Journal of Applied Sciences

ISSN 1812-5654

Journal of Applied Sciences 14 (1): 89-94, 2014
ISSN 1812-5654 / DOI: 10.3923/jas.2014.89.94

A Robust Optimization Model for the Vehicle Routing Problem under Uncertainty Based on Theory of RR-EP

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Abstract: The robustness measure in E-SDRM (expected value semi-deviation robust model) only pays attention to the measurement of risk part but ignores decision-makers' attitude towards some situations such as bidding or captical budget. Aiming to solve this problem in E-SDRM for the proposed problem, a new robust model referred to as the expected value-combinatorial semi-deviation robust model (E-CSDRM) is proposed. E-CSDRM can not only measure the potential risk of decision-makers' but also give enough consideration to the profits over expectation given a specific decision-makers' attitude towards some situations such as bidding or captical budget. Theoretical analysis shows that E-CSDRM is a generalization of E-SDRM and simulation results demonstrated the effectiveness of the new model.

Key words: Scheduling, robust optimization, vehicle routing problem, risk preference

INTRODUCTION

Solving the Vehicle Routing Problem (VRP) is the key to efficient transportation management and supply-chain coordination. One of the most commonly adopted vehicle routing formulations is the Capacitated Vehicle Routing Problem (CVRP). This concerns the minimum delivery cost of a single product from a depot to customers through a specified number of capacity constrained vehicles. If the demands of the clients are randomly varied, the problem is termed a vehicle routing problem with stochastic demands (VRP Under Uncertatinty) (unrevised).

To manage the transportation and coordinate the supply chain efficiently, it is important to solve the Vehicle Routing Problem (VRP). The vehicle routing problem can be presented in various forms. Among those vehicle routing formulations, the Capacitated Vehicle Routing Problem (CVRP) is most commonly used. If one or both of the demand and edge costs (distance, transportation cost, travel time, etc.) are uncertain, the variant of vehicle routing problem is termed vehicle routing problem under uncertainty (revised).

The optimal scheduling scheme used in existing Stochastic models for VRP is the Expectation-Value Model (EVM) (Goodson *et al*., 2012; Novoa and Storer, 2009). This takes the expected value of the total delivery expense (or time) as the optimization objective. Using EVM with stochastic demands ignores the negative impacts that possible adverse events can have on operation. At the same time, optimal solution of them is not feasible for all realizations of the data in a predetermined uncertainty set. In order to solve these problems, the recourse (Lei *et al*., 2011; Mendoza *et al*., 2010) and chance-constraint (Bertsimas and Simchi-Levi, 1996; Gendreau *et al*., 1996; Cordeau *et al*., 2007) models, two kinds of EVM, were proposed. However, like EVM, the recourse and chance-constraint models can not make a trade-off between the expectation value of the optimization objective and its variability when used in VRP Under uncertatinty Aimed at addressing these problems, robust optimization models for VRP Under Uncertatinty (List *et al*., 2003; Sungur *et al*., 2008) have received much attention in the last decade. E-SDRM is a kind of robust optimization model for VRP Under Uncertatinty E-SDRM is mainly applied in the routing of buses (a vehicle routing problem that takes the bus company as the depot, customer numbers that vary randomly and the time spent by customers for waiting buses as the demand). To incorporate the stochastic disturbances of daily passenger demand that occurs in actual operations, Yan and Tang (2008) established an expected value semi-deviation robust model (E-SDRM) for routing buses with stochastic demand (Sun *et al*., 2011; Yan and Tang, 2008).

E-SDRM This model combines the concept of semi-variance devised by Markowitz (1952) (unrevised) and Markowitz (1952) (revised), the robust optimization

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model proposed by Mulvey *et al.* (1995) and Mulvey and Ruszczynski (1995) and the actual characteristics of operation of a bus company. E-SDRM is an extension of expectation-value optimization (Tom and Mohan, 2003; Vuchic, 2005; Yan *et al*., 2006) to the routing of buses. Essentially, it consists of an optimization objective term and a robust measurement term. The method aims to minimize the sum of the expectation-values of the total transportation cost (the optimization objective term) and its variability (the robust measurement term) multiplied by a weighting value. Yan *et al.* (2006) proposed a reliable novel bus route schedule design solution by taking into account the uncertainty in the buses' travel times and the bus driver's schedule-recovery efforts. The aim was to minimize the sum of the expectation values of the random schedule deviations and its variability multiplied by a weighting value (Yan *et al*., 2012).

