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Intuitionistic Trapezoidal Fuzzy Multi-attribute Decision Making Method Based on Grey Relational Analysis

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Abstract: The study proposed a multiple attribute decision making method based on grey relational analysis, for the multiple attribute decision making problems with the evaluation information given in the form of intuition trapezoidal fuzzy numbers. First, give the definition of intuitionistic fuzzy numbers and distance formula and then obtained the grey relational coefficient about intuition trapezoidal fuzzy numbers based on grey relational analysis, to calculate the relational grade of each options by using relational coefficient and sort all the options by the size of this value in order to get the best option. Finally, it verified the feasibility and effectiveness of the proposed method through a numerical example.

Key words: Grey relational analysis, intuitionistic trapezoidal fuzzy numbers, multi-attribute decision making, hamming distance, grey relational coefficient

INTRODUCTION

Since the fuzzy set theory proposed by Zadeh (1965) and applied to multi-attribute decision making, the research has been developing rapidly. Atanassov (1986) proposed the concept of intuitionistic fuzzy sets based on fuzzy set theory, its characteristics mainly considering the three aspects of the information in membership, non-membership and hesitation. It better characterizes the three states information of the judgment on affirming, denying and hesitation extent and has stronger performance ability in dealing with uncertain information, thus causing wide attention in the academic field of technology and engineering. In recent years, the research of multi-attribute decision making theory and method based on intuitionistic fuzzy set has become a hot spot; the Intuitionistic Trapezoidal Fuzzy Numbers (ITFN) is also a special form of intuition fuzzy sets just as Interval Intuitionistic Fuzzy Sets (IVIFS) and Triangular Intuitionistic Fuzzy Numbers (TIFN). Currently, the research about ITFN focuses on the field of multi-attribute decision making and the research achievements is relatively small, to give more explore on this can describe thus decision-making problems more objectively. Wan and Dong (2010) defines the expectations, expected scores and the size comparison methods of intuitionistic trapezoidal fuzzy numbers and established the multi-attribute group decision making model based ITFN. Jianqiang and Zhong (2009) give studied on multiple criteria decision making problems with incomplete information and the guideline value is ITFN and proposed the corresponding decision-making methods. Wang and Nie (2012) proposed an improved ITFN arithmetic, defined several types of intuitionistic fuzzy set operators and used in group decision making and proposed the corresponding multi-criteria group decision making method. Wan and Zhang (2012) defined the new simple algorithms of intuitionistic trapezoidal fuzzy numbers, use the weighted possibility mean value of the membership and non-membership functions to give the new sorting method of ITFN, proposed ITFN matrix game problem and give the solution method. In view of the above analysis, this study give the intuitionistic fuzzy multiple attribute decision making problems making steps based on the basic idea of grey relational analysis, proposed the decision making method based on grey relational analysis and give numerical examples by the use of this method.

BASIC THEORIES

Intuitionistic trapezoidal fuzzy numbers

Definition 1: Assume the intuition trapezoidal fuzzy number on real number set is:

$$\tilde{a} \le ([a, b, c, d]; \mu_{\tilde{a}}), ([a', b, c, d']; \nu_{\tilde{a}})$$

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its membership function is:

$$\mu_{\tilde{a}}(x)=\begin{cases}\frac{x-a}{b-a}\mu_{\tilde{a}} & a\le x<b\\ \mu_{\tilde{a}} & b\le x\le c\\ \frac{d-x}{d-c}\mu_{\tilde{a}} & c<x\le d\\ 0 & \text{other}\end{cases}\quad (1)$$

The non-membership function is:

$$\nu_{\tilde{a}}(x)=\begin{cases}\frac{b-x+(x-a')\nu_{\tilde{a}}}{b-a'} & a'\le x<b\\ \nu_{\tilde{a}} & b\le x\le c\\ \frac{x-c+(d'-x)\nu_{\tilde{a}}}{d'-c} & c<x\le d'\\ 0 & \text{other}\end{cases}\quad (2)$$

