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## Review of Geomechanical Application in Reservoir Modeling

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Abstract: This study has reviewed the geomechanical considerations and applications in reservoir modeling. Geomechanical studies are applied in the reservoir to establish some features as field subsidence/inflation and stability. The reservoir stress alters with the change in the pressure and temperature either by production or EOR injection/thermal methods. The field subsidence/inflation can damage surface facilities. The change in field new stress state could lead to the failure in some rock types and faulting. The change in pressure and temperature leads to the change in stress. The resulted strain changes the porosity, permeability and the consequently the new pressure distribution will be obtained. Due to this change the stress state should be updated by coupling studies. There are some methods to investigate the effects of geomechanics on reservoir simulation, as implicit, explicit, iterative and psudo-coupling method. Many researchers have tried to model the stress in the reservoir for more than thirty years. They have governed various equations to investigate the flow, thermal and chemical effect in stress redistribution. They have done their studied for many different conditions in different flow phases. However, there are lots of reservoir studies that did not consider geomechanics.

**Key words:** Review, geomechanics, petroleum, reservoir, modeling

#### INTRODUCTION

Geomechanics is the study of the rock elastic or plastic behavior and has direct impact on flow models. Over the past two decades the geomechanical analysis has made major changes in the petroleum industry by maximizing production and increasing the life of the well. Rocks are generally composed of different materials and of course not homogeneous. But, the rocks have elastic response and fail in stresses etc., depend on their pore contents. In this section the void space would take into account which not only is essential for oil to be produced from a reservoir but also play an important role in rock mechanical behavior. The theory of thermo-poroelasticity (or porothermoelasticity) is developed by combining the influence of thermal stress and the difference between solid and fluid expansion to rock stresses and fluid diffusion. Poroelastic theory was initially applied in petroleum engineering to understand subsidence, estimate the stress evolution and predict production. With the development of computer techniques, coupled study of geomechanics and reservoir flow effects have become very popular. Thermal effect in the drilled well will cause additional stress and pressure changes and it will definitely affect the stability.

Enhanced Oil Recovery (EOR) refers to a variety of processes to increase the amount of oil extracted from a reservoir after primary and secondary recoveries, typically by injecting water or gas. The injected fluids might push the oil in the reservoir or rather interact with the reservoir rock/oil system to create favorable conditions for oil recovery. The thermo-poroelasticity can describe the effect of temperature and fluid flow change on the stress in the borehole and reservoir. The injection of water leads to the changes in temperature, pore pressure and stress in the reservoirs and also affects the reservoir permeability and porosity. Now-a-days, most reservoir simulators coupled with the stress changes and rock deformations within the production process, either one or two ways; this is because the physical impact of the geomechanical aspects of the behavior of reservoir is considerable.

If we want to deal with the geomechanics in the reservoir, first the equations should be defined. Then the methods of solving the equations should be defied. Finally the studies in the subject with their special characters and boundary conditions would be reviewed.

As time pasts, the researchers try to find a new concept in geomechanics science or to modify the past contributed theories. The improvement in each category could be divided into some distinct time steps.

#### THEORY

In order to analyze the stress state in the reservoir, the flow characteristics, the pressure and temperature should be known. These parameters would be defined by solving the following equations.

Also there are some different methods for coupling the geomechanics with the reservoir flow both in time and space coordination. The explicit coupling doesn't provide accurate results, because the flow characteristics doesn't update. The implicit procedure takes too much time to handle. The iteration method is good and also the iteration properties could be controlled to adjust the required accuracy. The pseudo-coupling method is used by the reservoir simulators and this method uses some correlations to explain the stress state.

