Journal of Applied Sciences

ISSN 1812-5654

Journal of Applied Sciences 15 (3): 524-530, 2015
ISSN 1812-5654

ansinet
Asian Network for Scientific Information

RESEARCH ARTICLE | OPEN ACCESS

DOI: 10.3923/jas.2015.524.530

Reliability and Failure Rate of the Electronic System by Using Mixture Lindley Distribution

[1]Medhat Ahmed El Damsesy, [2]Mohammed Mohammed El Genidy and [2]Ahmed Mohammed El Gazar
[1]Department of Mathematics, Faculty of Science, Tanta University, Tanta, Egypt
[2]Department of Mathematics, Faculty of Science, Port Said University, Port Said, Egypt

ARTICLE INFO
Article History:
Received: October 18, 2014
Accepted: December 11, 2014

Corresponding Author:
Mohammed Mohammed El Genidy,
Department of Mathematics,
Faculty of Science, Port Said University,
Port Said, Egypt

ABSTRACT
In this study, a statistical method was applied on the lifetime of electronic system contains 20 electronic unit, each unit include two electronic components having Lindley distributions in order to estimate the reliability and failure rate of the electronic system by using mixture Lindley distribution. The study dealt with the method of combining two Lindley distributions and produced the mixture Lindley distribution by including a mixing parameter which represents the proportions of the mixing of two components Lindley distributions. The maximum likelihood method was chose to estimate the values of the parameters for the mixture Lindley distribution. In addition the probability density function, cumulative distribution function, reliability function and failure rate of the mixture Lindley distribution were obtained. On the other hand, a numerical application was prepared to illustrate the implementation of mathematical procedures and obtain the required numerical results. Moreover this study enables the researchers and helps them to apply it in the field of their specialization.

Key words: Mixture Lindley distribution, mixture model, maximum likelihood estimation, reliability, failure rate

INTRODUCTION

In probability and statistics, a mixture distribution is the probability distribution of a random variable which derived form a set of other random variables. Mixture distributions used as a means of representing non-normal distributions and arise naturally where a statistical population contains two or more subpopulations and also concerning with statistical models involving mixture distributions called, mixture models. In this study the mixture distribution was produced from combining two Lindley distributions each includes a number of parameters, the first contains one parameter and the second includes two parameters to obtain a mixture Lindley distribution with four parameters, while the maximum likelihood estimation was applied to estimate all parameters of the mixture Lindley distribution. Nath and Bhattacharjee (2012) performed the comparison of different missing observation handling techniques with pattern mixture modeling applied on clinical trial data set. A mixture of two distributions; Weibull hazard rate with a power variance function frailty distribution to model hazard rate, where the proposed model included a mixture Weibull hazard rate with Gamma and inverse Gauss distribution was presented by Wasinart *et al.* (2013) to solve the problem of misleading conclusions when using the traditional method. Nugraha (2011) presented discussion on study of mixed logit model on multivariate binary response using maximum likelihood estimator and generalized estimating equations. Alharpy and Ibrahim (2013) considered the statistical problem in the parametric treatment comparison when partly interval-censored failure time data exit, constructed a score test and likelihood ratio test for failure time data under Weibull distribution using multiple imputation technique. Validity and reliability of Thai hospice quality of life index-revised version for cancer patients was carried out by Awikunprasert and Sittiprapaporn (2013). Pan *et al.* (2014) performed heat dissipation and thermo-mechanical reliability study for multi-chip module high power LED integrated packing with through silicon vias. Zaman *et al.* (2005) considered Chi-square mixture of Gamma distribution.

Reshid and Majid (2011) have done study a multi-state reliability model for a gas fueled co-generated power plant. Applied research on CMSR method used in reliability evaluation of the CNC machine tools have conducted study by Yu *et al.* (2013).

MATERIALS AND METHODS

Mixture models: The cumulative distribution functions in mixture model are defined as follows:

$$F(x)=\sum_{i=1}^{n}p_iF_i(x);\ \ p_i>0\ \text{ and }\ \sum_{i=1}^{n}p_i=1$$

where, p_i is a mixing parameter and $F_i(x)$ is cumulative distribution function of Lindley distribution. The statistical model involved more than one distribution, called subpopulations or mixture model. In the case of Lindley distributions, called Lindley mixture model.

Mixture Lindley distribution: The probability density function of Lindley distribution with one parameter was given by:

$$f(x,\theta_1)=\frac{\theta_1^{\,2}}{\theta_1+1}(1+x)e^{-\theta_1 x};\ \ x>0,\theta_1>0 \quad (1)$$

In case of two parameters, the probability density function of Lindley distribution will be as follows:

$$f(x,\alpha,\theta_2)=\frac{\theta_2^{\,2}}{\theta_2+\alpha}(1+\alpha x)e^{-\theta_2 x};\ \ x>0,\theta_2>0,\alpha>0 \quad (2)$$

where, θ_1 and θ_2 are shape parameters, while α is a scale parameter. The random variable X represents the values x of the lifetimes of the electronic unit in the system.