Among them, $0\le\mu_{\tilde{a}}\le 1$, $0\le\nu_{\tilde{a}}\le 1$, $\mu_{\tilde{a}}+\nu_{\tilde{a}}\le 1$; when b = c, intuition trapezoidal fuzzy number degenerate to instincts triangular fuzzy number, when a = a' and d = d', these intuitionistic trapezoidal fuzzy numbers can be written as:

$$\tilde{a} = ([a, b, c, d]; \mu_{\tilde{a}}, \nu_{\tilde{a}})$$

In this study, its all refers to this type of fuzzy numbers. The $\mu_{\tilde{a}}$ and $\nu_{\tilde{a}}$ represent the maximum membership degree and the minimum non-membership degree, $\pi_{\tilde{a}}(x) = 1-\mu_{\tilde{a}}(x)-\nu_{\tilde{a}}(x)$ is the hesitation function of ã, then the smaller the value, the fuzzy numbers were more determined.

Definition 2: Assume $\tilde{a}_1 = ([a_1, b_1, c_1, d_1]; \mu_{\tilde{a}1}, \nu_{\tilde{a}1})$ and $\tilde{a}_2 = ([a_2, b_2, c_2, d_2]; \mu_{\tilde{a}2}, \nu_{\tilde{a}2})$ are two intuition trapezoidal fuzzy numbers, then (Wan *et al*., 2012):

$$\tilde{a}_1+\tilde{a}_2 = ([a_1+a_2, b_1+b_2, c_1+c_2, d_1+d]2; \mu_{\tilde{a}1} \wedge \mu_{\tilde{a}2}, \nu_{\tilde{a}1} \vee \nu_{\tilde{a}2})$$

the symbols ∧, ∨ represent the calculation of the comparison between the two, to take a smaller or a larger value:

$$\tilde{a}_1\tilde{a}_2 = ([a_1a_2, b_1b_2, c_1c_2, d_1d_2]; \mu_{\tilde{a}_1} \wedge \mu_{\tilde{a}_2}, \nu_{\tilde{a}_1} \vee \nu_{\tilde{a}_2})$$

If $\lambda\ge 0$:

$$\lambda\tilde{a}_1 = ([\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1]; \mu_{\tilde{a}_1}, \nu_{\tilde{a}_1})$$

If $\lambda\ge 0$:

$$\lambda\tilde{a}_1 = ([\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1]; \mu_{\tilde{a}_1}, \nu_{\tilde{a}_1})$$

$$\tilde{a}_1^{\lambda} = ([a_1^{\lambda}, b_1^{\lambda}, c_1^{\lambda}, d_1^{\lambda}]; \mu_{\tilde{a}_1}, \nu_{\tilde{a}_1}),\ \lambda\ge 0$$

Definition 3: Assume $\tilde{a}_1 = ([a_1, b_1, c_1, d_1]; \mu_{\tilde{a}_1}, \nu_{\tilde{a}_1})$ and $\tilde{a}_2 = ([a_2, b_2, c_2, d_2]; \mu_{\tilde{a}_2}, \nu_{\tilde{a}_2})$ are two intuition trapezoidal fuzzy numbers, the Hamming distance of the fuzzy numbers $\tilde{a}_1$ and $\tilde{a}_2$ is:

$$\begin{aligned}d(\tilde{a}_1,\tilde{a}_2)=\frac{1}{8}(&|(1+\mu_{\tilde{a}_1}-\nu_{\tilde{a}_1})a_1-(1+\mu_{\tilde{a}_2}-\nu_{\tilde{a}_2})a_2|+\\ &|(1+\mu_{\tilde{a}_1}-\nu_{\tilde{a}_1})b_1-(1+\mu_{\tilde{a}_2}-\nu_{\tilde{a}_2})b_2|+\\ &|(1+\mu_{\tilde{a}_1}-\nu_{\tilde{a}_1})c_1-(1+\mu_{\tilde{a}_2}-\nu_{\tilde{a}_2})c_2|+\\ &|(1+\mu_{\tilde{a}_1}-\nu_{\tilde{a}_1})d_1-(1+\mu_{\tilde{a}_2}-\nu_{\tilde{a}_2})d_2|)\end{aligned}\quad (3)$$

