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Tooth Profile Design and Kinematic Mechanism Simulation of Higher-order Elliptic Gears Based on MAPLE

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Abstract: The analysis of tooth profile design and kinematical behavior of higher-order elliptical gears is very important to realize the multicycle output of variable speed. The design of high order elliptic gear based on tooth profile equation and the motion analysis process is relatively complicated. The method of tooth profile design and kinematic analysis of the higher-order elliptical gears based on the software of MAPLE is presented in the study. The results of dynamic simulation and numerical analysis of the kinematic pair can be obtained visually. The changes of motion parameters also can be gotten. It can be founded that the design and characteristic analysis of non-circle gear can be realized through this method.

Key words: Higher-order elliptical gears, envelope, gear profile, kinematical analysis, MAPLE

INTRODUCTION

In the transmission process of high order elliptic gears, when the driving wheel move a circuit, the driven wheel multiple symmetrical cycle changes will be occurred. As the evolution of the ordinary elliptical gear, the high order elliptic gear are adopted in occasions of more cycle variable outputting such as gear pump, flow meter, etc (Wuxutang, 1997; Yao, 2013; Li, 2004; Wu *et al*., 2008). The analytical method of high order elliptic gear transmission analysis, mainly involves the differential geometry, coordinate transformation, the numerical integral operation work, so the process is complicated (Fang, 2005). In order to realize visual expression in the process of the high order elliptic gear tooth profile design, the software of MAPLE which has the symbolic operation and numerical calculation function and geometric figure ability is adopted and the clear oval gear tooth meshing effect and the analysis results of dynamic movement can be obtained easily (Tan, 2002; Huang and Hui, 2011; Liu and Meng, 2010). This method is very effective for general dynamic simulation design of non-circular gear and the dynamic and static graphics information and other numerical results can be obtained at the same time.

DESIGN OF HIGH ORDER ELLIPTIC GEAR TOOTH ENVELOPE SHAPE

Polar equation of high order elliptic gear's pitch curve is as follows:

$$r = \frac{a(1-e^2)}{1-e\cos(n\varphi)} \qquad (1)$$

where, a represents elliptic semi-major axis type, e represents the eccentricity of the ellipses. In this study, in the design process of choosing the high order elliptic gears driving wheel, the order number n_1 is 3, driven wheel order number n_2 is 5. As it can be seen from the calculation process, the choice and do not break the design of general.

Uniform distribution of tooth shape on the center section: The number of teeth on the elliptical gear is set as z and the module number is set as m. In order to ensure the uniform distribution of tooth profile on the ellipse section curve, among the angle 2π, the total circumference length L should be equal to the z teeth space, namely that:

$$L = \pi m z \qquad (2)$$

According to the general curve arc length equation $r = r(\phi)$:

$$l = \int_{\varphi_0}^{\varphi} \sqrt{r^2 + \left(\frac{dr}{d\varphi}\right)^2}\, d\varphi \qquad (3)$$

Make derivative on the Eq. 1:

$$\frac{dr}{d\varphi} = -\frac{aen_1(1-e^2)\sin(n_1\varphi)}{1-e\cos(n_1\varphi)} \qquad (4)$$

Put Eq. 1 and 4 into Eq. 3 and adopt the MAPLE in the sum of calculated integral value, in the rectangular area between 0 to 2π, the rectangular number k is 1000 and then enough ellipse circumference accurate calculation results can be gotten.

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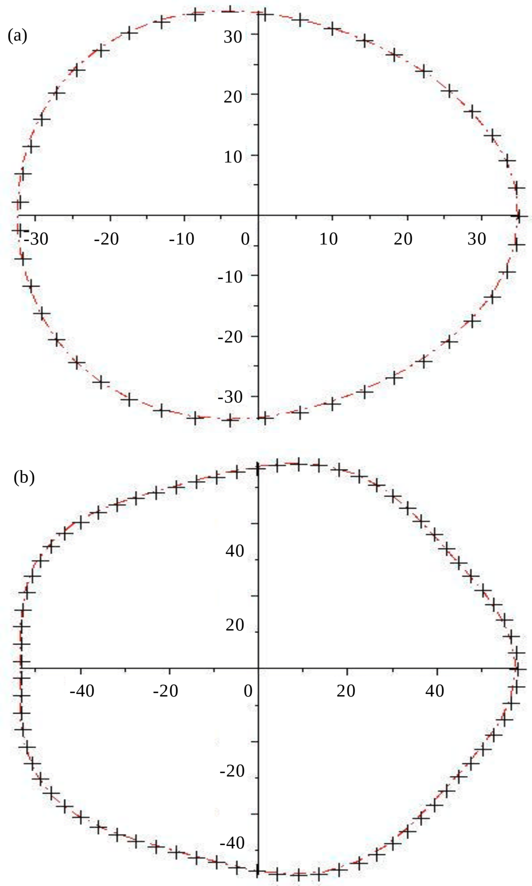

