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Zipf's Law and Urban Dynamics in an Indian State: Kerala (1951-2001)

[1]V. Christopher Amalraj and [2]A. Subbarayan
[1]Department of Community Medicine,
Sri Ramaswamy Memorial Medical College and Research Centre,
[2]Department of Master of Computer Applications, Sri Ramaswamy Memorial University,
Kattankulathur, Chennai, 603 203, Tamil Nadu, India

Abstract: This study examines the evolution of size distribution of cities and towns in a region viz., Kerala state in India during 1951-2001. An attempt is made to study in depth a region's multi-scale urban system evolution from the perspective of regional and sub-regional levels. The data in respect of urban size class under Indian Census is used for the study. The reference period of study is from 1951-1961. Urban evolutionary trend is analyzed using Zipf's exponent. The expansion methodology is used to investigate the dynamics of temporal dimension. The Zipf's law clearly captures main trends of urban system in the region and Zipf's plots overtime reflects the temporal dynamics of urban system evolution at the regional and sub-regional levels. The rank size clock is used to identify the trajectories.

Key words: Zipf's law, city-size distribution, rank-size distribution, trajectory, expansion methodology, rank-size clock

INTRODUCTION

Urbanization is a relatively recent but for the most dominant social transformation of our times. India's urbanization structure is marked by a high concentration of urban population in a few large cities and in towns. India's urban growth itself is high but is still significantly lower than many developing countries. The process of urban growth is closely related to size distribution of cities. The concept of rank size distribution of cities and the rank size rule was initially advanced by Zipf (1949). To describe the size distribution of cities urban researcher's use Zipf's law. One of the first attempts to measure empirical validity of Zipf's law in an international comparison is the influential study by Rosen and Resnick (1980). Dutt *et al*. (1994) made comparative study of rank size distributions for China and India.

The remarkable contributions by Eaton and Eckstein (1997) on France and Japan, by Dobkins and Ioannides (2001) on USA, with later studies by Black and Henderson (2003) and Ioannides and Overman (2003) on USA established some basic facts about the urban system and their developments in France, Japan and the USA over the last century. Gangopadhyay and Basu (2009) studied the size distribution of Indian cities and towns. Giesen and Suedekum (2011) based on the pioneering contribution made by Gabaix (1999) studied the size distribution of individual regions of a country, regardless of the type of defined regions. Ye and Xie (2012) examined the urban dynamics of China using Zipf's law at national and regional level.

Subbarayan (2009) studied the size distribution of cities for a region viz., an India state Tamilnadu for the period 1901-2001. In this study, he has concluded that Zipf's law is an appropriate one for explaining a regional city size distribution. Subbarayan *et al*. (2011) and Kumar and Subbarayan (2014) have attempted to study the temporal and spatial dynamics of regional city size distributions using Zipf's law.

These aspects have motivated the authors of this study to interpret Zipf's law from the perspective of urban dynamics in a systematic manner for a different region in an Indian state viz., Kerala for the period 1951-2001. Further the authors have also studied the urban dynamics of the sub-regions in the Kerala state. Based on non-parametric techniques (Giesen and Suedekum, 2011) they have concluded that Zipf's law exists at different geographical scales, no matter how these scales are defined. This clearly implies that Zipf's law appears to be a universal phenomenon irrespective of different concepts of a region.

Corresponding Author: V. Christopher, Department of Community Medicine, Sri Ramaswamy Memorial Medical College and Research Centre, Sri Ramaswamy Memorial University, Kattankulathur, Chennai, 603 203, Tamil Nadu, India Tel: +91 9443239837

MATERIALS AND METHODS

City size distribution: Auerbach (1913) found that the product of the population size and the rank in the distribution of a city approximate a constant. Zipf (1949) used the rank size relationship to examine a wide variety of issues. The simplest representation of this relationship is:

$$a = pr^{-b} \tag{1}$$

where, a is a constant, p is the population of a particular city and r is its rank according to population size. When the exponent b equals -1, it is referred to Zipf as the rank size rule. In a natural logarithmic form this relationship can also be expressed as:

$$\ln p = a+b \ln r \tag{2}$$

where, a is the estimate of the intercept value, which is also the estimate of the natural logarithm of the population of the largest city and b is the estimate of the slope coefficient of the rank size curve. The b-coefficient is the derivative of the logarithmic function. It evaluates the percentage rate of change in population size associated with the percentage rate of change in rank. On a doubly logarithmic paper, the rank size rule suggested by Zipf appears as a straight line descending from left to right at an angle of 45, indicating a slope of -1.