Grey relational analysis: Grey system theory (Liu *et al*., 2008) is a system engineering disciplines based on the mathematical theory and proposed and created at first by Professor Deng Julong of China Huazhong University of Science and Technology in 1980s. Among them, the grey relational analysis serves as an important method of grey system theory have been widely used by many experts and scholars. The basic idea of the method is to use the similarity of the geometric relationship and the curve geometric shape between the program sequences to determine the degree of association between the programs. If the change trends of the two sub-programs (factor) have consistency, it considered to a greater degree of association between the two; contrary, the degree of association is smaller.

DECISION MAKING METHOD BASED ON GREY RELATIONAL ANALYSIS

Decision making problem description: For a fuzzy multi-attribute decision-making problems, assume the program set is:

$$A = \{A_1, A_2,\ldots, A_m\}$$

the attribute set is:

$$C = \{C_1, C_2,\ldots, C_n\}$$

the attribute weights vector:

$$w = \{w_1, w_2,\ldots, w_n\} \text{ and } w_j\ge 0,\ \sum_{j=1}^{n} w_j = 1$$

The evaluation value of the program A_i under the attribute C_j can be represented by the intuitionistic trapezoidal fuzzy number:

$$\tilde{a}_{ij} = ([h_{1j}(a_i), h_{2j}(a_i), h_{3j}(a_i), h_{4j}(a_i)]; \mu_j(a_i), \nu_j(a_i))$$

where, $\mu_j(a)$ and $\nu_j(a_i)$ represent the satisfaction or dissatisfaction $\tilde{a}_{ij}$ degree of the evaluation value of the program A_i under the property C_j and $0\leq\mu_j(a_i)\leq1$, $0\leq\nu_j(a_i)\leq1$, $\mu_j(a_i)+\nu_j(a_i)\leq1$.

Algorithm steps: This study discusses the grey relational analysis based on ITFN multi-attribute decision problem; the specific steps are as follows:

Step 1: Given the intuitionistic trapezoidal fuzzy number evaluation value under each attribute of the programs, construct its decision matrix:

$$\tilde{A} = (\tilde{a}_{ij})_{m\times n}$$

Step 2: Standardized the decision matrix, in order to avoid the affect on results due to the different dimensions between the attributes, we can use the formula of the standardized fuzzy decision matrix to transform the trapezoidal fuzzy numbers decision matrix $\tilde{A}$ into standardized decision matrix:

$$\tilde{R} = (\tilde{r}_{ij})_{m\times n}$$

Where in:

$$\tilde{r}_{ij} = ([r_{1j}(a_i), r_{2j}(a_i), r_{3j}(a_i), r_{4j}(a_i)]; \mu_j(a_i), \nu_j(a_i))$$

In the multi-attribute decision-making problems, the common attribute types have efficiency and cost type, the value of efficiency attribute is the larger the better and the value of cost attribute is the smaller the better. The method to build the standardization intuitionistic fuzzy number decision matrix is as follows:

For efficiency attribute is:

$$r_{lj}(a_i) = \frac{h_{lj}(a_i) - \min_j(h_{1j}(a_i))}{\max_j(h_{4j}(a_i)) - \min_j(h_{1j}(a_i))} \quad (4)$$

For cost attribute is:

$$r_{lj}(a_i) = \frac{\max_j(h_{4j}(a_i)) - h_{lj}(a_i)}{\max_j(h_{4j}(a_i)) - \min_j(h_{1j}(a_i))} \quad (5)$$

Where:
i = 1, 2,..., m
j = 1, 2,..., n
l = 1, 2,..., 4

Step 3: Determine the reference sequence. The rule is to constitute the elements in the reference sequence by the optimal solution of intuitionistic trapezoidal fuzzy number standardized attribute value of the programs. Among them, the optimal solution under the attribute C_j relative to the membership degree of the maximum fuzzy number is 1; the non-membership degree is 0, that is:

$$\tilde{U}_j^0 = ([\max(r_{1j}(a_i)), \max(r_{2j}(a_i)), \max(r_{3j}(a_i)) \max(r_{4j}(a_i))]; 1, 0)$$

Where:
i = 1,2,..., m
j = 1, 2,..., n

The reference sequence is $\tilde{U}^0 = (\tilde{U}_1^0, \tilde{U}_2^0, \cdots, \tilde{U}_n^0)$ (Zhang *et al*., 2010).