By adopting a balance factor, E-SDRM for VRP under uncertainty considers both the average performance of the optimization objective term subject to stochastic demands and the average deviation between the optimization objective and the expectation value. It is obvious that E-SDRM makes some improvements compared to EVM. However, it still has some aspects related to its optimization objective and robust measurement that require further investigation.

Validity of the robust measurement: The value of the optimization objective in response to different possible events ('relative risks') may get worse compared to the expected value because of stochastic demands, or it may get better (i.e., 'excess benefits'). Total transportation cost in different scenarios may be greater than the expected value of total transportation cost (i.e., 'potential risks'), or it may be less than expected value of total transportation cost (i.e., 'incidental benefits'). When expected value of total transportation cost as the reference point from which deviation are measured in E-SDRM, potential risks and incidental benefits cancel out. Hence, the risk preference of the decision-maker should be determined by considering the loss of incidental benefits or the reduction in the potential risk. When the decision-maker chooses different reference level of expected total transportation cost about which variability will be measured, the risk preference of the decision-maker should be determined by considering the loss of incidental benefits as well as the reduction in the potential risk rather than just potential risk or incidental benefits alone.

Contrary to the existing contributions in this field, we propose a new robust optimization model (termed E-CSDRM) for VRP Under Uncertatinty based on E-SDRM (unrevised). On basis of theory of RR-EP model, a new robust optimization model (termed E-CSDRM) for VRP under uncertainty is proposed (revised) based on RR-EP model (Niu and Wang, 2013). The semi-deviation in E-SDRM is substituted for robust measurement of combinatorial semi-deviation in the new method. This has the effect of reducing both relative risk and the loss of excess benefit, so the adaptability of the robust measurement under stochastic demands is also strengthened.

MATERIALS AND METHODS

Basic concepts

Definition 1: Risk preferences: This refers to the attitude of the decision-maker with regard to the loss of incidental benefit while restraining the potential risk. If the decision-maker thinks he or she will still gain from the incidental benefit while restraining the potential risk, then it is an optimistic risk preference, otherwise it is a pessimistic one.

Assumptions: Put concisely, these are:

- d_j^λ does not exceed the vehicle's capacity Q
- The costs c_{ij}^λ are assumed to be symmetric (although, results can easily be modified to hold even in the non symmetric case) and they satisfy the triangle inequality:

  $$c\,(\lambda, (i,\ j)) \le c\,(\lambda, (i, q)) + c\,(\lambda, (q, j))$$

  for each λ in Λ
- The stochastic demands of different customers are independent

Formulation: VRP under uncertainty is defined on a complete undirected graph G = (V, A), where V = $\{v_0, \ldots, v_n\}$ is the vertex set and A = $\{v_i, v_j\} \in V \times V$, $i \neq j$ is the edge set. Vertex $v_0 \in V$ is the depot at which *m* identical vehicles of capacity Q are based, whereas the remaining vertices represent customers.

For a set of vehicles K, a cost function c: $\Lambda \times A \rightarrow R^+$ for traveling along the edges of G and a demand function d: $\Lambda \times N \rightarrow R^+ \cup \{0\}$ are defined for V. Here, $c_{ij}^\lambda = c(\lambda, (i, j))$ denotes the cost to deliver the commodity from customer i to customer j using vehicle k $\{k = 1, \ldots, m\}$ and $d_i^\lambda = d\,(i, \lambda)$ denotes the demand on customer i from the possible event λ in Λ (Λ is a set of possible events representing the stochastic demands of customers). The total demand on some commodity from possible event λ is:

$$\sum_{i\in V} d_i^{\lambda}$$

Based on commercial and social considerations, operators are expected to minimize their expected total transportation costs. On this basis, the operator needs to choose a scheduling scheme to ensure that every customer be visited exactly by exactly one vehicle as the demands of the customers change. Each vehicle should start from the depot and eventually return to the place of its departure. A robust optimization model is formulated for VRP under uncertainty. The aim is to minimize the sum of the expected value of the total transportation costs and the weighted variability.