**Equations:** The fundamental formulas which will be used as the basic of these studies are the equilibrium, continuity, energy balance and equilibrium equations. Also some other formulas should be used to make relation between the formulas and corporate the phenomena as capillary pressures, phase saturations, average pressures, relative permeabilities, porosity and permeability change. At first we obtain the mass conservation equation:

$$\frac{\partial(\phi\rho)}{\partial t} = -\nabla \cdot (\rho u) + q$$

Then we state the momentum conservation in the form of Darcy's law:

$$u = -\frac{1}{\mu} k (\nabla p - \rho \, \wp \, \nabla z)$$

After including the effect of pressure on density and porosity change, we will obtain this formula:

$$\phi \rho c_{_{t}} \frac{\partial p}{\partial t} = \nabla \bullet \! \left( \frac{\rho}{\mu} k (\nabla p - \rho \, \wp \, \nabla z \, ) \right) \! + q$$

For gas flow, the compressibility cg of gas is a very important factor. By considering real gas formula:

$$\rho \frac{pW}{ZRT}$$

Gas flow equation could be obtained:

$$\frac{\phi\mu c_{\rm g}}{k}\frac{\partial p^2}{\partial t} = \Delta p^2 + \frac{2ZRT\mu}{Wk}q$$

In order to solve the two/three phase saturation equations other equations should be solved as in two phase water-oil system there are four unknown to define  $S_o$ ,  $S_w$ ,  $P_o$ ,  $P_w$  so four equations are needed to be solved either implicitly, explicitly or implicit-explicit methods. These equations are capillary equation, total saturation equation, oil flow and water flow equations.

The conservation of energy equation can be derived for the mass conservation, using the statement of the energy balance or first law of thermodynamics.

Net rate of energy transported+Rate of energy production = Rate of energy accumulation.

Using this law, the overall energy balance equation is:

$$\begin{split} & \frac{\partial}{\partial t} \Bigg( \rho_t U + \frac{1}{2} \sum_{\alpha = w}^g \rho_\alpha \big| u_\alpha \big|^2 \Bigg) + \nabla \cdot E + \\ & \sum_{\alpha = w}^g \left( \nabla \cdot p_\alpha u_\alpha \right) - (\rho_\alpha u_\alpha) - \rho_\alpha u_\alpha \cdot \wp \, \nabla z ) = qH - qL \end{split}$$

The equilibrium equation is as followed; it should also contain pressure in it as the effective stress formula:

div 
$$\sigma$$
-f = 0

Different boundary conditions and reservoir characteristics had applied in different studies to solve for the specific conditions which will be discussed later.

Time and spatial coupling methods: In order to solve the equations we can use four methods as implicit, explicit, iterative and psudo-coupling method. The coupling iteration is controlled by a convergence criterion that is normally based on pressure or stress changes between the last two iterates of the solution.

**Time-based coupling:** A flow simulation is based on time which gets initial conditions and goes through time by a defined time step. Geomechanical calculations are not base on time (except for some phenomenon as creep which usually ignored) but the deformation and pore volume changes feed back to the time based flow results. The degree of frequency of this updating procedure (implicitness) has a strong impact on running speed and the result accuracy. It and can be categorized as follows:

**Full coupling:** Flow variables and geomechanics variables (pressure, temperature, stresses and strains) are solved simultaneously. It gives accurate solutions; however, this approach requires the solution of a large matrix and it is very time consuming.

**Iterative coupling:** Flow variables and geomechanics variables are solved separately and in the sequence. This method has the accuracy close to the full implicit coupling, with better speed.

**Explicit coupling:** Required data is called from the flow simulator to the stress calculations but it wouldn't return. This method is the one-way coupling. This coupling method is very fast and the geomechanics simulator is a post-processing step. However, the accuracy is low because the flow characteristics do not depend on geomechanics and are not updated.

**Pseudo coupling:** Some correlations between porosity and stress are used in flow calculations to find compaction and dilation. However, this method does not process geomechanics (e.g., stress field) and simple formulas are used in a reservoir simulator to compute subsidence or inflation during the process. Running speed in this method is high.

**Spatial coupling:** Both the geomechanics and flow simulators deal with space. This discretized into some grids, so it is required to do the spatial coupling in such studies. In order to model the phenomena, simulators might need gridding in separate places or different mesh sizes.

There are two types of grid coupling:

**Single-grid system:** The geomechanics and flow simulators use the same grid. The geomechanics grid may have shape deformation but it still correspond with the flow grids. This affects the running speed and it takes too long to process.

**Dual-grid system:** The geomechanics and flow simulators use different grids. This coupling method requires an algorithm between the grids. When the dual-grid system is used with the iterative time-base coupling, the running speed is high and it is use for large field scale studies.

These two coupling methods can be used with any time-based coupling methods.

