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## Finite Element Numerical Analysis of Unsteady Temperature Field of Rock Mass under Change of Fractures Flow Temperature

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**Abstract:** The numerical simulation for temperature of rock mass is one of the most important issues in geotechnical engineering. When there is a temperature difference between water flow and rock mass, the heat flux will remove from high temperature to low temperature. The paper derived a control equation of the temperature field of fractured the rock mass and its boundary conditions based on the differential equation of heat conduction. The calculation format of the finite element method with a triangle unit is deduced and the relevant program codes are developed to compute for the node temperature in the temperature field. Finally, a case study is analyzed. The results demonstrate that the heat exchanges between flow and rock mass are accordance with the actual situation and varied boundary will influence the surrounding area first, the temperature field will be changed simultaneously which is equal to the change of temperature field. The hysteresis effect does really exist within the rock mass when the boundary condition varies. If the research area is insulated from the outside and the temperature of water flow is uniform and without considering the temperature distribution of fractured rock mass, the trends will become identical when it reaches the stable status.

**Key words:** Fractured rock mass, temperature field, finite element method, numerical analysis

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### INTRODUCTION

Fractured rock mass becomes an integral part of most infrastructure construction projects and the exploitation of deep groundwater resources (Wu and Zhang, 1995; Zhou and Wang, 2004; Chai, 2001). Therefore, coupling analysis of the seepage and temperature fields have become an important research subject, particularly in the fields of groundwater exploitation of rock mass, petroleum development, heat mining and nuclear waste disposal (Zhuo and Sun, 1982; Adams and Rogers, 1982; Jing and Feng, 2006; Zhao, 2004). Most fractured rock masses are currently treated as equivalent continuum media, because it has a very good theoretical foundation and practical experience (Xu *et al.*, 2006; Zhu, 1999). Pehme *et al.* (2010) showed that high-resolution temperature ( $\pm 0.001$  degrees C) profiles could be used to distinguish the active fractures of rock mass under natural groundwater environment. Built the governing equations of the Thermo-hydro-mechanical (THM) coupling of freezing rock based on the dual-porosity medium theory. Huang and Yang (1999) built a mathematical model of the coupling of the temperature field with the seepage field in

the surrounding rock mass using the equivalent continuum method. Gao *et al.* (2013) analyzed the influence of fracture on the distribution of the freezing temperature and seepage fields by using the finite-element analysis software. However, the application of equivalent continuum media cannot fully reflect the actual situation of rock mass with sparse fracture; thus, fissure network method has become a logical alternative. In order to solve the temperature distribution rock mass with water temperature variation when water flow only occurs in the rock, the mass fracture has also become a core problem (Wu and Zhang, 1995; Ding and Wang, 2005; Liu *et al.*, 2011).

In this study, the fractured rock mass is assessed by using the fissure network method under the supposition that water flow occurs only in the fracture. Based on the heat conduction equation of rock mass, the temperature distribution of the fracture flow is regarded as the boundary condition in the calculation of the temperature field using the finite element method. A preliminary study on the distribution of 2-D unsteady temperature field in a fractured rock mass was conducted to explore the distribution of the temperature field in the fracture

network of the rock mass and establish a certain foundation for the coupling analysis of seepage and temperature fields in the fractured rock mass.

**COMPUTING METHODS**

**Governing equation:** The heat conduction equation is the internal relations of temperature with time and space. The 2-D heat conduction equation of rock mass can be obtained from heat conduction theory (Dai, 1999):

$$a \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{w}{cp} = \frac{\partial T}{\partial t} \tag{1}$$

where,  $T$  is the temperature of rock mass,  $^{\circ}\text{C}$  is the unit;  $a$  is the thermal diffusivity which can be expressed as  $a = \lambda/cp$ , where,  $\lambda$  is the thermal conductivity,  $\text{KJ m}^{-1} \text{sec}^{-1}\text{C}$ ;  $c$  is the specific heat of rock mass,  $\text{kJ kg}^{-1}\text{C}$ ;  $\rho$  is the bulk density of rock mass,  $\text{kg m}^{-3}$ ;  $t$  is the time,  $s$  and  $w$  are the heat emitted from unit volume in unit time.