Incorporating dynamic components in the rank size function: Since 1970s, some researchers have made an improvement in city size distribution studied by incorporating dynamic components viz., time and space in the rank size function (Casetti, 1972; Fan, 1999). To investigate the changes in an urban system through time, Eq. 2 is defined as the initial model. Both the parameters 'a' which is the estimate of the natural logarithm of the largest city's population and the slope coefficient 'b' can vary through time and they can be redefined by the following expansion equation:

$$a = a_0+a_1t \tag{3}$$

$$b = b_0+b_1t \tag{4}$$

where, t is time. By replacing the parameters in Eq. 2 by the right hand side of Eq. 3 and 4, the following terminal model is obtained:

$$\ln p = a_0+a_1t+b_0 \ln r+b_1t \ln r \tag{5}$$

It is to be noted that the signs of the estimates for the expansion parameters a_1 and b_1 show the direction of the linear trend of 'a' and 'b' over time.

Urban size class under Indian census: India has very rich source of information for urban studies. The census volumes, both at the National and State and district levels, provide a mine of information for rural and urban places for period of 100 years. It is also main source of information for temporal studies focusing in the recent past. The census periods covered for the study are: 1951, 1961, 1971, 1981, 1991 and 2001.

Urban population by size classification is presented in Table 1.

Regional city size distribution: In this study, an Indian state viz., Kerala was considered as a region and city size distribution for each census year under six classes for Kerala is given in Table 2. It is important to note that process of urbanization in Kerala primarily consists of class II, class III, class IV and class V cities/towns. The share of these three classes has been growing up systematically during 1951-2001. It is also observed that the share of class VI is negligible. It was also noted that the share of class I cities has considerably increased during 1991 and 2001.

Fit measures to test the deviations from rank-size rule: Cities and towns at regional and sub-regional levels are ranked in descending order with rank one meaning the largest population. The log of ranks in the x-axis and log of population in y-axis on a double logarithmic graph are

Table 1: Classification of urban population with respect to size

Class	Population
I	Greater than 1,00,000
II	50,000-1,00,000
III	20,000-50,000
IV	10,000-20,000
V	5,000-10,000
VI	Less than 5,000

Table 2: Size distribution of cities and towns in Kerala (1951-2001)

Census year	>1,00,000	50,000-1,00,000	20,000-50,000	10,000-20,000	5,000-10,000	<5,000	Total
2001	4	3	10	21	6	1	45
1991	4	4	22	17	4	1	52
1981	5	8	32	11	3	1	60
1971	6	8	55	14	4	1	88
1961	9	17	69	34	10	1	140
1951	10	24	72	37	15	1	159

Source: Census of India volumes-Kerala

presented. To illustrate the fit of Zipf's law, log of ranks as the dependent variable against the log of population as the independent variable were regressed. Following Dutt *et al*. (1994) have classified three terms of goodness-of-fit to test the deviations from rank size rule:

- Deviation from the Rank Size Rule (DRS):

$$DRS = \frac{(b_r - 1)^2}{n}$$

- Standard deviation of slopes of the sub-regional city size distributions (SDL):

$$SDL = \frac{(b_r - b_m)^2}{n}$$

- Deviation from the regional city size distribution:

$$DRD = \frac{(b_r - b_n)^2}{n}$$

Where:

b_r = Absolute value of the slope of the sub-regional city size distribution (Zipf's exponent)
b_m = Mean value of the b_r values
b_n = Slope of the regional city size distribution
n = Number of sub-regions

The indicator of DRS (deviation from the rank size rule) shows to what varying degree of sub-regional city size distribution deviates from the rank size rule across the region. If the value of DRS is zero, it indicates that Zipf's law for city sizes tends to hold good in every sub-region. A smaller SDL (Standard Deviation of the slopes of the sub-regional city size distributions) suggests a lesser degree of sub-regional city size distribution deviates from each other. The values of DRD (Deviation from the regional city size distribution) demonstrate how the sub-regional city size distributions diverge away from the regional urban system. A decreasing value of DRD suggests an increasing similarity between sub-regional and regional urban size distributions.