### RESERVOIR STRESS REDISTRIBUTION

After defining the general equations and the methods, the related studies in field stress simulation could be discussed. The stress situation of the reservoir will practically change after production. This stress redistribution changes the state of reservoir properties as permeability and porosity; as the consequence, the

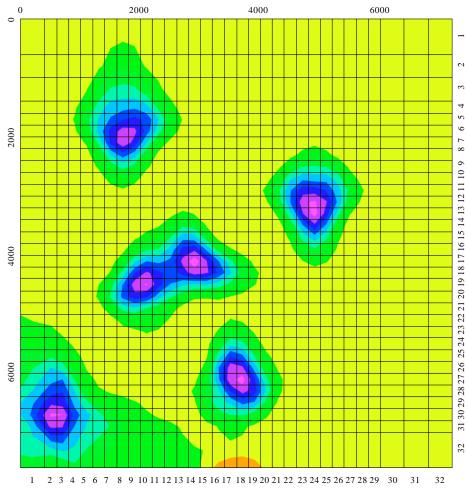
reservoir simulators started to act coupled with geomechanics. There are some studies tried to explain the stress redistribution in the field after some EOR processes (which is mostly focused on the SAGD process). The idea of geomechanical application in the field was first ignited in Geertsma (1973), after observing the field subsidence after some years of production. Researchers tried to find the relation of the change in pressure and the change in rock properties until 1998. In order to improve the reservoir simulation, the geomechanical features has been applied to estimate the better situation for the reservoir and different geomechanical programs had coupled with the reservoir simulators. As the result different cases of production and EOR processes have been simulated with the more accurate results.

Methods of flow and stress coupling: An early attempt of applying the stress to the reservoir state had done by Espinoza (1983). Some formula that relates the pressure and temperature to the pore compressibility had used in the study to obtain the compaction in the steam injection model. He changes the rock properties and reach to some results; as the change in permeability resulted from pore volume alteration is not considerable. The results were primeval but it was an ignition for further studies.

In Lewis *et al.* (1986) had published the first edition of the book of deformation in porous media. They had collected the formula of flow and stress and provide mathematical models to solve the equations. They had modified their work some years later.

Settari et al. (1999) expressed the idea of reservoir flow analysis coupling with reservoir geomechanics. As they had explained, reservoir simulators ignore the geomechanical aspect of porous rocks. They had considered the effect of pressure and temperature on stress and had applied it in a field study. They had also studied different coupling and compaction models. They used the experimental studies to calibrate the rock characteristics and input the parameter into the simulator. Figure 1 shows their simulation compaction results. Their attempts were the basis of the many works afterwards. Although their results were interesting, the study suffers of obsolete formula and boundary conditions.

Chin et al. (2002) had used an iterative procedure for coupled analysis of multi-phase flow and geomechanics in reservoir simulator. They had examined the rocks with other compaction behaviors. They had presented the descriptions of formulations, solution procedures and strategies for enhancement of computational efficiency in their paper. Also some different boundary conditions had been applied as large-scale field examples.



Layer 1, time = 3650 days

Fig. 1: Compaction distribution-case with thermal compaction effects

Table 1: Runtime information for different methods			
Technique	CPU time (s)	Time steps	Iterations
Explicit	7.8	50.0	53
Iterative	10.7	50.0	52
Full	12.4	50.0	51

Dean et al. (2003) had compared the three different techniques of flow-stress coupling: The explicit, iterative and fully coupling method. Figure 2 shows the result of pressure obtained by the three methods and the Table 1 compares the running time of the methods. They concluded that the methods have nearly the same results, of course with high sensibility with the iterative characteristics and time step.

Tran et al. (2004) developed new iterative coupling method and had applied it in CMG reservoir simulator. They had also corrected the porosity formula for the

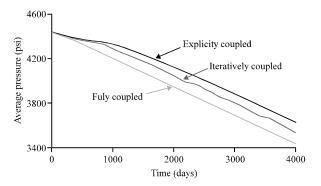


Fig. 2: Comparison between average reservoir pressure obtained by different methods

method. They call it psudo-coupling and their study had shown that the result of this model is like the fully-coupling method but with higher speed. One year later (2005), they compare the different methods of coupling and their results. Figure 3 is the triangle that they had used to represent the status of the methods. This figure implies that there is a balance between the speed, adaptability and accuracy for the methods. Also they did some examples using their new method. Their study on SAGD problem is shown in Fig. 4.

