin\lambda_2 - \lambda_2\cos^2\delta\cos\lambda_2$$

$$D_2 = \cos\delta\cos\lambda_2\cos\theta_2 - \frac{\sin\delta\sin\lambda_2 - \lambda_2\cos^2\delta\sin\delta\cos\lambda_2}{\sqrt{\sin^2\delta + \left(\lambda_2\cos^2\delta\right)^2}}\sin\theta_2$$

$$E_2 = \lambda_2\sin\delta\cos\delta$$

$$F_2 = \sin\delta\cos\theta_2 - \frac{\lambda_2\cos^3\delta}{\sqrt{\sin^2\delta + \left(\lambda_2\cos^2\delta\right)^2}}\sin\theta_2$$

Front and back cones: In hyperboloidal-type normal circular-arc gear, there are a front cone at the small end and a back cone at the big end. Because the datum surface is not a cone surface, the cone angles of front and back cones can be chosen freely by the requirement of processing technology and usually they are same. For brevity, here only front and back cones of gear are described. Once the tooth width W is determined, the relationship between front and back cones can be deduced. In the coordinate sytem $\{O_2;\ X_2Y_2Z_2\}$, the parametric equation of front and back cones of gear can be uniformly described as:

$$\begin{cases} x_2 = u_b \sin\delta_b \cos\phi \\ y_2 = u_b \sin\delta_b \sin\phi \\ z_2 = u_b \cos\delta_b + z_0 - w/\sin\delta_b \end{cases} \quad (10)$$

where, u_b is the length of generatrix of cone, δ_b is the cone angle, ϕ is the rotation angle of generatrix about Z_2-axis; z_0 is the Z_2-coordinate component of the vertex of front cone. When w = 0, Eq. 10 represents the front cone and when w = W, Eq. 10 represents the back cone.

DETERMINATION OF BOUNDARIES AND DISCRETIZATION OF SURFACE ELEMENT

In 3D parametric modeling, a basic procedure is: (1) Acquiring the discrete data defining the structural surfaces, (2) Fitting those discrete points into a 3D model. In this section, a surface element, including a single tooth surface and a part of datum surface, is extracted from the gear model. As illustrated in Fig. 2, taking the gear as an example, the surface element is surrounded by the lines $l_1, l_2, \ldots, l_7$, thereinto, the zone enclosed by the lines l_1, l_2, l_3 and l_4 is the single tooth surface and the zone enclosed by the lines l_4, l_5, l_6 and l_7 is a part of datum surface (i.e., addendum face). The line l_1 is the upper boundary of tooth surface (i.e., the intersecting line of the tooth surface and the front cone); the line l_2 is the lower boundary of tooth surface (i.e., the intersecting line of the tooth surface and the back cone); the lines l_3 and l_4 denote the right and left boundaries of tooth surface (i.e., the intersecting lines of the tooth surface and the datum surface). Similarly, the lines l_4, l_5, l_6, l_7 represent the boundaries of datum face in this surface element. As a precondition of 3D modeling, the boundary of surface element should be firstly determined. Additionally, because the front and back cone are simple, only the discretization of datum surface and tooth surface is presented. In order to conveniently represent the calculating procedure of discrete points at different positions, the tooth surface is divided into three areas by dashed lines as I, II, III and the datum surface as IV, V, VI.

For the datum surface in the surface element, the boundaries l_6, l_7 can be obtained by the simultaneous equations of datum surface, front and back cones and the boundaries l_4, l_5 can be acquired by the right and left boundaries of adjoining tooth surfaces, respectively. Hence, only the boundaries of tooth surface are discussed.