Fig. 1(a-b): Location of tooth shape center, (a) $n_1 = 3$ and (b) $n_2 = 5$

As the requirement of uniform distribution of tooth profile, the results must be equal to L.

Therefore, the solution of equation is as follows:

$$\text{rightsum}\ (l(\varphi),\ \varphi = 0..2\pi,\ 1000) = \pi nz \tag{5}$$

The value a conforms to the requirements can be obtained. By the same token, i can be set as 1…z , take central angle of tooth profile Ω_i as variables and solving the equation:

$$\text{rightsum}\ (l(\varphi),\ \varphi = 0..\Omega\pi,\ 1000) = i\pi z \tag{6}$$

The central Angle of tooth profile Ω_i (I = 1…z) can be obtained. Through the differential geometry, the center curvature $Q(\alpha, \beta)$ of the point on the curve is as follows:

$$\begin{cases} \alpha = x - \dfrac{y'(1+y'^2)}{y''} \\ \beta = y + \dfrac{1+y'^2}{y''} \end{cases} \tag{7}$$

Radius of curvature is as Eq. 8:

$$\rho = \frac{(1+y'^2)^{\frac{3}{2}}}{|y''|} \tag{8}$$

The Eq. 1 can be written in rectangular coordinate equation form as Eq. 9:

$$\begin{aligned} x &= r\cos(\varphi) \\ y &= r\sin(\varphi) \end{aligned} \tag{9}$$

Through the MAPLE derivation of implicit function command and through the Eq. 9, y', y" can be obtained by type and if substitute them into the Eq. 7, 8, at the same time substitute $\omega i(i = 1 \ldots z)$ into the Eq. 9, the center point $P_i(xe_i, ye_i)$ of ellipse curve and radius of curvature ρ_i can be obtained. And the related curve are as shown in Fig. 1.

Design of tooth profile: The basic principle of elliptic gear tooth profile map is using the pure rolling rack cutter and elliptical gear billet, through the rack tooth envelope the elliptical gear tooth profile. As for the high order elliptic gears, under the condition of outer convex section curve, the method of using enveloping drawing to form of the gear rack is effective. As to the outside convex problem of the high order elliptic section line, the condition is as follows:

$$e \le \frac{1}{n^2 - 1} \tag{10}$$

In the design e is set as 0.04, through the method of "translation and rotation" the high order elliptic gears tooth profile can be obtained. The circumference of the circle is divided into n equal parts, i.e., the polar angle is $\phi = 2\pi/n$. The angle between the gear and rack pole diameter section line is as Eq. 11:

$$\mu = \arctan\left(\frac{r}{r'}\right)_{\varphi=\varphi_i} \tag{11}$$

Through Eq. 3 and 11, the location of each cycle rack can be determined. If set n = 150 and perform the programmable cycle calculation, after 30 times calculations, the cycle graphics is as shown in Fig. 2.

When 150 cycle calculations are finished, elliptical gear enveloping graph is as shown in Fig. 3.

According to the feature points in the process of enveloping the tooth shape, the outline of elliptical gear is as shown in Fig. 4.

In the calculation above, m = 1.5, z1 = 45, z2 = 75, e = 0.04.