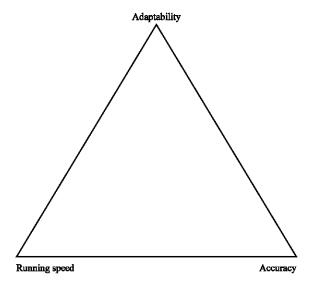


Fig. 3: Balancing of coupling aspects

Mendes *et al.* (2012) had done a study of coupling with heterogeneity. They get their special boundary conditions and solve the two-phase flow problem using Monte Carlo algorithm. They reach to the result of locally conservative numerical solution and impress that there is an obvious change in production resulted by heterogeneity.

Studies of reservoir simulation coupled with geomechanics: The thermo-poroelasticity can describe the effect of temperature and fluid flow change on the stress in the borehole and reservoir. The injection of water leads to the changes in temperature, pore pressure and stress in the reservoirs and also affects the reservoir permeability and porosity. Now-a-days most reservoir simulators coupled with the stress changes and rock deformations within the production process, either one or two ways; this is because the physical impact of the geomechanical aspects of the behavior of reservoir is considerable. There are some programs such as visage and CMG who start using co-worker programs. Some better reservoir description works using coupled programs and many geomechanical studies had been done but there is lots of reservoir simulation and EOR studies which the geomechanics is ignored in them. Also some researchers still prefer not to use simulator software and solve the problems by the numerical solutions to

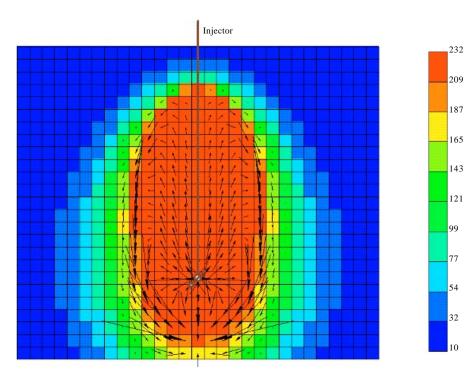


Fig. 4: Circulation of steam in a heated chamber

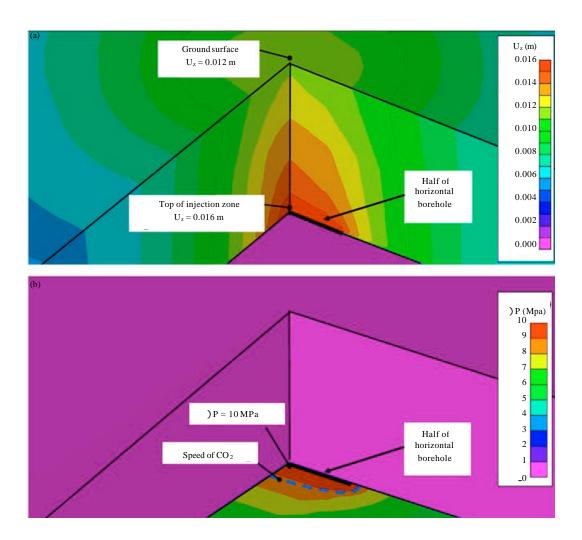


Fig. 5(a-b): (a) Simulated vertical displacement and (b) Changes in fluid pressure after 3 years of injection with an impermeable caprock

obtain the results in specific cases. These studies were important as they show the effect of stress on the reservoir characterization and flow properties.

Hansen *et al.* (1995), Pattillo *et al.* (1998) and Fredrich *et al.* (2000) had done some finite element studies to obtain the reservoir compaction and determine the surface maximum subsidence. They did the case study but their equations models and considerations were not satisfying.