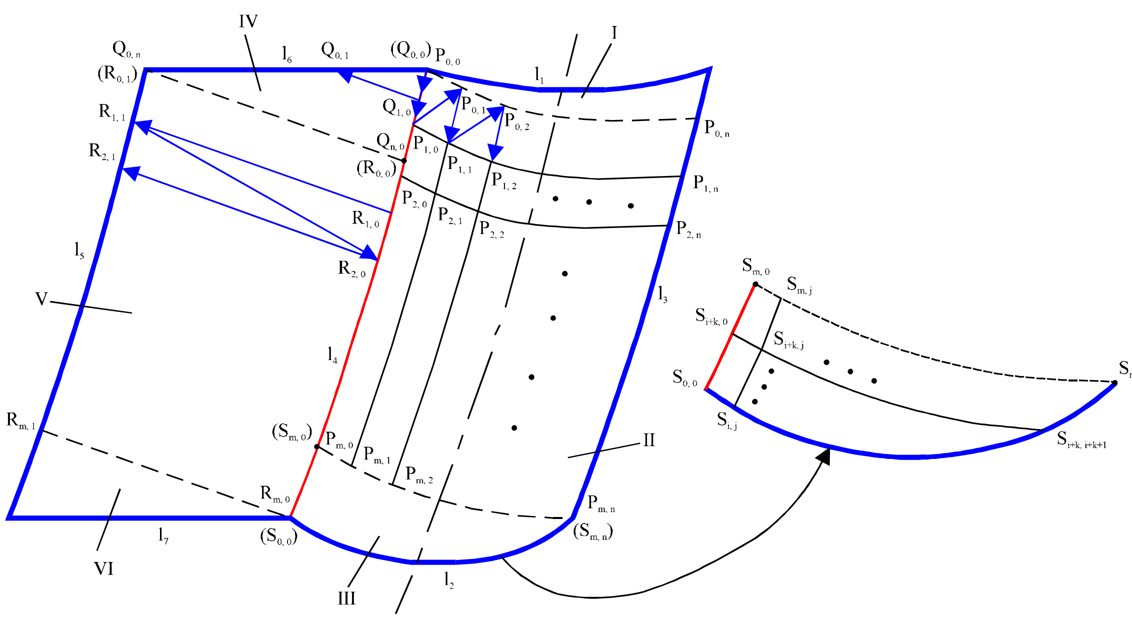

Fig. 2: Surface element of 3D modeling of gear

Left and right boundaries of tooth surface: Usually, the concave tooth is used for the gear and the convex tooth for the pinion which means that the tooth of gear is inside the datum surface of gear and the tooth of pinion is outside the datum surface of pinion. Because the spiral directrix of tooth surface lies on the datum surface and the meshing line is vertical to this directrix, the normal tooth profile is just a semi-circle.

For the gear, the arc angle of tooth profile ranges from 180-360°, thus the right and left boundaries l_3, l_4 of gear can be described as:

$$\begin{cases} x_2 = R_2\left(\cos\lambda_2 + \lambda_2\cos^2\delta\sin\lambda_2\right) \pm r\cos\delta\sin\lambda_2 \\ y_2 = R_2\left(\sin\lambda_2 - \lambda_2\cos^2\delta\cos\lambda_2\right) \mp r\cos\delta\cos\lambda_2 \\ z_2 = R_2\lambda_2\sin\delta\cos\delta \pm r\sin\delta \end{cases} \tag{11}$$

In the signs "±" and "∓", the upper signs and the lower signs are used for the boundaries l_3, l_4, respectively.

Upper and lower boundaries of tooth surface: It is difficult to directly acquire the upper and lower boundaries by solving the simultaneous equations of tooth surface, front and back cones, thus the method of numerical solution is adopted here. Because the upper and lower boundaries l_1, l_2 are similar, only the solution of l_2 is presented.

When λ_2 in Eq. 9 is set as a constant, Eq. 9 will represent the normal circle-arc tooth profile of tooth surface. So, the boundary l_2 can be regarded as a set of intersecting points between a series of normal sections of tooth surface and the back cone.

According to Eq. 10, the scalar expression of back cone can be described as:

$$f(x_2, y_2, z_2) = \left(\frac{z_2 - z_0 + w/\sin\delta_b}{\cot\delta_b}\right)^2 - x_2^2 - y_2^2 \tag{12}$$

For the area III of tooth surface, its normal section is not a whole tooth profile and the points at the boundary l_4 can be regarded as the starting points of normal section at this position and the points at the boundary l_2 can be regarded as the end points of normal section. When the points are searched along the normal section by Eq. 12, $f(x_2, y_2, z_2)<0$ means that the point is inside the back cone and $f(x_2, y_2, z_2)>0$ means that the point is outside the back cone. It can be deduced that a single peak interval of $|f(x_2, y_2, z_2)|$ exists at the vicinity of the boundary l_2 which indicates that one-dimensional search can be used to obtain the discrete points of l_2.