Step 4: According to definition 3, calculated the Hamming distance Δ_{ij} between the corresponding elements of reference sequence and standardized attribute value sequence:

$$\Delta_{ij} = d(\tilde{U}_j^0, \tilde{r}_{ij}), \; i = 1, 2,..., m; \; j = 1, 2,..., n \quad (6)$$

Step 5: Calculate the maximum difference Δ_{max} and minimum difference Δ_{min}

$$\Delta_{max} = \max_{i,j} \Delta_{ij}, \quad \Delta_{min} = \min_{i,j} \Delta_{ij} \quad (7)$$

Step 6: Calculate the grey relational coefficient ε_{ij} between the attribute value sequence and reference sequence of the programs and then get the grey relational coefficient matrix $(\varepsilon_{ij})_{m\times n}$:

$$\varepsilon_{ij} = \frac{\Delta_{min} + \lambda\Delta_{max}}{\Delta_{ij} + \lambda\Delta_{max}} \quad (8)$$

where, λ is distinguish coefficient, $\lambda\in[0, 1]$; when λ is smaller, the distinguish ability will be greater and generally assume $\lambda = 0.5$.

Step 7: Calculate the correlation degree under different decision-making status between the attribute value sequence and reference sequence of the programs r_i:

$$r_i = \sum_{i=1}^{m} \varepsilon_{ij} \bullet w_j \quad (9)$$

In the Eq. 9, taking into account the association with intuitionistic trapezoidal fuzzy numbers variable weight multi-attribute decision theory, weight vector w_j can appear in different decision status and give different values, it can fully reflect the flexibility in decision-making process. Accordingly, use the related variable weight multi-attribute decision making method, to calculate the weight of each attribute weights w_j under different decision status by solving the single-objective optimization model and get their correlation degrees.

Step 8: Sort the programs according to the size of the correlation degree values under different decision making status, the larger the value, the program is more superior, that resulting in the optimal program

NUMERICAL EXAMPLE

In order to enhance the market competitiveness of a company, select three peer enterprises $\{A_1, A_2, A_3\}$ and choose a best enterprise to reach Cooperation Union. The company invited experts to evaluate three attributes as the production capacity (C_1), research and development capabilities (C_2) and cash flow capacity (C_3) and all attributes are efficiency attributes (Gao *et al*., 2011). The experts have given the intuitionistic trapezoidal fuzzy numbers evaluation value of the program as shown in Table 1, then try to determine the best cooperative enterprise.

- According to Eq. 4-5, give standardized processing on Table 1 and the results shown in Table 2
- Determine the reference sequence $\tilde{U}^0$:

$$\tilde{U}^0 = (([0.286, 0.571, 0.714, 1]; 1, 0)\ ([0.286, 0.429, 0.714, 1]; 1, 0)\ ([0.333, 0.500, 0.833, 1]; 1, 0))$$

- According to Eq. 6-7, calculate the hamming distance Δ_{ij}, the maximum difference Δ_{max} and the minimum difference Δ_{min}, the results shown in Table 3

It can be seen from Table 3 that the minimum difference Δ_{min} = 0.1665 and the maximum difference Δ_{max} = 0.4499.