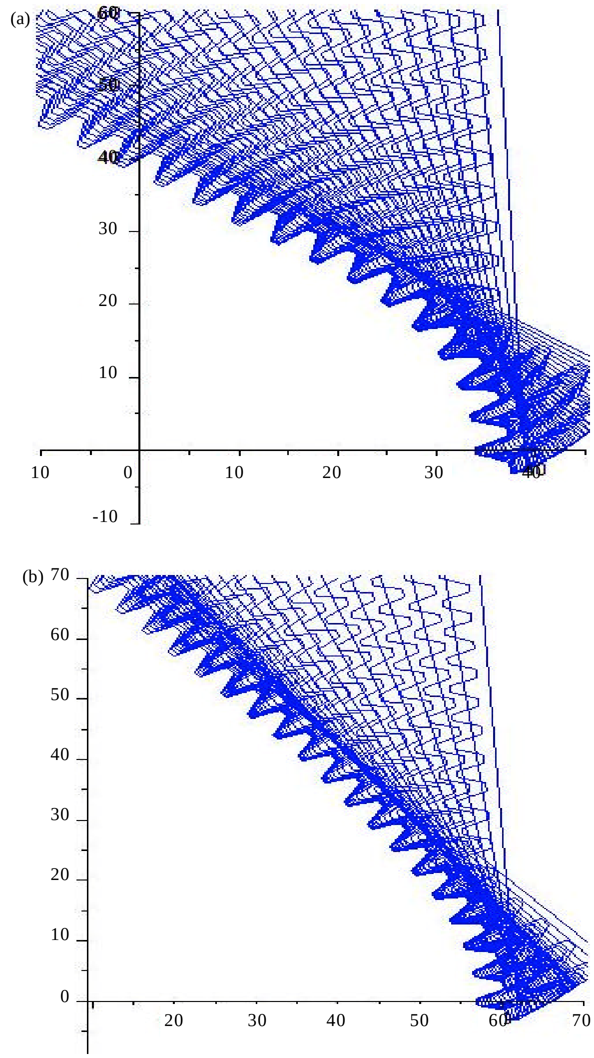

Fig. 2(a-b): Design process of tooth envelope, (a) n_1 = 3 and (b) n_2 = 5

Meshing of high elliptic gears: Closed n_1 order elliptic gear's pitch curve is within the scope of φ~2π, r_1 changes n_1 cycles, r_2 changes n_2 cycles, closed condition of the driven wheel section curve is Eq. 12:

$$\frac{2\pi}{n_2}=\int_0^{\frac{2\pi}{n_1}}\frac{r}{S-r}d\varphi \qquad (12)$$

If substitute Eq. 1 into the solution of high order elliptic gears meshing center distance S. Take center distance S as distance translation and make the transform translation of high order elliptic gears, effects of static of elliptic gears meshing are obtained which is as shown in Fig. 5.

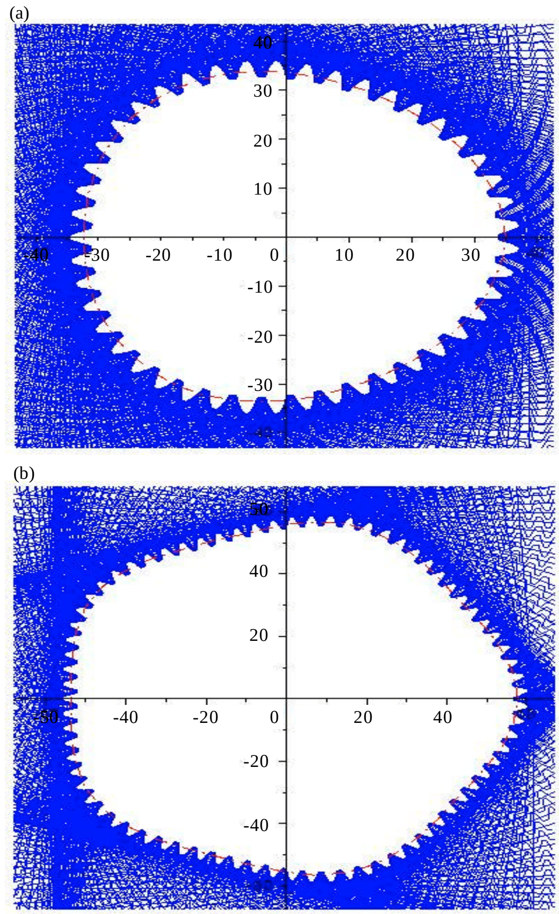

Fig. 3(a-b): Design results complete tooth envelope, (a) n_1 = 3 and (b) n_2 = 5

The design program of high order elliptic gear generated by wire-cutting processing of physical elliptical gears is shown in Fig. 6.