Optimization objective term: The optimization objective term of E-CSDRM is the same as optimization objective term of E-SDRM.

Reference level of optimization objective: ω (ω>0) is a reference level of Eq. 2. Thus, ω.EV is the reference point from which deviation are measured. Besides ω = 1, ω can reflect some matters of concern to decision-makers of the depot, such as bidding (ω<1) and capital budget situations(ω<1).

Robust measurement term: The optimization objective term under the stochastic demands is the expected value of the total transportation costs. Therefore, the relative risks and excess benefits of the total transportation costs caused by demand uncertainty are defined, respectively, as follows.

Definition 2: If h (s, λ) for some scenarios is greater than EV (please see appendix in detail), the expectation excess over EV is called the relative risk. This can be expressed as:

$$RR(h(s,\lambda)) = \sum_{\lambda\in\Lambda} \max(0, h(s,\lambda) - \omega\cdot EV)p^{\lambda} \tag{1}$$

Definition 3: If h (s, λ) for some scenarios is less than EV, the expectation deficiency below EV is called the incidental benefit. This can be expressed as:

$$EP(h(s,\lambda)) = \sum_{\lambda\in\Lambda} \min(0, h(s,\lambda) - \omega\cdot EV)p^{\lambda} \tag{2}$$

Combining Eq. 1 and 2, the robustness measure term of E-CSDRM is defined by:

$$\Gamma(s) = \mu RR(h(s,\lambda)) - (1-\mu)\,|\,EP(h(s,\lambda))\,| \tag{3}$$

As for E-CSDRM, this robustness measure term is still a deviation expectation, but it considers both reducing the relative risk and lessening the loss of excess benefit. In Eq. 3, μ (0≤μ≤1) denotes the interplay between the loss of excess benefit and the reduction in the relative risk-it is determined by the decision maker.

Combining Eq. 10 and 3, the robust optimization model can be expressed as:

(CE-CSDRM) minimize:

$$f_{E-CSDRM} = \omega\cdot E(h(s,\lambda)) + \Gamma(s)$$

subject to:

$$\sum_{k=1}^{m} y_{ik} = 1,\ \forall v_i \in V\setminus\{v_0\} \tag{4}$$

$$\sum_{k=1}^{m} y_{0k} = m \tag{5}$$

$$\sum_{j\in V} x_{ijk} = \sum_{j\in V} x_{jik} = y_{ik}, \forall v_i \in V, k = 1,......,m \tag{6}$$

$$\text{Prob}(\sum_{i\in V} d_i y_{ik} > Q) \le \alpha, k = 1,2,3,......,m \tag{7}$$

$$\sum_{i\in S}\sum_{j\in S} x_{ijk} \le |S|-1, \forall S \subseteq V\setminus\{v_0\}, |S| \ge 2, k = 1,2,......,m \tag{8}$$

$$\text{Prob}(\sum_{i\in V} d_i y_{ik} > Q)$$

is a probability that in proposed routes the total demand will be greater than the vehicle capacity given the maximal service ability of the depot was observed. The parameter α is a positive constant (0≤α≤1).

Constraint (11) is referred to a chance-constraint.

The constraint in Eq. 4-8 impose that each customer is visited exactly once, that m vehicles leave the depot and that the same vehicle enters and leaves a given customer, respectively.

Equation 8 is a sub tour elimination constraint, which imposes that for each vehicle k at least 1 arc leaves each vertex set S visited by k and not containing the depot.

RESULTS

Analysis of the properties of C-CSDRM for the proposed problem

Theorem 1: (Validity analysis of the robustness measure term). With the aid of μ, the robustness measure term in E-CSDRM can reflect different kinds of risk preferences under a given decision preference.