The determination of the boundary l_2 can be divided into following steps as:

- **Setting the initial search interval:** As mentioned previously, the polar angle θ_2 of whole tooth profile ranges from 180-360° and the range of λ_2 depends on the tooth width. Once λ_2 is determined, the position of normal section of tooth surface can also be deduced and Eq. 9 can be utilized to search the points along the normal section beginning with θ_2 = 180 by the increment $\Delta\theta_2$ = 1. Farther, the coordinates of obtained points are substituted into Eq. 12 and the value of θ_2 is recorded when $f(x_2, y_2, z_2)$ turns to negative from positive. Thus, $[\theta_2\text{-}1, \theta_2]$ can be used as the initial search interval of $f(\theta_2) = |f(x_2, y_2, z_2)|$. Similarly, all of initial search intervals at different positions can be obtained when λ_2 is changed continuously
- **Determining the polar angle of boundary point:** As the precondition that a point is determined, its polar angle in the coordinate system of tooth profile should be obtained. As illustrated in Fig. 3, the golden section search is used to obtain the polar angle of boundary point and its procedure follows:

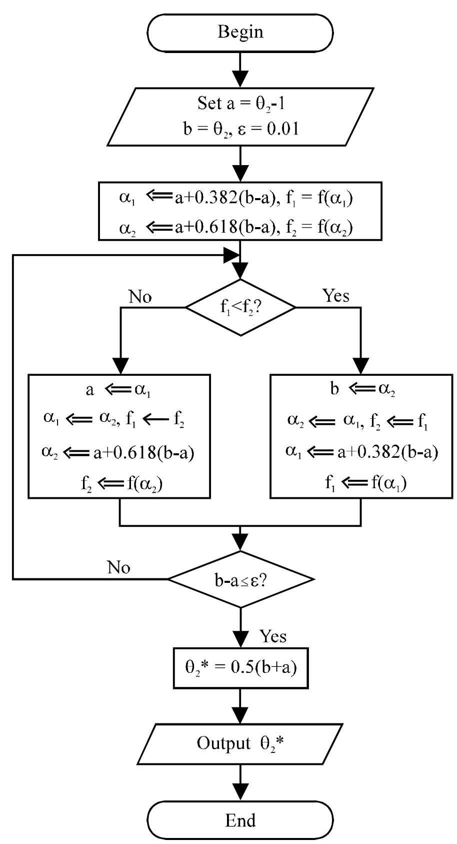

Fig. 3: Flowchart of golden section search for the polar angle

- Two internal points of division are selected in an interval [a, b] obtained by rewriting the interval [θ_2-1, θ_2] and then f (θ_2) can be calculated by:

$$\alpha_1 = a+0.382(b-a),\ f_1 = f(\alpha_1) \tag{13}$$

$$\alpha_2 = a+0.618(b-a),\ f_2 = f(\alpha_2) \tag{14}$$

- Comparing f_1 and f_2

If $f_1<f_2$, α_1 is used for the first trial point in a new interval [a, α_2], i.e.:

$$b = \alpha_2,\ \alpha_2 = \alpha_1,\ f_2 = f_1 \tag{15}$$

Another trial point can be calculated by:

$$\alpha_1 = a+0.382(b-a),\ f_1 = f(\alpha_1) \tag{16}$$

If $f_1 \geq f_2$, α_2 is used for the first trial point in a new interval [α_1, b], i.e.:

$$a = \alpha_1,\ \alpha_1 = \alpha_2,\ f_1 = f_2 \tag{17}$$

Another trial point can be calculated by:

$$\alpha_2 = a+0.618(b-a),\ f_2 = f(\alpha_2) \tag{18}$$

- If b-a$\leq\varepsilon$ (the convergence accuracy ε is 0.01), proceeding to next step, otherwise returning to step 2 to execute a new iteration for shortening the search interval
- In order to guarantee the precision, the optimal polar angle of boundary point is determined as:

$$\theta_2^* = \frac{a+b}{2} \tag{19}$$

- **Determining the boundary point:** With the position change of normal section, a series of parameter combination ($\lambda_{2(i)}$, $\theta_{2(i)}$) (i = 0, 1, 2...m), corresponding to the boundary points, can be gained and further, the boundary l_2 can be obtained after these parameters ($\lambda_{2(i)}$, $\theta_{2(i)}$) are substituted into the equation of tooth surface