MOTION SIMULATION OF HIGH ORDER ELLIPTIC GEARS

The dynamic design can be intuitive to see the whole cycle and get a clear understanding of the mechanism movement. Especially for variable transmission ratio movement, the simulation is very necessary to determine whether the design agencies can meet the actual demand.

Motion simulation approach: In order to get high order elliptic motion simulation, the driving wheel of a cycle is divided into n equal parts, in each of the driving wheel position the MAPLE is adopted as the plot tools. In order to ensure the elliptical gear pair and maintain the correct meshing relationship, the driven wheel angle must satisfy transmission ratio function. The whole program with loop

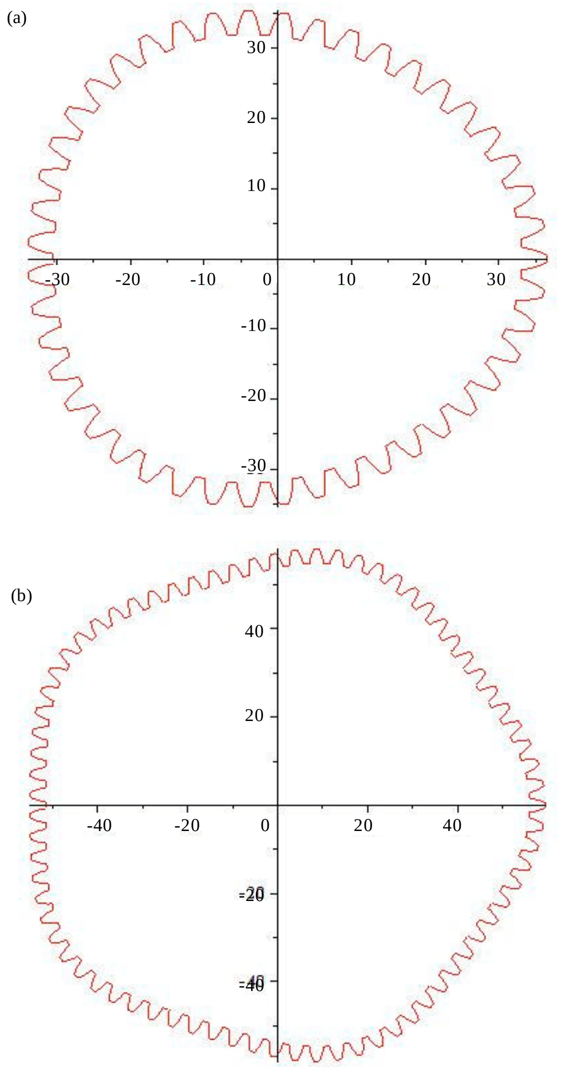

Fig. 4(a-b): Full uniform distribution of high order elliptic gears, (a) $n_1 = 3$ and (b) $n_2 = 5$

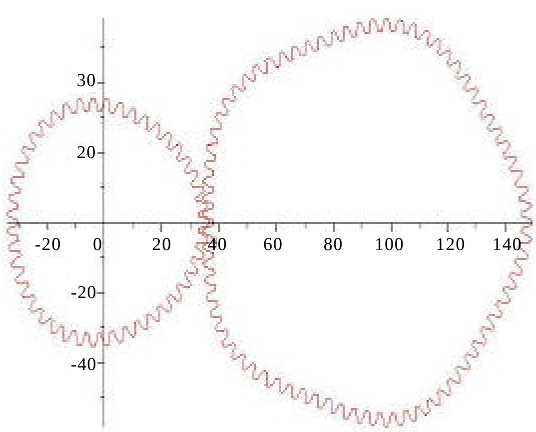

Fig. 5: Meshing elliptical gear pair

Fig. 6: Physical structure of elliptical gears

statements can draw graphics for all position, after running the program, all of the graphics, according to the order and the dynamic effect of elliptic gears transmission can be seen. If change the value of n, it is actually changed the number of frames in MAPLE mapping, so it can founded that the driving wheel angular velocity does not equal to the movement of elliptical gear at the same time.