A mixture distribution arise by combining lifetimes distributions of two or more components in the electronic unit, therefore the probability density function of mixture distribution was written as follows:

$$f(x)=p_1f_1(x)+p_2f_2(x)+...+p_nf_n(x);p_i>0,\sum_{i=1}^{n}p_i=1$$

where, p_i is the mixing parameter which represents the proportion of mixing for the components distribution i and the function $f_i(x)$ is the probability density function of the components distribution i.

Consider a simple case of two components distributions and the mixing parameter for each components was given by p and 1-p. The probability density function of the mixture Lindley distribution of the two distribution in the Eq. 1 and 2 was given as follows:

$$\begin{aligned}f(x)&=pf_1(x)+(1-p)f_2(x)\\&=p\left(\frac{\theta_1^2}{\theta_1+1}(1+x)e^{-\theta_1 x}\right)+(1-p)\left(\frac{\theta_2^2}{\theta_2+\alpha}(1+\alpha x)e^{-\theta_2 x}\right)\end{aligned}$$

Such that θ_1, θ_2, $\alpha>0$ and $0<p<1$. Therefore two-fold Lindley model was given by:

$$F(x)=pF_1(x)+(1-p)F_2(x)$$

Thus:

$$p=\frac{F(x)-F_2(x)}{F_1(x)-F_2(x)}$$

where, $F_1(x)$ and $F_2(x)$ are the cumulative distribution functions of Lindley distribution with one and two parameters respectively. The probability density function f(x), failure rate h(x) and reliability function R(x) of two-fold Lindley mixture were given by:

$$f(x)=pf_1(x)+(1-p)f_2(x),\ h(x)=\sum_{i=1}^{n}p_i(x)h_i(x)$$

Where:

$$p_i(x)=\frac{p_iR_i(x)}{\sum_{i=1}^{n}p_iR_i(x)}\ ;\ \sum_{i=1}^{n}p_i(x),\ \ R_i(x)=1-F_i(x),\ \ R(x)=1-F(x)$$

$R_i(x)$ is the reliability function of components i; i = 1,2 and R(x) is the reliability function of two components 1,2 therefore:

$$h(x)=\frac{pR_1(x)}{pR_1(x)+(1-p)R_2(x)}h_1(x)+\frac{(1-p)R_2(x)}{pR_1(x)+(1-p)R_2(x)}h_2(x)$$

where, $h_i(x)$ is the failure rate function of the components i; i = 1,2.

Maximum likelihood estimation: Maximum likelihood estimation method was applied on the mixture Lindley distribution to estimate its parameters. Let $x_1, x_2,..., x_n$ be a sample of values of size n were taken from the random variables $X_1, X_2,..., X_n$ which represent the lifetimes of the electronic unit in the system, thus the likelihood function L was given as follows:

$$L=f(x_1).\,f(x_2)\,...\,f(x_3)=\prod_{i=1}^{n}f(x_i)$$

Then the Log likelihood function will be as follows:

$$\mathrm{Ln}(L)=\mathrm{Ln}\left(\prod_{i=1}^{n} f(x_i)\right)=\sum_{i=1}^{n}\mathrm{Ln}\left(f(x_i)\right)$$

Therefore the Log likelihood function derivatives with respect to each parameter θ_1, θ_2, α and p were given as follows:

$$\frac{d\mathrm{Ln}(L)}{d\theta_1}=\sum_{i=1}^{n}\frac{p}{f(x_i)}\frac{df_1(x_i)}{d\theta_1}$$

$$=\sum_{i=1}^{n}\frac{p}{f(x_i)}(1+x)\left[\frac{\theta_1^2+2\theta_1}{(\theta_1+1)^2}e^{-\theta_1 x}-\frac{\theta_1^2}{\theta_1+1}xe^{-\theta_1 x}\right]$$

$$\frac{d\mathrm{Ln}(L)}{d\theta_2}=\sum_{i=1}^{n}\frac{(1-p)}{f(x_i)}\frac{df_2(x_i)}{d\theta_2}$$

$$=\sum_{i=1}^{n}\frac{(1-p)}{f(x_i)}(1+\alpha x)\left[\frac{\theta_2^2+2\theta_2\alpha}{(\theta_2+\alpha)^2}e^{-\theta_2 x}-\frac{\theta_2^2}{\theta_2+\alpha}xe^{-\theta_2 x}\right]$$

$$\frac{d\mathrm{Ln}(L)}{d\alpha}=\sum_{i=1}^{n}\frac{(1-p)}{f(x_i)}\frac{df_2(x_i)}{d\alpha}$$

$$=\sum_{i=1}^{n}\frac{(1-p)}{f(x_i)}e^{-\theta_2 x}\left[\frac{-\theta_2^2}{(\theta_2+\alpha)^2}(1+\alpha x)+\frac{x\theta_2^2}{\theta_2+\alpha}\right]$$

$$\frac{d\mathrm{Ln}(L)}{dp}=\sum_{i=1}^{n}\frac{f_1(x_1,\theta_1)-f_2(x_i,\theta_2,\alpha)}{pf_1(x_1,\theta_1)-(1-p)f_2(x_i,\theta_2,\alpha)}$$

Make the Log likelihood function derivatives equal to zero and then apply numerical methods and computer facilities to solve the resulting equations to obtain the maximum likelihood estimator for each parameter.