Discretization of tooth surface: For the area II of tooth surface, the discrete points $P_{(i,j)}$ (i = 0, 1, 2...m; j = 0, 1, 2...n) are selected along the normal section and its orthogonal direction. Because the equation of tooth surface contains the variables λ_2, θ_2, any point on the tooth surface corresponds to a parameter combination (λ_2, θ_2). Here, the parameter combination ($\lambda_{2(i,j)}$, $\theta_{2(i,j)}$) is described as:

$$\begin{aligned} \lambda_{2(i,j)} &= \lambda_{2(0,j)} + \frac{\lambda_{2(m,j)}-\lambda_{2(0,j)}}{m}\cdot i \\ \theta_{2(i,j)} &= \theta_{2(i,0)} + \frac{\theta_{2(i,n)}-\theta_{2(i,0)}}{n}\cdot j \end{aligned} \tag{20}$$

In order to fit the discrete points into a surface, the triangular patch mesh generation method is used in the computer graphic programming. Considering this method, the discrete points are arranged in the order of ($P_{(0,0)}$, $P_{(1,0)}$, $P_{(0,1)}$), ($P_{(0,1)}$, $P_{(1,1)}$, $P_{(0,2)}$),..., ($P_{(m,n-1)}$, $P_{(m-1,n)}$, $P_{(m,n)}$), as illustrated in Fig. 2.

The normal section at the area III of tooth surface is not a whole tooth profile (i.e., semi-circle). Though the discrete points $S_{i,j}$ (i = 0, 1, 2,..., m; j = 0, 1, 2...n; n = m+1) are also determined along the normal section and its orthogonal direction, the polar angles of end points of normal sections at different positions are different and have been obtained in the determination of the boundary l_2. Thus, the description of the parameter combination ($\lambda_{2(i,j)}$, $\theta_{2(i,j)}$) of the discrete points on the area III of tooth surface is different from Eq. 20 and can be described as:

$$\begin{aligned} \lambda_{2(i,j)} &= \lambda_{2(0)} - \frac{\lambda_{2(0)}-\lambda_{2(m)}}{m}\cdot i \\ \theta_{2(i,j)} &= \theta_{2(i,0)} + \frac{\theta_{2(i)}-\theta_{2(i,0)}}{i+1}\cdot j \end{aligned} \tag{21}$$

where, $\lambda_{2(0)}$, $\lambda_{2(m)}$, $\theta_{2(i)}$ are the components of the parameter combination s($\lambda_{2(i)}$, $\theta_{2(i)}$) which have been obtained by golden section search.

Discretization of datum surface: The datum surface is generated by the meshing line rotating the axis of gear, thus the discrete points of datum surface can be selected along the direction of meshing line and this surface can be fitted by only using the discrete points at the boundaries. Because the areas IV and VI of datum surface are similar, the discretizations of the areas IV and V are represented.

For the area IV of datum surface, the discrete points $Q_{0,j}$ (j = 0, 1, 2,..., n) at the boundary l_6 can be acquired by changing θ in Eq. 5 at u = 0; the discrete points $Q_{i,0}$ (i = 0, 1, 2,..., n) can be gained at the boundary l_4 by changing λ_2 in Eq. 11. As illustrated in Fig. 2, the direction in fitting the discrete points is planned as from $Q_{0,0}$ to $Q_{0,1}$, $Q_{1,0}$, $Q_{0,2}$... $Q_{0,m}$, $Q_{m,0}$.

For the area V of datum surface, the discrete points $R_{i,0}$ (i = 0, 1, 2.....m) can also be obtained from the boundary l_4; the solution of the discrete points $R_{i,1}$ (i = 0, 1, 2,..., m) includes two steps: (1) The equation of l_3 is utilized to acquire some points which correspond the part from $R_{0,1}$ to $R_{m,1}$ and (2) The obtained-above points are rotated with the indexing angle $2\pi/n_2$ along the axis of gear. In the fitting process, the discrete points are programmed as the order from $R_{0,0}$ to $R_{0,1}$,..., $R_{m,0}$, $R_{m,1}$.

THREE-DIMENSIONAL PARAMETRIC MODELING METHOD AND APPLICATION

In order to obtain the models of pinion and gear, the programming tool VC^{++} 6.0 and the professional graphics program interface OpenGL are used in this study and a software package for the parametric modeling of hyperboloidal-type normal circular-arc gear is developed out. In this software package, VC^{++} 6.0 is applied to design the interface of parameter input, construct the framework of the software package and conduct related calculations. Some functions in OpenGL, such as the modeling and the transformation, are called to generate the simulation model by using the data from the calculations.