Numerous solutions exist for the heat conduction equation. In order to identify the temperature field that can satisfy the engineering requirements, a corresponding set of initial and boundary conditions must be determined.

The initial condition refers to the distribution of the temperature field in the inner of solid at initial time. When  $t = 0$ , the initial condition can be expressed as:

$$T = T_0(x, y) \tag{2}$$

Boundary condition which regulates the interaction between the solid surface and the surrounding media, can be divided into three categories as follows:

First boundary condition (boundary condition with a given temperature):

$$T|_{\Gamma_1(x,y,t)} = T_1(t) \tag{3}$$

Second boundary condition (boundary condition with a given heat flux):

$$-\lambda \frac{\partial T}{\partial n} |_{\Gamma_2(x,y,t)} = q(t) \tag{4}$$

Third boundary condition: when the solid surface comes in contact with the fluid, the heat flux is proportional to the difference between the solid surface temperature  $T$  and the fluid surface temperature  $T_f$  in the boundary  $\Gamma_3$ , such that:

$$\lambda \frac{\partial T}{\partial x} l_x + \lambda \frac{\partial T}{\partial y} l_y = -\beta(T - T_f) \tag{5}$$

where,  $l_x$  and  $l_y$  represent the direction cosine of the outward normal on the boundary surface and  $\beta$  is the surface heat transfer coefficient,  $\text{kJ}/(\text{m}^2 \cdot \text{s} \cdot ^{\circ}\text{C})$ .

**NUMERICAL SOLUTION**

Assuming  $w = 0$  in Eq. 1, according to the variation principle, the temperature field can be solved by transforming it into the external problem of a functional (Zhu, 1998), such that:

$$I(T) = \iint_R \left\{ \frac{a}{2} \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right] + \frac{\partial T}{\partial t} T \right\} dx dy + \int_{\Gamma_3} \frac{\beta}{cp} \left( \frac{1}{2} T^2 - T_f T \right) d\Gamma \tag{6}$$

where,  $\Gamma_3$  is the boundary that can satisfy the third boundary condition and  $R$  is the research region of temperature field. Evidently, the second term of Eq. 6 only exists when the element comes in contact with the boundary  $\Gamma_3$ . A finite element scheme can be derived as:

$$[K]\{T\} + [C] \left\{ \frac{\partial T}{\partial t} \right\} + [Q]\{T\} = [P] \tag{7}$$

Where the element  $q_{ij}$  of  $[Q]$  is unequal to zero only when the nodes  $i$  and  $j$  of the finite element mesh are on the boundary  $\Gamma_3$  simultaneously and  $[P]$  is the known constant term.

Taking implicit finite difference to the time term:

$$\left\{ [K] + [Q] + \frac{1}{\Delta t} [C] \right\} \{T\}_{t+\Delta t} - \frac{1}{\Delta t} [C] \{T\}_t = [P] \tag{8}$$

Equation 8 is the system of linear algebraic equation that can yield the final solution. When the temperature of the previous time is known, the temperature of the subsequent time can be solved. Based on this equation, a corresponding program that can calculate the temperature value of each node in the research region is written.

**ENGINEERING EXAPMLE**

**Calculation model and boundary condition:** A rock mass area of  $30 \times 30 \text{ m}$  in dam site was considered as the

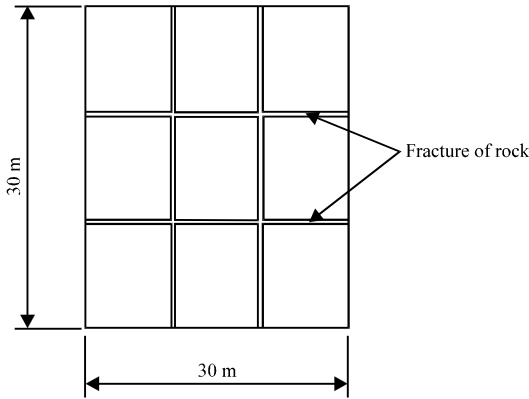


Fig. 1: Computed model map with a size of 30×30 m, there are two fractures along horizontal and vertical direction, respectively

