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New Approach for Quantization of Second Class Constraint Systems

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Abstract: A disintegration approach to second class constraint systems in a covariant manner is studied. Also, the quantization of the brackets is proposed. In particular, the brackets for coordinates are non-vanishing and after quantization, we obtain the second class constraint systems in emerged coordinates.

Key words: Constraint system, dirac bracket, poisson braket

INTRODUCTION

Recently, Batalin and Tyutin (1992) have constructed an interesting algebraic scheme in which the constraints are formally kept on the same footing. Their starting point is that all the constraint systems are first class if one uses the Dirac bracket. They focused on developing a powerful formalism that is in corporate to the covariance properties of the Dirac bracket under redefinition of the constraint systems. The standard quantization rules implement first and second class constraints quite differently (Gitman and Tyutin, 1990; Hadjialieva and Jafarov, 2007; Gavrilov and Gitman, 2000; Deriglazov and Kuznetsova, 2007; Batalin and Marnelius, 2001). However, this feature is somewhat unfortunate because it requires an explicit separation of the constraint systems into first and second class constraint systems before going to the quantization. This is always possible in principle but in particular, the split may be cumbersome or may spoil manifest Lorentz invariance. For this reason, various efforts have been devoted to question of quantizing the constraint systems in a more uniform manner. In this article we have studied a disintegration method for second class constraints in a covariant manner.

According to Dirac's procedure for dealing with constrained systems, Dirac brackets must replace those constraints or their Poisson brackets which do not commute with all others (the second-class constraints), must be solved explicitly but the first and second class constraint systems are mixed up. Convincing ways have been found to disentangle them in a covariant manner in spinning particles (Ghosh, 1994; Chou *et al*., 1993; Farmany *et al*., 2009a-c; Farmany, 2005; Farmany, 2010; Farmany, 2011a, b).

Let we begin with the Lagrangian of an anyon:

$$L=(M^2u^\mu u_\mu+J^2\Sigma^a\Sigma_a+2MJu^\mu e_{\mu a}\Sigma^a)^{\frac{1}{2}} \qquad (1)$$

Where:

$$u^a=\frac{dx^a}{d\tau},e^\mu{}_a e_{\mu b}=\eta_{ab},e_\mu{}^a e_{\nu b}=g_{\mu\nu}$$

and $g_{\mu\nu}$ is the space-time metric.

In the non-Abelian gauge interactions the canonical momenta are:

$$P_\mu=\frac{\partial L}{\partial u^\mu}=(\frac{M}{J}e_\mu{}^a-w_\mu{}^a)J_a \qquad (2)$$

$$\frac{\partial L}{\partial\sigma^a}=2MJu^\mu e_{\mu a}+2J^2\Sigma_a=2LJ_a \qquad (3)$$

from the four primary constraints we can write:

$$J^aJ_a=J^2 \qquad (4)$$

$$\prod_\mu=\frac{\partial L}{\partial L^\mu}-w_\mu^aJ^a=\lambda e_\mu^aJ^a \qquad (5)$$

$$V^\mu=\varepsilon^{\mu\nu\lambda}\prod_\nu e_\lambda^aJ^a \qquad (6)$$

$$C=\prod^\mu\prod_\mu\text{-}M^2 \qquad (7)$$

where, λ = M/J. Using analytical methods, from four primary constraints we can construct further constraint set. We focus on the second-class constraint set (v^μ, χ^ν). The Poisson bracket matrix of the constraint set (v^μ, χ^ν) is:

$$m=\{V^\mu\text{-}\chi^\nu\} \qquad (8)$$

In addition, the inverse Poisson bracket is defined by:

$$m^{-1}=\frac{1}{M^2}\begin{pmatrix}\frac{-1}{N}[(F-\lambda T)]^{\mu\nu} & -g^{\mu\nu}\\ g^{\mu\nu} & S^{\mu\nu}\end{pmatrix} \qquad (9)$$

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where, $m = \{V^\mu, \chi^\nu\}$, $M = M^2\text{-}1/2\varepsilon^{\mu\nu\lambda}J_\mu(F\text{-}\lambda T)_{\nu\lambda}$ and $T^{\mu\nu}$ is the torsion term. According to Dirac's procedure for dealing with constrained systems, those constraint systems that do not commute with all other ones the second-class constraint system must be solved explicitly or their Poisson brackets must be replaced by Dirac brackets. The Poisson brackets are,

$$\{x_\mu, \Pi_\nu\} = g^\mu{}_\nu \tag{10}$$

$$\{\Pi_\mu, \Pi_\nu\} = -F^a{}_{\mu\nu}J_a \tag{11}$$

$$\{J_a, J_b\} = \varepsilon_{abc}J^C \tag{12}$$

$$\{J_a\Lambda_{bc}\} = \varepsilon_{acd}\Lambda^d{}_b \tag{13}$$

Let x^μ and x^ν be the coordinates. Using Eq. 8 we can write the generic Dirac bracket for coordinates as:

$$\{x^\mu, x^\nu\}^* = \{x^\mu, x^\nu\} - (\{x^\mu, V^\mu\}\{x^\mu, \chi^\nu\})m^{-1}\begin{pmatrix}\{V^\mu, x^\nu\}\\ \{\chi^\nu, x^\nu\}\end{pmatrix} \tag{14}$$

Inserting Eq. 10-13 to Eq. 14 we obtain the:

$$\{x^\mu, x^\nu\}^* = -\frac{1}{N}S^{\mu\nu} \tag{15}$$

$$\{x^\mu, \Pi_\nu\}^* = g^\mu{}_\nu - \frac{1}{N}S^{\mu a}(F_{a\nu} + \lambda D_\nu J_a) \tag{16}$$

$$\{\Pi_\mu, \Pi_\nu\}^* = -F_{\mu\nu} + \frac{1}{N}(F_{\mu a} + \lambda D_\mu J_a) \times S^{ab}(F_{b\nu} - \lambda D_\nu J_b) \tag{17}$$

The brackets for coordinates are non-vanishing and after quantization, the coordinates are emerging. The main method for quantization is following:

$$\{A, B\}^* \to \frac{[A, B]^*}{i\hbar} \tag{18}$$

We can write:

$$\{x^\mu, x^\nu\}^* \to \frac{[x^\mu, x^\nu]^*}{i\hbar} \tag{19}$$

$$\{x^\mu, \Pi_\nu\}^* \to \frac{[x^\mu, \Pi_\nu]^*}{i\hbar} \tag{20}$$

$$\{\Pi_\mu, \Pi_\nu\}^* \to \frac{[\Pi_\mu, \Pi_\nu]^*}{i\hbar} \tag{21}$$

As an example Eq. 19 could be simplified as Eq. 19:

$$[x^\mu, x^\nu]^* = \frac{-i\hbar}{N}S^{\mu\nu} \tag{22}$$

where, $S^{\mu\nu}$ is a anti-symmetric matrix. Eq. 22 shows the geometric non-commutativity of space-time.

CONCLUSION

In this study, in a coordinate system, the second class constraint system is studied in a covariant manner. In continue, a quantized bracket of coordinate system is obtained in an emerged form as Eq. 22.

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