The following are examined in this study: First examined the general fit of the Zipf's law for region's urban system by plotting the Zipf's curves for cities over the region and in three sub-regions (Fig. 1), respectively. Second exploring the applicability of Zipf's law at different scales among the three different sub-regions. Third investigating the temporal dynamics of urban systems of Zipf's law fittings at 6 census years (1951, 1961, 1971, 1981, 1991 and 2001). Fourth the evolution of trajectories of sub-regional urban systems is used to illustrate the space-time applicability of Zipf's law in a region viz., Kerala state in India.

RESULTS

Fitting of Zipf's law for top 45 cities and all cities in the region: Observed rank size logarithmic graphs and fitted Zipf's law lines for top 45 cities and towns (Fig. 2a) and all cities and towns (Fig. 2b) are illustrated. The basic relations on a log-log scale are ploted, which provides a direct comparison among the selected years 1951, 1961, 1971, 1981, 1991 and 2001.

The results based on Fig. 2a are noted. The primacy of the urban system increased upto 1981 and in 1991 and 2001 it has decreased. The slope coefficient has shown that the distribution of the population was more even after 1951. Remarkable consistency between R^2 values were observed for the study period.

The results in respect of Fig. 2b are given. The primacy of the urban system has increased right from 1951 in the region. The slope coefficient indicated that the distribution of the population was less even upto 1971 and thereafter became uneven. The number of cities have steadily increased since 1951 and witnessed a big jump between 1981 and 1991 as shown by the tail in Fig. 2b. Consistency between R^2 values were also observed for the study period.

Table 3 has revealed the following results: Rank changes for top 10 cities and towns indicated three interesting phenomena: First seven cities (Thiruvananthapuram, Kochi, Kozhikode, Kollam, Thrissur, Allappuzha and Palakkad) remained in the top seven ranks with stable nature. Kottayam was closer to the top 10 ranks. Except in 1951, Kanhangad was also closer to the top ranks.

Rank size clock for selected cities: Batty (2006) has proposed a rank clock where rank orders are plotted for each city in a temporal clockwise direction with the highest at the centre and the lowest on the circumference. This provides an immediate visualization of the dynamics. A city that remains at a given rank traces a perfectly circular orbit, while one that drops in ranks shows a clockwise spiral from a more central to a less central location. A city, which moves up the hierarchy over time, also traces out a spiral trajectory but moves inwards from edge of the space towards the center.

Three cities located in North, Central and South sub-regions among the top 10 in 2001 were chosen to illustrate a trend of urban system emergence in the region during the period 1951-2001 (Fig. 3). The status of