research object. The horizontal and vertical directions are assumed to have four fissures, such that the research region is divided into nine sub-regions, as shown in Fig. 1. In this study, the rock mass is regarded as impervious and the groundwater flow only occurred in the fracture. Therefore, the triangular element is used to determine the discrete research region which is solved using the finite element method. Every sub-region is divided into eight triangular elements, resulting in a total of 49 nodes and 72 elements within this research region. Five calculation cases are designed, as shown in Table 1. Finally, the program codes written in this study are used to calculate the distribution of the temperature field. The thermal conductivity of rock mass equals to  $0.002575 \text{ KJ m}^{-1} \text{ sec}^{-1} \text{ } ^\circ\text{C}$ . Density of rock mass is  $3000 \text{ kg m}^{-3}$ . Specific heat of rock mass equals to  $0.9672 \text{ KJ kg}^{-1} \text{ } ^\circ\text{C}$ . Surface heat transfer coefficient of rock mass equals to  $0.01163 \text{ KJ m}^2 \text{ sec}^{-1} \text{ } ^\circ\text{C}$ .

The related parameters used are conventional values in engineering.

**Calculation of unsteady temperature field:** The unsteady temperature field is calculated under the five conditions illustrated as Table 1 and the temperature field of each case at different times is drawn by the surfer software. Considering the space limitations, the contour of the temperature field is only given at 10 and 1000 h under each condition, as shown in Fig. 2-11. For case 1, the temperature in whole region will stabilize gradually at  $2^\circ\text{C}$  due to  $2^\circ\text{C}$  of temperature water (Fig. 3). For case 2, the temperature decreases in upper part and increases in lower part (Fig. 4). For case 3, the temperature increases gradually and stabilize at relative high temperature

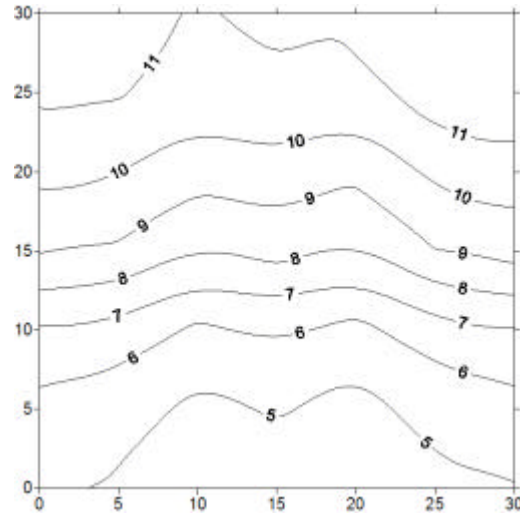


Fig. 2: Contour of temperature field in case 1 at 100 h ( $^\circ\text{C}$ )

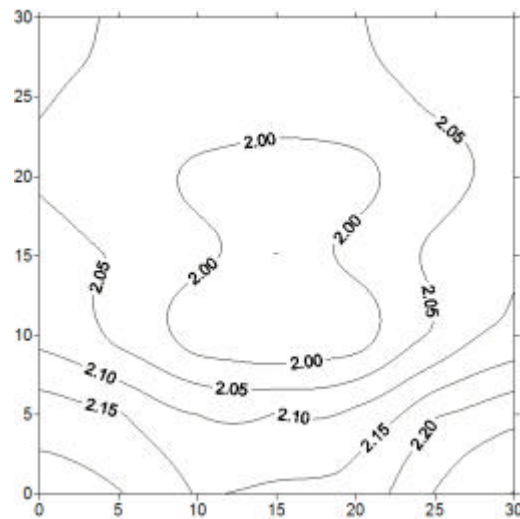


Fig. 3: Contour of temperature field in case 1 at 1000 h ( $^\circ\text{C}$ )

(Fig. 6 and 7). For case 4, the temperature could stabilize at  $20^\circ\text{C}$  (Fig. 9). For case 4, the temperature could stabilize at  $2^\circ\text{C}$  (Fig. 11).