Proof: Since, the robustness measure in CE-SDRM measures the conditional expected value of total transportation cost given that failure event has occurred. It is still essentially an expected value, so we have RR (s)≠0 and EP (s) ≠ 0. so we have (unrevised). The proof is shown by considering the following 3 cases:

- When:

$$\mu > \max\left\{\frac{|EP(s)|}{|EP(s)|+RR(s)}\right\}$$

Γ (s) 0, so it is obvious that $f_{ce\text{-}csdrm}$>EV.

This means that reducing the potential risk is at the cost of heavy loss of incidental benefit. The decision-maker cannot make the value of the objective function of the optimization function less than the value of the optimization objective, even with the aid of incidental benefit, so the risk preference is pessimistic

- When:

$$\mu < \min\left\{\frac{|EP(s)|}{|EP(s)|+RR(s)}\right\}$$

Γ(s)<0, so $f_{ce\text{-}csdrm}$<EV.

This means that although reducing the potential risk leads to the loss of incidental benefit to some extent, the decision-maker can still gain some incidental benefit. So, there is the possibility that the decision-maker can achieve better optimization performance by means of incidental benefit. In this case, ω is for the decision-maker to choose available and reasonable incidental benefit, so the risk preference is optimistic

- When:

$$\mathrm{Min}\left\{\frac{|EP(s)|}{|EP(s)|+RR(s)}\right\} \le \mu \le \max\left\{\frac{|EP(s)|}{|EP(s)|+RR(s)}\right\}$$

The decision maker's attitude towards risk change with solutions.

Theorem 2: (Relationship between E-CSDRM and E-SDRM). E-CSDRM will degrade to E-SDRM if μ = 1 and ω = 1.

Proof: When μ = 1 and ω = 1, E-CSDRM becomes:

$$f_{ce-csdrm} = E(h(s,\lambda)) + RR(h(s,\lambda))^{+}$$

Hence, it is obvious that Theorem 2 holds

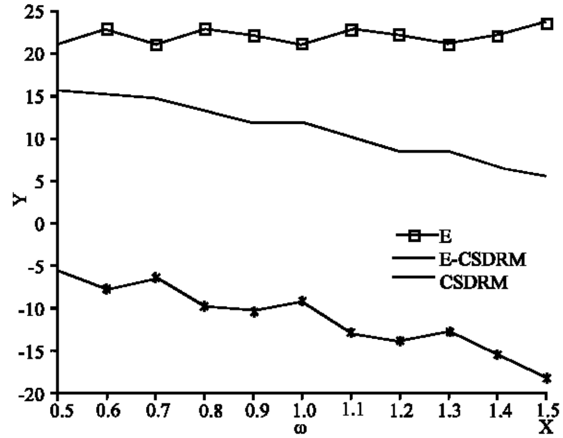

Fig. 1: Variation with ω when μ = 0.85

Theorems 2 shows that E-CSDRM includes E-SDRM. It is clear that E-CSDRM is a generalization of E-SDRM.

Numerical results: Analysis of the influence on risk preference of ω and μ in CE-CSDRM.

Operators are to route for 5 clients and one commodity. There are 2 vehicles in the depot with maximum loads of 400 and 500. The transportation cost is assumed to be symmetrical between the depot and demanding nodes (the cost from customer i to customer j equals the cost from customer j to customer i, ∀i, j∈{1, 2, 3, 4, 5,}. The amount of demand for each client under various possible events is produced using pseudo-random numbers in the interval [200, 500]. In addition, random numbers were randomly generated from the interval [70, 100] for the transportation costs between the depot and the clients. The influence of the maximum service capacity of the depot on the total transportation costs of various possible events is subject to the normal distribution [4, 0.2]. Sets of possible events were generated using the above scheme comprising 50 possible events.

X axes are value of ω. Y axes are values with regard to optimization objective, objective function, robustness measure.