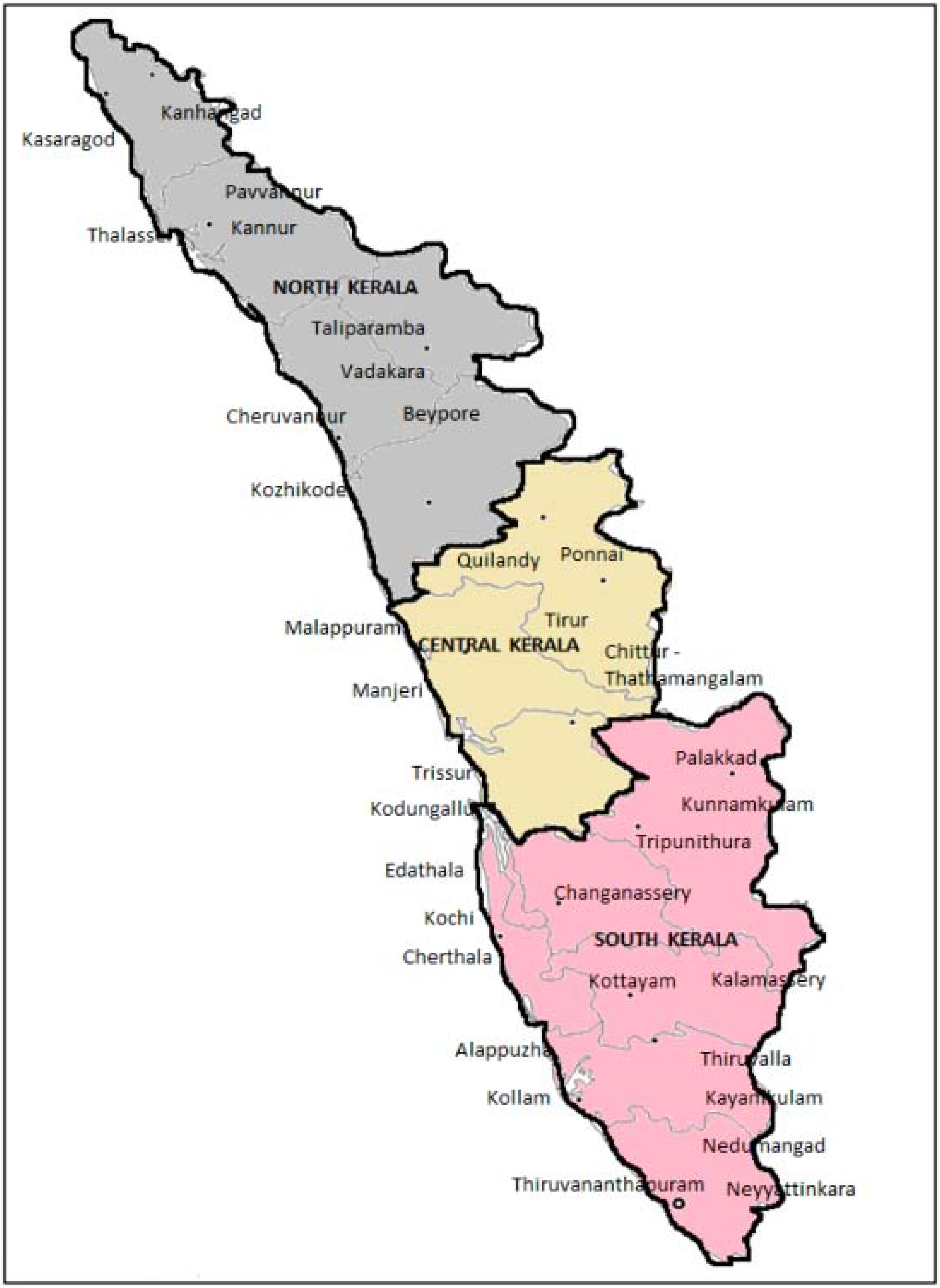


Fig. 1: Three macro regions in Kerala and city distribution in 2001

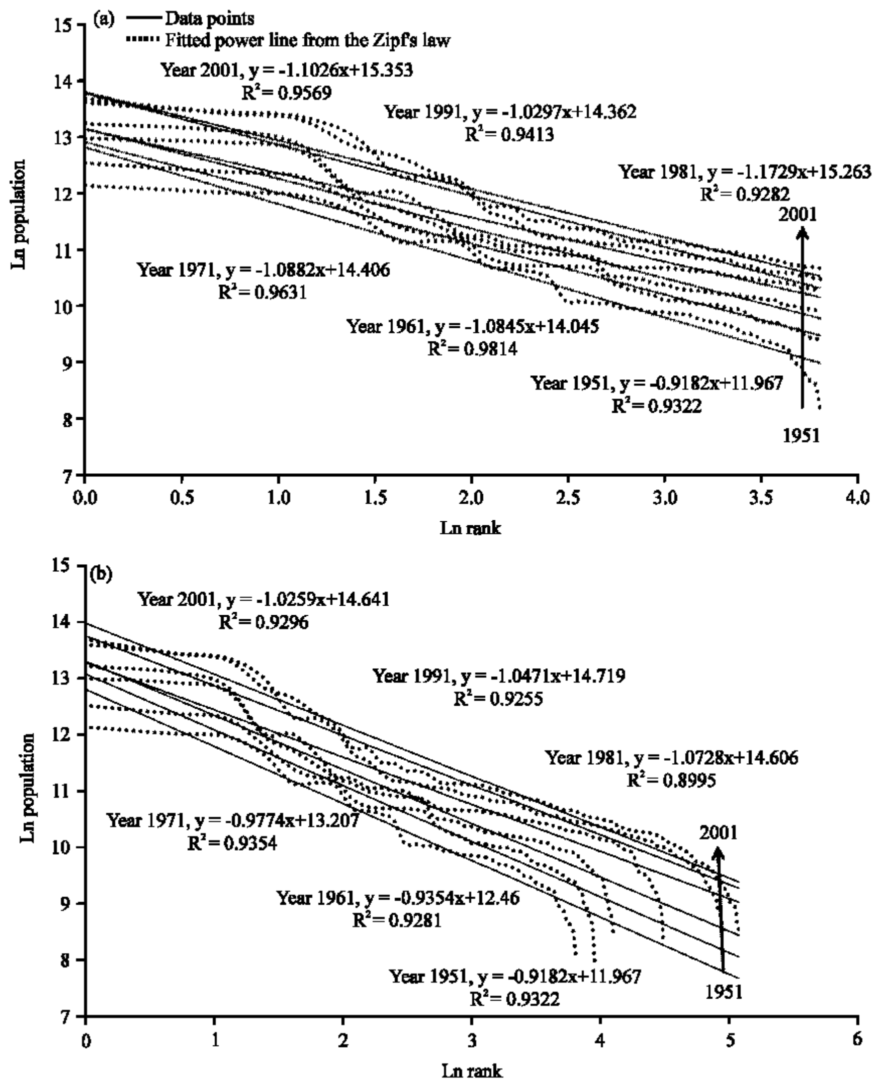


Fig. 2(a-b): Observed rank-size logarithmic graphs and fitted Zipf's law lines of (a) Top 45 and (b) All cities in Kerala 1951-2001. The style applies to all figures, if not specified

Table 3: Key cities in Kerala 1951-2001 census

Census year	Thiruvananthapuram	Kochi	Kozhikode	Kollam	Thrissur	Alappuzha	Palakkad	Kottayam	Kanhangad	Cherthala
2001	1	2	3	4	5	6	7	8	9	10
1991	2	3	4	6	1	5	7	10	8	32*
1981	2	1	3	5	7	4	6	9	-	25*
1971	2	1	3	5	7	4	6	9	10	16*
1961	2	1	3	5	7	4	6	8	11*	16*
1951	1	2	3	7	5	4	6	8	20*	41*

-: Data missing. *The city is not in the top ten ranks at that time

Kozhikode in North Kerala sub-region is one of the main commercial cities of the region and it is the major trade hub with good connectivity through road, rail and air. Its status in rank size clock has been almost unchanged between 1951 and 1981. During 1991 it has lost its advantage and in 2001 it has restored its third rank position. The trajectory of Kozhikode in the rank size clock was slow with inward moving spiral.

Kollam is an old seaport and town on the Laccadive Sea Coast in the South Kerala sub-region. It continues to