**Analysis of result:**

- The comparison of case 2 with case 4 (or case 3 with case 5) shows that the temperature of water flow is uniform and does not change with time. Thus, when the heat exchange reaches the stable status, regardless of the initial temperature of the rock mass, the temperature distribution becomes identical and the temperature is near to the temperature of water flow

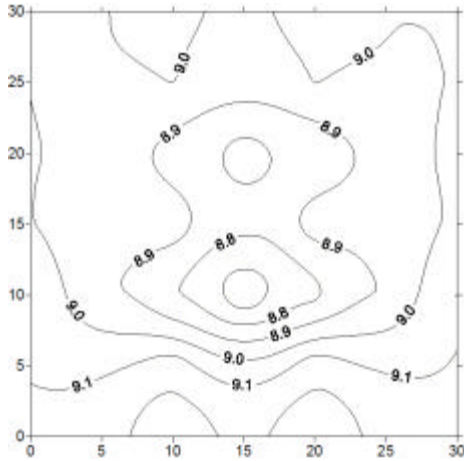


Fig. 4: Contour of temperature field in case 2 at 100 h (°C)

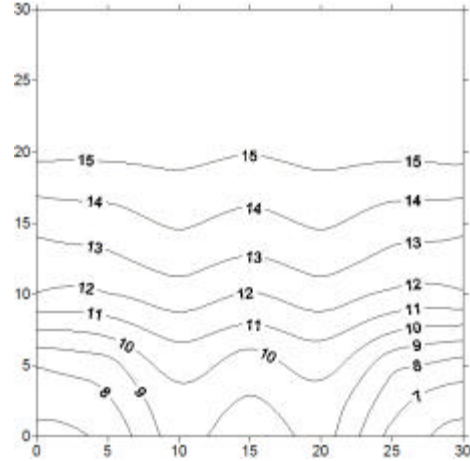


Fig. 7: Contour of temperature field in case 3 at 1000 h (°C)

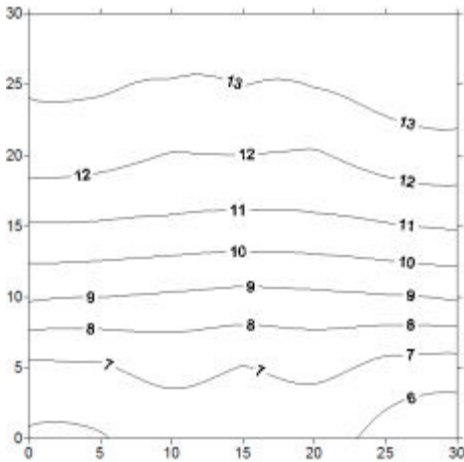


Fig. 5: Contour of temperature field in case 2 at 1000 h (°C)

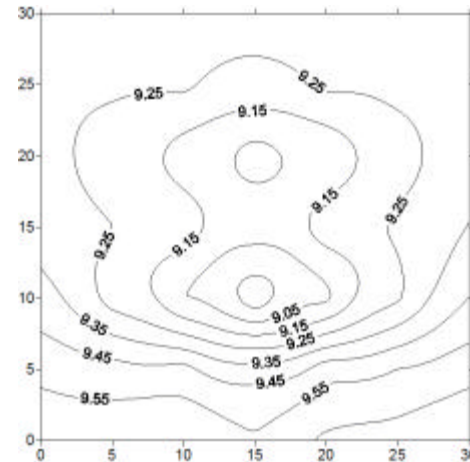


Fig. 8: Contour of temperature field in case 4 at 100 h (°C)

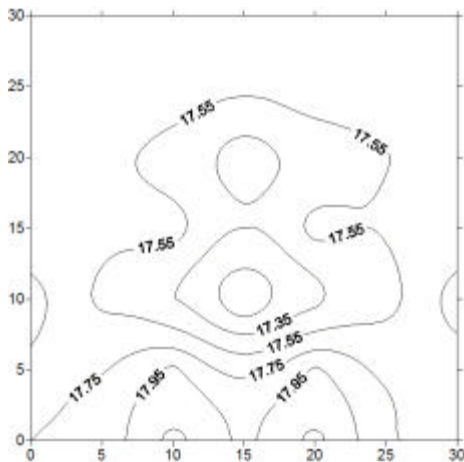


Fig. 6: Contour of temperature field in case 3 at 100 h (°C)