- Take the above values into Eq. 8, calculate the grey relational coefficient ε_{ij} of the programs and take λ = 0.5:

$$(\varepsilon_{ij})_{3\times3} = \begin{bmatrix} 0.7614 & 0.7164 & 0.9216 \\ 0.5801 & 0.7856 & 1.0000 \\ 0.6831 & 0.7717 & 0.6415 \end{bmatrix}$$

- Use the data in example analysis of literature, by solving the single-objective optimization model, considering three objective functions is fair competition and take appropriate balance coefficient, then get each attribute weights under different decision making status, as shown in Table 4
- Calculate the correlation degree under different decision-making status of the programs according to Eq. 9 and the results are shown in Table 5

Thereby get the sorting results of the programs: $A_1 \succ A_2 \succ A_3$, the best cooperative enterprise is A_1. In this study, the results are basically the same with the literature (Gao *et al*., 2011), it proved that this method is feasible and effective and compared to the existing literature of the

Table 1: Intuitionistic trapezoidal fuzzy number decision making matrix

Program	C_1	C_2	C_3
A_1	([3, 5, 6, 8]; 0.5, 0.4)	([2, 3, 4, 5]; 0.8, 0.2)	([2, 4, 5, 7]; 0.7, 0.1)
A_2	([1, 2, 3, 4]; 0.8, 0.0)	([3, 4, 6, 8]; 0.5, 0.4)	([3, 4, 6, 7]; 0.7, 0.2)
A_3	([2, 3, 4, 6]; 0.7, 0.2)	([1, 3, 5, 8]; 0.6, 0.2)	([1, 2, 4, 6]; 0.7, 0.2)

Table 2: Intuitionistic trapezoidal fuzzy number decision making matrix after standardized processing

Program	C_1	C_2	C_3
A^1	([0.286, 0.571, 0.714, 1.000]; 0.5, 0.4)	([0.143, 0.286, 0.429, 0.571]; 0.8, 0.2)	([0.167. 0.500. 0.667. 1.000]; 0.7, 0.1)
A^2	([0.000, 0.143, 0.286, 0.426]; 0.8, 0.0)	([0.286, 0.429, 0.714, 1.000]; 0.5, 0.4)	([0.333, 0.500, 0.833, 1.000]; 0.7, 0.2)
A^3	([0.143, 0.286, 0.429, 0.714]; 0.7, 0.2)	([0.000, 0.286, 0.571, 1.000]; 0.6, 0.2)	([0.000, 0.167, 0.500, 0.833]; 0.7, 0.2)

Table 3: Hamming distance between the corresponding elements of reference sequence and attribute value sequence

Parameters	C_1	C_2	C_3	$\underset{j}{\text{Max}}\,\Delta_{ij}$	$\underset{j}{\text{Max}}\,\Delta_{ij}$
Δ_{1j}	0.2892	0.3215	0.1998	0.1998	0.3215
Δ_{2j}	0.4499	0.2733	0.1665	0.1665	0.4499
Δ_{3j}	0.3481	0.2823	0.3853	0.2823	0.3853

Table 4: Attribute weights under different decision making status

Parameters	C_1	C_2	C_3
α_1	0.346	0.209	0.233
α_2	0.359	0.472	0.376
α_3	0.173	0.420	0.336

Table 5: Correlation degree and sorting results under different decision-making status

Parameters	r_1	r_2	r_3	Correlation sorting
α_1	0.6279	0.5979	0.5471	$r_1>r_2>r_3$
α_2	0.9580	0.9551	0.8507	$r_1>r_2>r_3$
α_3	0.7423	0.7663	0.6578	$r_2>r_1>r_3$

proposed method based on the processes and steps of the calculation and analysis, this method contain more decision-making information and it will make the decisions more consistent with reality.

CONCLUSION

This study proposesd a multi-attribute decision making method based on grey relational analysis according to research and evaluate the multi-attribute decision making problems which the information in the forms of intuitionistic trapezoidal fuzzy numbers. The study discussed the implementation steps in detail and give numerical examples to prove the rationality of this approach. This decision making method have a clear concept, simple calculation process and it is easy to understand, it also have good application value and the actual decision-making value and effectively extends the theory and application of grey relational analysis method. This provides a new way to solve the intuition trapezoidal fuzzy multi-attribute decision making problems which the attribute weights is known and showed the different decision making status.

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