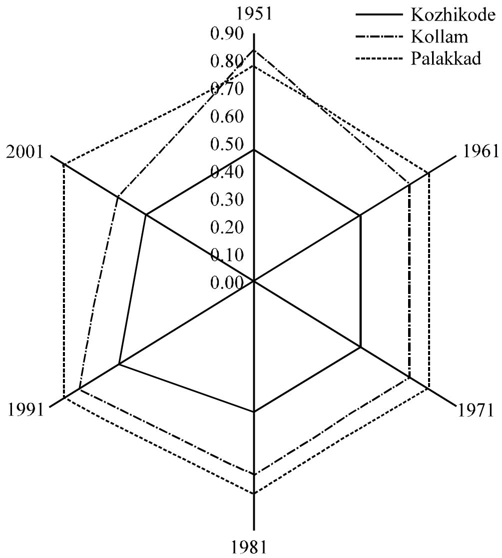
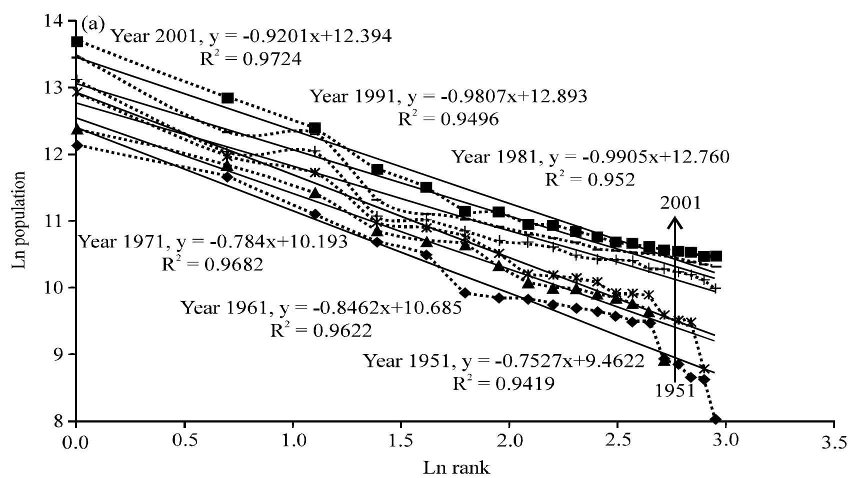


Fig. 3: Rank-size clock in log for three selected cities in Kerala (1951-2001)

be a major and commercial centre. It is status in the rank size clock almost unchanged between 1961-1981. The trajectory of Kollam in the rank size clock was slow with inward moving spiral. Palakkad in the Central Kerala sub-region is the gateway to the region from the North. It is a land of valleys, hillocks, rivers, dams and irrigation projects. The status of Palakkad in the rank size clock remains unchanged between 1951 and 1981. The trajectory of Palakkad in the rank size clock has shown an outward moving spiral.

Urban system evolution at the sub-regional level: The rank size graphs for the sub-regions (Fig. 4a-c; Table 4) has shown diverse patterns, but their Zipf's exponents were moving closer in general as shown in Fig. 5. Due to apparent differences in convergence or divergence, sub-regional level systems indicated sub-regional forces interact with regional forces to shape regional city size distributions. The following two types of evolution trajectories were identified based on primacy, shape and size.