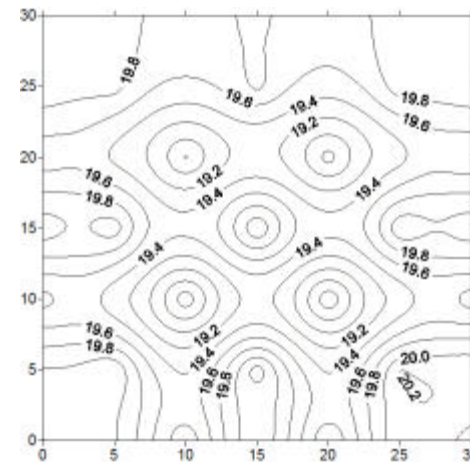


Fig. 9: Contour of temperature field in case 4 at 1000 h (°C)

**Table 1: Computed cases of unsteady temperature field, there are 5 cases with the same boundary conditions and different initial conditions**

Case	Initial condition	Boundary condition
1	Temperature of rock mass is in linear distribution from the upper boundary (20°C) to the lower boundary (2°C) temperature of water in the fissure is 2°C	Four boundary of the research region are adiabatic boundary which doesn't change with time
2	Temperature of water in the fissure is 10°C	
3	Temperature of water in the fissure is 20°C	
4	Temperature of rock mass is 20°C temperature of water in fissure is 10°C	
5	Temperature of rock mass is 2°C temperature of water in fissure is 20°C	

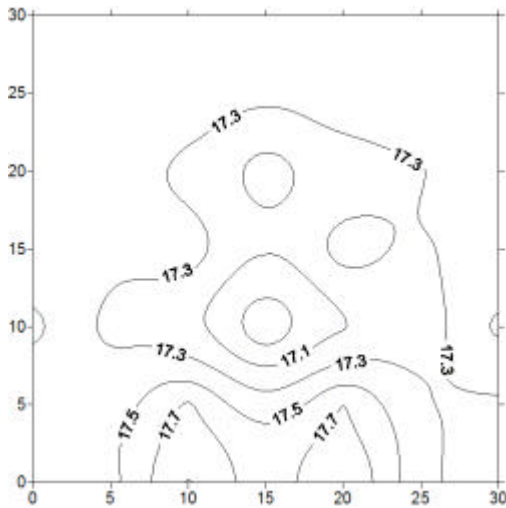


Fig. 10: Contour of temperature field in case 5 at 100 h (°C)

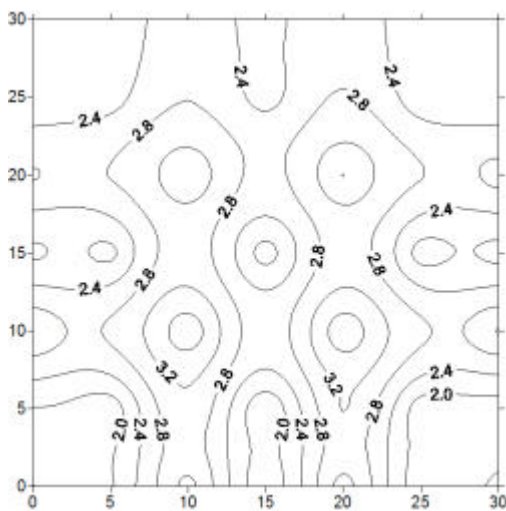


Fig. 11: Contour of temperature field in case 5 at 1000 h (°C)

- Figure 2, 4 and 6, show that when the difference of initial temperature between rock mass and water is greater, the speed of heat exchange is faster and the variation of the temperature contour is more distinct

- Figure 8 and 10 show that heat exchange starts from the contact surface of the rock mass and water flow and that a hysteresis effect exists for the inner rock mass

These results showed basic laws of heat transfer (Zhuo and Sun, 1982). The temperature of rack mass can be influenced by water flow temperature in the fissure (Pehme *et al.*, 2010).

### CONCLUSION

In this study, a 2-D unsteady temperature field in fractured rock mass is analyzed based on the temperature difference between rock mass and water flow in the fracture network of rock mass. The following conclusions can be drawn from the numerical analysis:

- The heat exchange starts from the contact surface of rock mass and water flow, a hysteresis effect exists for the inner rock mass and when the difference of initial temperature between rock mass and water flow is greater, the speed of heat exchange is faster
- If the research area is insulated from the outside and the temperature of water flow is uniform, regardless of the temperature distribution of fractured rock mass, when tends to become identical when the stable status is reached
- For the fractured rock mass, the heat exchange between water flow and rock mass is more in line with the actual situation

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