Influence of ω on the optimistic risk preference of the decision-maker in CE-CSDRM: With μ = 0.85, calculations were carried out using 11 values of ω taken from the interval [0.5, 1.5] (and the step size is again 0.1). From Fig. 1, it is seen that the objective function decreases with an increase in ω. This means there is a better performance with an increase in ω. In addition, it is easy to see from Fig. 1 that the optimization objective and robustness measure terms always change conversely. This means that the robustness measure term will correspondingly decrease when the optimization objective increases and vice versa. This demonstrates that besides realization of the decision-maker's optimistic risk preference, the CE-CSDRM can also obtain incidental benefit while reducing the potential risk. This conforms with the analysis of Theorem 3.

CONCLUSION

In this study, E-CSDRM was proposed for VRP under uncertainty of a single depot, multiple vehicles and a single commodity. The model considers the stochastic demands of clients and encapsulates the relative risk and excess benefits of the variation of the total transportation costs under the stochastic demands. The optimization objective of CE-CSDRM is the minimization of the total transportation costs. We can draw the following conclusions through theoretical analysis (unrevised). The advantages of E-CSDRM are as follows:

- E-CSDRM of VRP under uncertainty is a generalization of E-SDRM
- As well as considering both reducing relative risk and lessening the loss of excess benefit under stochastic demands, the robustness measure term in CE-SDRM also reduces the degree of deviation of h (s, λ) from the optimization objective term (in a mean square error sense). Expected total edge costs as reference point from which deviation are measured in robustness measure of E-SDRM. Thus, E-SDRM can not take some situations that relevant to bidding or capital budget situations into consideration
- Contrary to E-SDRM for the proposed problem, E-CSDRM can take into account these situations using a reference level of EV. Thus, it improves the adaptability of the optimization objective term with respect to variation of stochastic demands

Although, E-CSDRM does have some significant improvements over E-SDRM, as described above, there are still some problems that are left to address further via research: As a generalization of E-SDRM, E-CSDRM has some limitations in common with E-SDRM. For example, although it is possible to find an optimal scheduling scheme for the objective function concerned, the feasibility of this scheme is not guaranteed for all of the possible events without any assumptions on stochastic demands. So, methods of improving the adaptability of the optimal scheduling scheme need further investigation.

Appendix

Brief introduction to E-SDRM

Optimization objective term: The total transportation cost of the schedule scheme s due to possible event λ can be represented as:

$$h(s,\lambda)=\sum_{i\in V}\sum_{j\in V}\sum_{k\in K}c_{ij}^{\lambda}x_{ijk} \tag{9}$$

Let p^λ be the probability of the possible event λ in Λ. Then, the expectation value of the total transportation cost for all possible events, E (h (s, λ)), is:

$$E(h(s,\lambda))=\sum_{\lambda\in\Lambda}p^{\lambda}\sum_{i\in V}\sum_{j\in V}\sum_{k\in K}c_{ij}^{\lambda}x_{ijk} \tag{10}$$

where, $p^\lambda \geq 0$ and:

$$\sum_{\lambda\in\Lambda}p^{\lambda}=1$$

Let:

$$EV = E(h(s, \lambda))$$

Robust measurement term (multiplied by a weighting value): E-SDRM treatment of VRP Under Uncertatinty not only considers optimization of the expectation value of the optimization objective term, but also the expected excess over E (h (s, λ)). If the values of the optimization objective for different possible events are higher than expected, then it will have some impact on the minimization of the total transportation costs. The robust measurement term is given by:

$$\begin{aligned}&E(h(s,\lambda)-E(h(s,\lambda)))^{+}\\&=\sum_{\lambda\in\Lambda}p^{\lambda}\max\{0,h(s,\lambda)-E(h(s,\lambda))\}\end{aligned} \tag{11}$$

Formulation: Combining the definitions of optimization object term and robustness measurement term, E-SDRM is described as follows:

(E-SDRM) minimize:

$$f_{E-SDRM}=E(h(s,\lambda))+w\cdot E(h(s,\lambda)-E(h(s,\lambda)))^{+} \tag{12}$$

In Eq. 4, w denotes the influence of the expectation value deviation on the expected value, w≥0. The constraints and decision variables are the same as in the E-CSDRM case.

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