Transmission relationship of high order elliptic gears: According to the curve equation of non-circular gear driven wheel:

$$\begin{cases} r_2 = S - r_1(\phi_1) \\ \phi_2 = \int_0^{\phi_1} \frac{r_1(\phi_1)}{S - r_1(\phi_1)} d\phi_1 \end{cases} \tag{13}$$

Substitute Eq. 1 into the Eq. 13, the relationship between the driven wheel and driving wheel is as Eq. 14:

$$\phi_2 = \int_0^{\phi_1} \frac{a(1-e^2)}{S(1-e\cos n_1\phi_1) - a(1-e^2)} d\phi_1 \tag{14}$$

Transmission ratio function is as follows:

$$f(\varphi_1) = \frac{e(e-\cos n_1\varphi_1) + (1-e\cos n_1\varphi)\sqrt{n^2-(n^2-1)e^2}}{1-e^2} \tag{15}$$

where, $n = n_2/n_1$, through the type Eq. 14 and 15, motion simulation graphics the high order elliptic gears meshing can be gotten.

Examples of elliptic gears dynamic motion graphic: Through the geometric figure function of MAPLE can draw the ellipse gears meshing transmission motioned graphics. According to the above methods, through the store display graphics and graphics rotation transformation function of MAPLE, the dynamic simulation of high order elliptic gears movement cab be obtained, in the Fig. 7, it give out eight locations of the a motion period. If using the loop playback function, you can see continuous movement effect in the MAPLE environment.