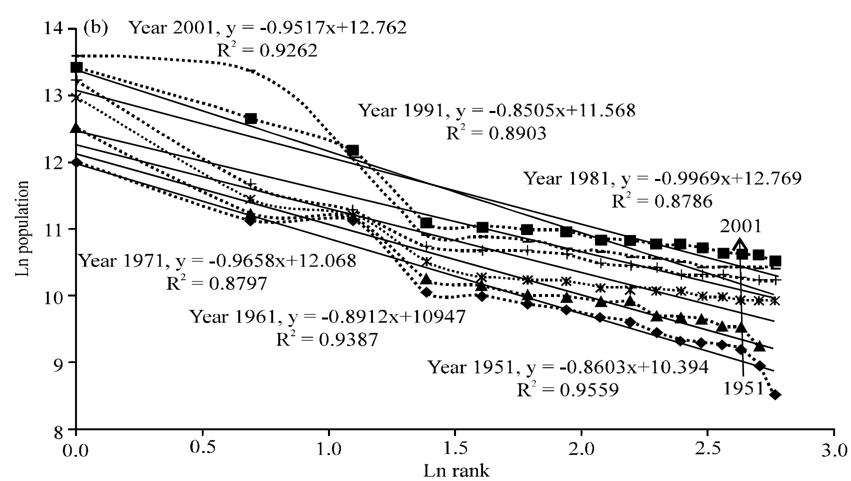


Fig. 4(a-c): Continue

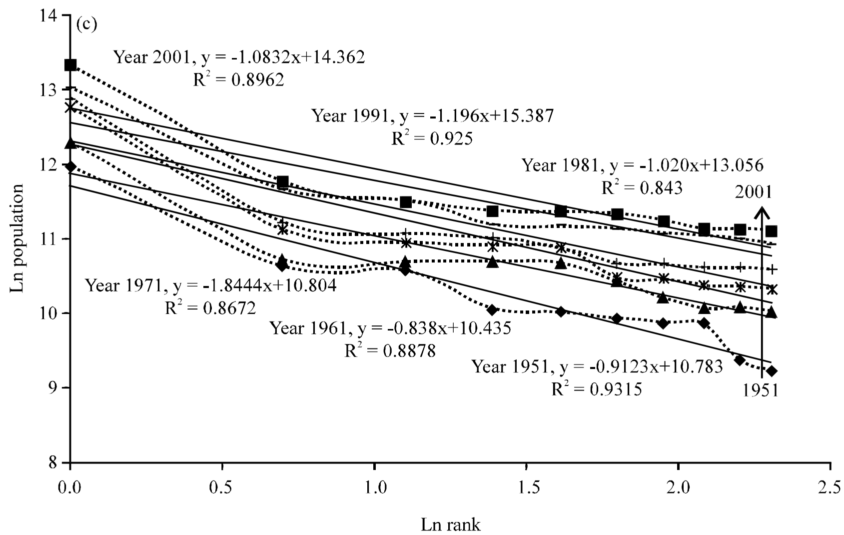


Fig. 4(a-c): Rank-size graph in (a) South, (b) Central and (c) North Kerala 1951-2001

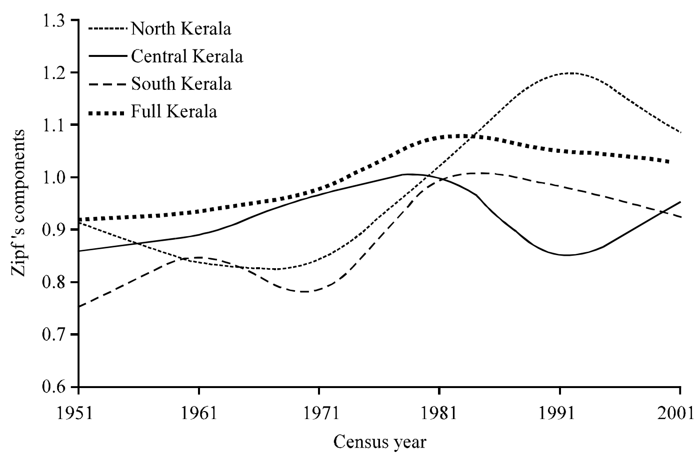


Fig. 5: Zipf's exponent at regional and sub-regional level 1951-2001

Table 4: Zipf's components in Kerala 1951-2001

Year	North Kerala	Central Kerala	South Kerala	Full Kerala	Top 45
1951	0.913	0.860	0.753	0.918	0.918
1961	0.838	0.891	0.846	0.935	1.085
1971	0.844	0.966	0.784	0.977	1.088
1981	1.020	0.997	0.991	1.073	1.173
1991	1.196	0.851	0.981	1.047	1.029
2001	1.083	0.952	0.920	1.026	1.103
1951- 2001	-0.170	-0.092	-0.167	-0.108	-0.185

1951-2001 means the deduction of 1951 from 2001, which indicates the extent of Zipf's component move away (positive) or closer to (negative) rank size rule

Group I trajectory (North Kerala and South Kerala sub-regions): The economy of the South Kerala sub-region is much better than the economy of the North Kerala sub-region because of the regional development policies adopted by the governments from time to time. Both are traditionally wealthy sub-regions with advanced industry and economy. North Kerala sub-region slightly moved away from the rank size rule upto 1971 and thereafter fit the hypothetical one. South Kerala sub-region also moved away significantly from the rank size rule upto 1971 and thereafter declined.