Parametric modeling software package: In the design of modeling software system, the flow is planned as follows:

- **Creating the interface of parameter input:** In order to realize the parametric modeling, a parameter input interface, including the transmission ratio, the modulus and the tooth number and so on, is provided
- **Calculating the discrete points of surface element:** The discrete points of gear model are calculated by the presented methods
- **Fitting the surface element:** By using the discrete point data, the surface element is generated with the triangular patch command glBegin(GL_TRIANGLE_STRIP) in OpenGL
- **Generating the simulation model by the array of surface element:** The array is made about the axis of gear by the command glRotatef ($2\pi/n_2$, 0.0, 0.0, 1.0) in OpenGL and the tooth number determines the time of array
- **Real-time rendering of the model:** Some real-time rendering operations, such as the lighting and antialiasing, are adopted to make the model have a real three-dimensional quality

Application example: Figure 4 illustrates the parameter input window after the software package run. When the parameters $m_n = 5$, $Z_1 = 7$, i = 1:3 and $\alpha = 30°$ were inputted, a pair of 3D models of pinion and gear of hyperboloidal-type normal circular-arc gear in mesh are generated, as shown in Fig. 5a, b.

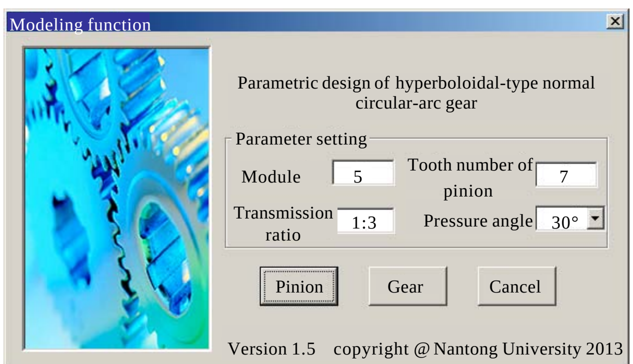

Fig. 4: Input data for hyperboloidal-type normal circular-arc gear

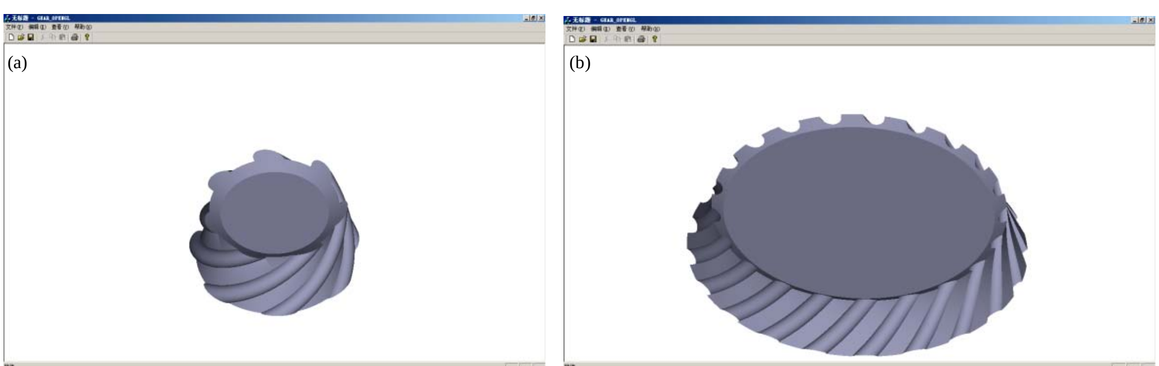

Fig. 5(a-b): Modeling results of the sofware package, (a) Pinion and (b) Gear

CONCLUSION

Hyperboloidal-type normal circular-arc gear is a new type of gear, in which the circular-arc tooth profile is extended to use in the gear pair with non-parallel and non-intersecting axis. In this study, based on the mathematical representation of structural surfaces in hyperboloidal-type normal circular-arc gear, a surface element was utilized as the basic unit of gear modeling and the surface boundaries were determined by using golden section search method. The surface element was discretized to produce the data for generating the gear model. By using VC^{++} 6.0 and OpenGL, a 3D parametric modeling software package for this gear was developed out. The results lay foundation for computer aided design and analysis of hyperboloidal-type normal circular-arc gear.

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