Rutqvist et al. (2008) had governed all the equation required for the modeling the phenomena in partially saturated porous media (2000). They govern all the equations as for thermal studies, mechanical force applied and for the fractured medium, although they did not applied them for any models at the time. Two years later

they studied the model of multi-phase flow in pours media with considering the heat transfer in order to model the CO<sub>2</sub> sequestration. They use the FLAC 3D software to model the stress, coupled with the THOUGH simulator. They did the simulation study for CO<sub>2</sub> in different fields between 2008 and 2012. Figure 5 shows the result displacement in their coupled reservoir-geomechanical model for CO<sub>2</sub> injection and storage. Figure 6 shows the result of their study and the obtained the stress changes due to the change in pressure and the temperature in a field. However, the method which they had used is the one-way coupling (regarding to their simulator software). Pao *et al.* (2001) provide a solution for the multi-phase flow in porous media. They derive all the equations and provide suitable environment for the modeling. Although

they had gone through the problem with proper equations, their models are not good representative of the real reservoir. Also there are some small defects in some formula.

Bostrom and Skomedal (2004) studied the coupled hydro-mechanical behavior of a HPHT gas-condensate field. They had implemented the finite element study

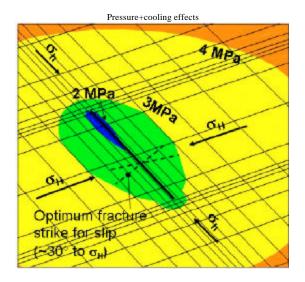


Fig. 6: Strength-to-stress margin after about 3 years of injection

using the ABAQUS software. Their model lacks some important aspect of reservoir modeling. Also the study with the ABAQUS software are implemented by the one-way coupling method which is not considers as high accuracy method. Capasso and Mantica (2006) had done another study by ABAQUS software. They had obtained the reservoir compaction and field subsidence and explained the results properly for a case study.

The study of geomechanics in SAGD process is important because of the higher contribution of temperature and its effect in stress. Chalaturnyk and Li (2004), Tran et al. (2005) and Freeman et al. (2009) and Bao et al. (2010) had studied the geomechanics of the SAGD process. Chalaturnyk and Li (2004) had implemented the coupling of geomechanics and flow simulation for SAGD using CMG software. Some factors that are important in calculating the stress had been studied and their effects are obtained in SAGD process as initial in-situ stress, initial pore pressure and steam pressure and temperature. All these effects had been done for three different cases: Shallow, medium and deep reservoir. Freeman et al. (2009) studied the geomechanics of bitumen formations. They had used two different simulators and compare the results of them. Bao et al. (2010) had done a project with special condition for SAGD process. They had used a compositional model and studied the effect of different solvent on results. Figure 7 shows the result of the steam profile for different solvents.

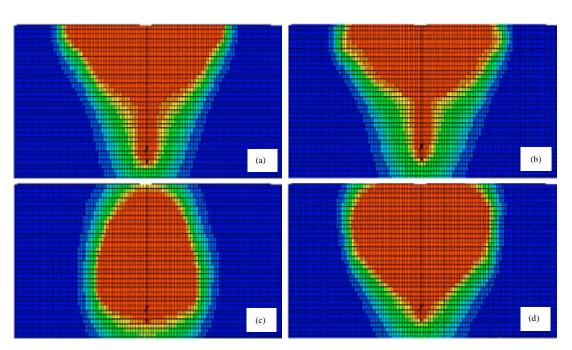


Fig. 7(a-d): Steam chamber profiles of different solvent mixture. (a) IC<sub>4</sub>-NC<sub>5</sub>, (b) C<sub>6</sub>-C<sub>8</sub>, (c) C<sub>9</sub>-C<sub>11</sub> and (d) C<sub>4</sub>-C<sub>11</sub>

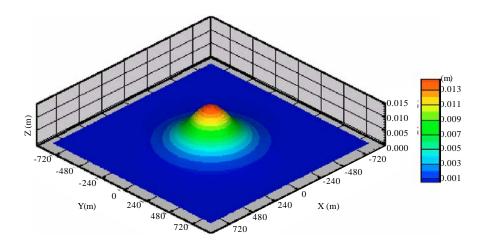


Fig. 8: Ground surface uplift at 30 days

Du and Wong (2005) had developed the coupled geomechanical thermal flow simulator. They had used the finite element method to express the reservoir model (which in most of studies the flow is modeled by the finite difference method). The finite element formulation is explained well but the result was not comprehensively expressed.

A very good example of the coupling study had done by Yin *et al.* (2009). They provide a finite element model of stress coupled with finite difference model for the flow in reservoir and done some examples related to the model. Although some of examples are not applicable for the reservoir and one or two formula was obsolete, the results are good representative of what is actually and accurately is happening in the reservoir. Figure 8 represent the ground surface uplift after 30 days of injection.