Group II trajectory (Central Kerala sub-region): The economy of the central Kerala sub-region is primarily

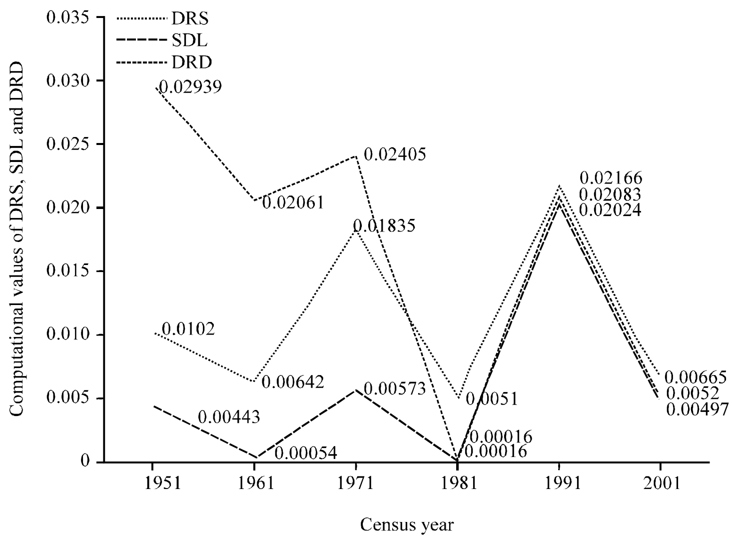


Fig. 6: Variance indicators of Zipf's components in Kerala 1951-2001

Table 5: Variance indicators of Zipf's component in Kerala 1951-2001

Census year	DRS	SDL	DRD
1951	0.02939	0.00443	0.01020
1961	0.02061	0.00054	0.00642
1971	0.02405	0.00573	0.01835
1981	0.00016	0.00016	0.00510
1991	0.02033	0.02024	0.02166
2001	0.00520	0.00497	0.00665

based on commercial activities carried out at Kochin. This region has moved away from the rank size rule upto 1981 and thereafter exhibited a mixed pattern.

Zipf's law at regional-sub regional scale: We have computed the three "fit" measures DRS, SDL and DRD based on the census data in respect of the region for the census years 1951, 1961, 1971, 1981, 1991 and 2001 (Table 5). It is clear that the region moved closer to the rank size rule overtime; based on deviation from rank size (DRS) mentioned section 2. The DRS sharply decreased from 0.0294-0.0052, which indicated Zipf's theory becomes closer to its validity in the region.

Standard deviation of the slopes of the sub-regional city size distributions (SDL) were smaller in general over years which revealed a more balanced development of urban system in the sub-regions. By using Deviation from the regional City Size Distribution (DRD) the difference between slopes of sub-regional distributions and the slope of the regional distribution overtime were detected. The DRD indicator has shown a very similar trajectory as the SDL indicator (Fig. 6).

DISCUSSION

In this study, a region's (an Indian state viz., Kerala) multi-scale urban system evolution from the perspective of regional and sub-regional levels using Zipf's law was discussed in depth. The results clearly reveal that the Zipf's law captures main trends of urban system emergence in Kerala. This is in consistent with the findings by Subbarayan (2009). The Zipf's plots over time reflect temporal dynamics of urban system evolution at regional and sub-regional levels.

The above finding of the study also supports the results obtained by Kumar and Subbarayan (2014) in respect of temporal dynamics of a regional city size distribution based on Zipf's law.

Cities are changing their position in the rank size distribution at multi-scale levels: some are moving up; some are dropping down; some are moving up and then dropping down and some displayed complex spiral centre. The above findings are also are in consistent with the findings of Ye and Xie (2012). These authors have also used evolution trajectories to determine the shape of the rank size distribution. Further in this study, we have used evolution trajectories and identified two sub-regions for determining the shape of rank size distribution. We conclude that by tracking the evolution of urban system at the regional level over time and space it is possible to detect the irregular patterns in the process of urban growth which will be beneficial for framing regional policies for economic development.

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