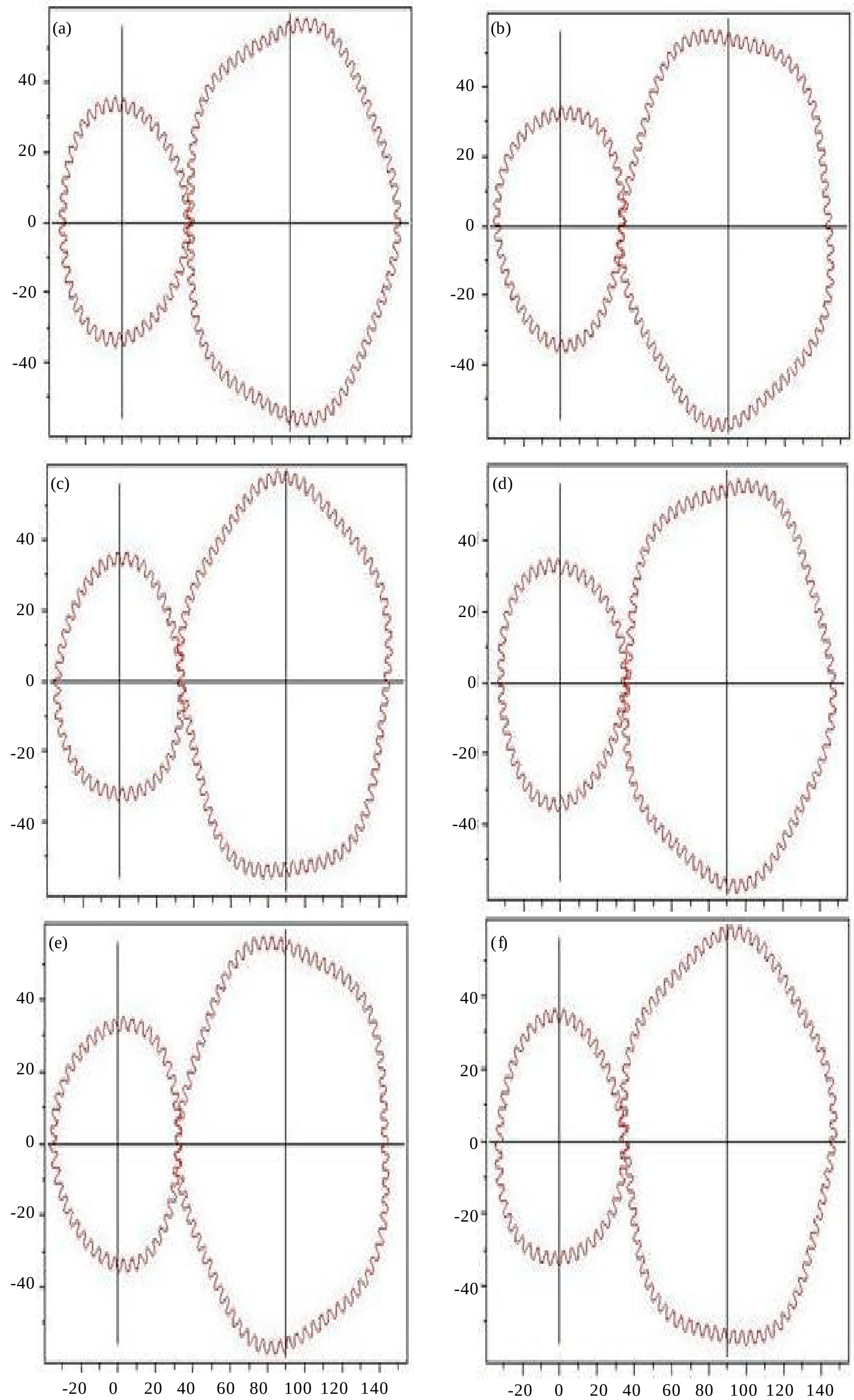

Fig. 7(a-h): Continue

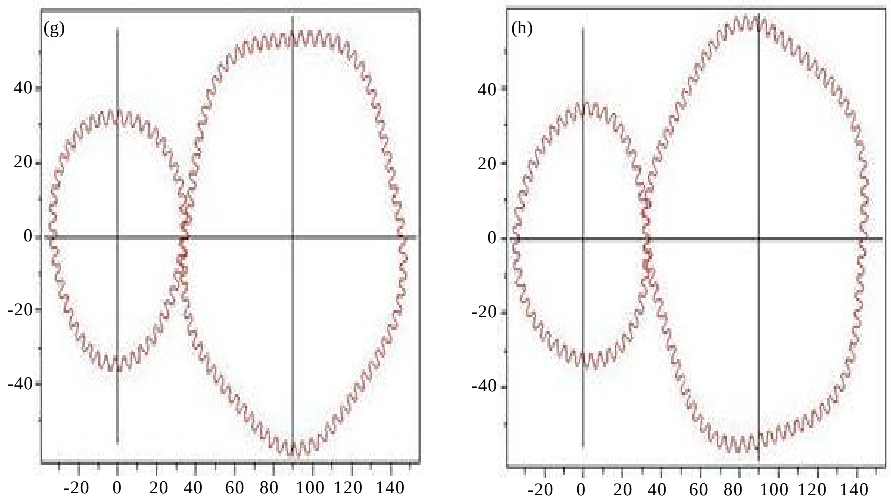

Fig. 7(a-h): Transmission signal of elliptic gears dynamic, (a) ϕ_1 = 0°, (b) ϕ_1 = 45°, (c) ϕ_1 = 90°, (d) ϕ_1 = 135°, (e) ϕ_1 = 180°, (f) ϕ_1 = 225°, (g) ϕ_1 = 270° and (h) ϕ_1 = 315°

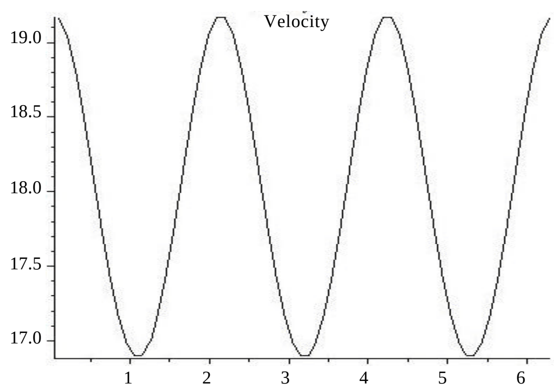

Fig. 8: Changes of driven wheel angular velocity

CHANGE RULE DRIVEN WHEEL SPEED

Through dynamic simulation results of high order elliptic gears, it can easily get the driven wheel angular velocity change rule used numerical method, as shown in Fig. 8. It can be seen from the diagram, when the 3 order elliptic gear of driving wheel change one cycle, the 5 order elliptic gears of driven wheel will change three cycles accordingly. Thus it reflects the high order elliptic gear transmission characteristics of multiple cycle variable output.

CONCLUSION

The Software of MAPLE can adopted in high order elliptic gear tooth envelope design and motion analysis, it can get uniform distribution tooth profile design effect of high order elliptic and the it can dynamic display the movement effect of high order elliptic gears, at the same time. And on the basis of dynamic simulation, the angular velocity curve of driven wheel can be obtained. Through the analysis of process, we can find that this method can be applied in the general design of non-circular gear.

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