Morris et al. (2011) had studied the geomechanical aspects of large CO<sub>2</sub> sequestration. They had studied the different in-situ stresses, fault inclination and conditions. The simulation study had used the iterative coupling method and study the result of stability with the different initial values of parameters.

Safari and Ghassemi (2011) analyzed the geomechanical aspect of huff and puff process. They had done the study for a fractured geothermal reservoir. Their model had shown a good agreement with the field measurements. They had analyzed different geomechanical and flow behavior of the fractures after some years.

Chiaramonte *et al.* (2012) had published some books on the subject of reservoir geomechanics and CO<sub>2</sub> sequestration simulation. They had done another

CO<sub>2</sub>-EOR simulation project in a fractured reservoir (2012). They had investigated the mobility of CO<sub>2</sub> in a fractured field.

#### CONCLUSION

Beside of the necessity of failure study in reservoir, the coupling between reservoir simulation and geomechanics is important because the flow will alter the stress and the porosity-permeability. As a result this will change the flow pattern. So this procedure should be studied in a coupling procedure.

There are different methods of coupling methods and each one has its specific characteristics. Some geomechanical programs, (such as ABAQUS and FLAC) provide a one-way analysis for the reservoir simulators. Some reservoir simulators have module for geomechanical study, so they provide the two-way co-operation with more accurate results.

Some popular studies had been reviewed. Some of them had small defects but they provide good results and let us understand what is exactly happening in reservoir. Nowadays, EOR coupled geomechanical studies are very popular. Also some of EOR methods had been simulated coupled with geomechanical analysis. Although lots of field stress studies had been done, the stress change in different reservoir conditions and characters should be investigated.

### **NOMENCLATURE**

k: permeability (m<sup>2</sup>)

S $\psi$ : Saturation of phase  $\psi$ -g, 1 (dimensionless)

t: Time (seconds)

T: Absolute temperature (K)

z: Elevation (m)

α: Biot's constant for a porous media (dimensionless)

ε: total strain tensor (dimensionless)

 $\phi$ ;  $\phi$ f: porosity in general and porosity (dimensionless)

ρl; ρg: liquid and gas density (kg/m3)

σ: macroscopic total stress tensor (tension positive) (Pa)

r: radius (m)

P: Pressure (Psi)

EOR: Enhanced oil recovery

SAGD: Steam-assisted gravity drainage

#### REFERENCES

- Bao, X., Z.J. Chen, Y. Wei, C.C. Dong, J. Sun, H. Deng and S. Yu, 2010. Geostatistical modeling and numerical simulation of the SAGD process: Case study of an Athabasca reservoir with top water and gas thief zones. Proceedings of the Canadian Unconventional Resources and International Petroleum Conference, October 19-21, 2010, Calgary, Alberta, Canada.
- Bostrom, B. and E. Skomedal, 2004. Reservoir geomechanics with ABAQUS. Proceedings of the Abaqus User's Conference, May, 2004, Boston, USA., pp: 117-131.
- Capasso, G. and S. Mantica, 2006. Numerical simulation of compaction and subsidence using ABAQUS. Proceedings of the ABAQUS User's Conference, May 2006, Boston, USA., pp:125-144.
- Chalaturnyk, R.J. and P. Li, 2004. When is it important to consider geomechanics in SAGD operations? J. Can. Pet. Technol., 43: 53-61.
- Chiaramonte, L., M.D. Zoback, J.S. Friedmann and V.W. Stamp, 2012. 3D stochastic reservoir model and fluid flow simulation of a co2-eor pilot in a fractured reservoir. Proceedings of the Carbon Management Technology Conference, February 7-9, 2012, Orlando, Florida, USA.
- Chin, L.Y., L.K. Thomas, J.E. Sylte and R.G. Pierson, 2002. Iterative coupled analysis of geomechanics and fluid flow for rock compaction in reservoir simulation. Oil and Gas Sci. Technol., 57: 485-497.
- Dean, R.H., X. Gai, C.M. Stone and S.E. Minkoff, 2003.

  A comparison of techniques for coupling porous flow and geomechanics. Proceedings of the SPE Reservoir Simulation Symposium, February 3-5, 2003, Houston, Texas.

- Du, J. and R.C.K. Wong, 2005. Development of a coupled geomechanics-thermal reservoir simulator using finite element method. Proceedings of the Canadian International Petroleum Conference, June 7-9, 2005, Calgary, Canadian.
- Espinoza, C.E., 1983. A new formulation for numerical simulation of compaction, sensitivity studies for steam injection. Proceedings of the Society of Petroleum Engineers Reservoir Simulation Symposium, November 15-18, 1983, San Francisco, California.
- Fredrich, J.T., J.G. Arguello, G.L. Deitrick and E.P. De Rouffignac, 2000. Geomechanical modeling of reservoir compaction surface subsidence and casing damage at the belridge diatomite field. SPE Reservoir Eval. Eng., 3: 348-359.
- Freeman, T.T., R.J. Chalaturnyk and I.I. Bogdanov, 2009.
  Geomechanics of heterogeneous bitumen carbonates.
  Proceedings of the SPE Reservoir Simulation
  Symposium, February 2-4, 2009, The Woodlands,
  Texas.
- Geertsma, J., 1973. Land subsidence above compacting oil and gas reservoirs. J. Pet. Technol., 25: 734-744.
- Hansen, K.S., M. Prats and C.K. Chan, 1995. Modeling of reservoir compaction and surface subsidence at South Belridge. SPE Prod. Facilities, 10: 134-143.
- Lewis, R.W., C.E. Majorana and B.A. Schrefler, 1986. A coupled finite element model for the consolidation of nonisothermal elastoplastic porous media. Transp. Porous Media, 1: 155-178.
- Mendes, M.A., M.A. Murad and F. Pereira, 2012. A new computational strategy for solving two-phase flow in strongly heterogeneous poroelastic media of evolving scales. Int. J. Numer. Anal. Methods Geomech., 36: 1683-1716.
- Morris, J.P., R.L. Detwiler, S.J. Friedmann, O.Y. Vorobiev and Y. Hao, 2011. The large-scale geomechanical and hydrogeological effects of multiple CO<sub>2</sub> injection sites on formation stability. Int. J. Greenhouse Gas Control, 5: 69-74.
- Pao, W.K., Lewis and I. Masters, 2001. A fully coupled hydro-thermo-poro-mechanical model for black oil reservoir simulation. Int. J. Numer. Anal. Geomech., 25: 1229-1256.
- Pattillo, P.D., T.G. Kristiansen, G.V. Sund and R.M. Kjelstadli, 1998. Reservoir compaction and seafloor subsidence at valhall. Proceedings of the SPE/ISRM Rock Mechanics in Petroleum Engineering, July 8-10, 1998, Trondheim, Norway.

- Rutqvist, J., J.T. Birkholzer and C.F. Tsang, 2008. Coupled reservoir-geomechanical analysis of the potential for tensile and shear failure associated with CO<sub>2</sub> injection in multilayered reservoir-caprock systems. Int. J. Rock Mech. Min. Sci., 45: 132-143.
- Safari, M.A. and A. Ghassemi, 2011. 3D Analysis of huff and puff and injection tests in geothermal reservoirs. Proceedings of the 36th Workshop on Geothermal Reservoir Engineering. January 31-February 2, 2011, Stanford University, Stanford, CA., USA.
- Settari, A., D. Walters and G. Behie, 1999. Reservoir geomechanics: New approach to reservoir engineering analysis. Proceedings of the Technical Meeting/Petroleum Conference of The South Saskatchewan Section, October 18-21, 1999, Regina, Canada.

- Tran, D., A. Settari and L. Nghiem, 2004. New iterative coupling between a reservoir simulator and a geomechanics module. SPE J., 9: 362-369.
- Tran, D., L. Nghiem and L. Buchanan, 2005. Improved iterative coupling of geomechanics with reservoir simulation. Proceedings of the SPE Reservoir Simulation Symposium, January 31-Feburary 2, 2005, The Woodlands, Texas, 10.2118/93244-MS.
- Yin, S., M.B. Dusseault and L. Rothenburg, 2009. Multiphase poroelastic modeling in semi-space for deformable reservoirs. J. Pet. Sci. Eng., 64: 45-54.