RESULTS

In this section, consider the lifetimes of 20 electronic unit per year each of which contains two components having Lindley distributions and were processed by a designed program on the computer to estimate the parameters θ_1, θ_2, α at different values of the mixing parameter p for mixture Lindley distribution. Estimated values $\hat{\theta}_1, \hat{\theta}_2, \hat{\alpha}$ are showed in Table 1.

The values of the probability density function, cumulative distribution function, reliability function and failure rate of two Lindley distributions at the parameters $\theta_1 = 1, \theta_2 = 1, \alpha = 2$ were obtained in Table 2.

While, the values of the probability density function, cumulative distribution function, reliability function and failure rate of mixture Lindley distribution at the parameters $\theta_1 = 1, \theta_2 = 1, \alpha = 2$ in the case of $p = 0.1$, $p = 0.5$, $p = 0.3$ were obtained in Table 3-5, respectively.

Graphical representation and linear regressions of the previous numerical results concerning two Lindley distributions are shown in the Fig. 1a-d.

Similarly, graphical representation and linear regressions of the previous numerical results concerning mixture Lindley distribution are shown in the Fig. 2a-d.

DISCUSSION

The main objective of this study is to clarify how to combine between two Lindley distributions containing different parameters and obtain a mixture Lindley distribution included a mixing parameter. Moreover, estimating the unknown parameters at different values of the mixing

Table 1: Estimated values $\hat{\theta}_1, \hat{\theta}_2, \hat{\alpha}$ at mixing parameter p of mixture Lindley distribution

	Estimated values		
Mixing parameter	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\alpha}$
p = 0.1, 1- p = 0.9	0.771	0.771	2.072
p = 0.3, 1- p = 0.7	0.771	0.771	2.666
p = 0.5, 1- p = 0.5	0.771	0.771	4.741

Table 2: Two Lindley distributions at the parameters $\theta_1 = 1$, $\theta_2 = 1$, $\alpha = 2$

x	$f_1(x)$	$f_2(x)$	$F_1(x)$	$F_2(x)$	$R_1(x)$	$R_2(x)$	$h_1(x)$	$h_2(x)$
0.07	0.4988	0.3543	0.0349	0.0240	0.9651	0.9760	0.5169	0.3630
0.19	0.4920	0.3804	0.0944	0.0682	0.9056	0.9318	0.5433	0.4082
0.34	0.4768	0.3985	0.1672	0.1268	0.8328	0.8732	0.5726	0.4565
0.65	0.4306	0.4002	0.3082	0.2517	0.6918	0.7483	0.6226	0.5348
0.83	0.3989	0.3866	0.3829	0.3226	0.6171	0.6774	0.6446	0.5708
0.99	0.3697	0.3690	0.4444	0.3831	0.5556	0.6169	0.6655	0.5983
1.32	0.3098	0.3241	0.5565	0.4977	0.4436	0.5023	0.6987	0.6453
1.57	0.2673	0.2871	0.6286	0.5742	0.3714	0.4258	0.7198	0.6742
1.75	0.2389	0.2606	0.6741	0.6234	0.3259	0.3766	0.7333	0.6923
1.97	0.2070	0.2296	0.7231	0.6773	0.2769	0.3227	0.7481	0.7118
2.19	0.1785	0.2007	0.7655	0.7246	0.2346	0.2754	0.7613	0.7289
2.45	0.1488	0.1697	0.8079	0.7727	0.1921	0.2278	0.7752	0.7468
2.77	0.1181	0.1366	0.8505	0.8216	0.1496	0.1784	0.7903	0.7658
2.98	0.1010	0.1178	0.8735	0.8482	0.1266	0.1518	0.7991	0.7767
3.17	0.0875	0.1027	0.8914	0.8692	0.1086	0.1308	0.8065	0.7858
3.49	0.0684	0.0811	0.9162	0.8985	0.0838	0.1015	0.8178	0.7995
3.86	0.0511	0.0612	0.9382	0.9247	0.0618	0.0753	0.8293	0.8134
4.13	0.0412	0.0496	0.9507	0.9396	0.0493	0.0604	0.8368	0.8223
4.77	0.0244	0.0297	0.9712	0.9645	0.0358	0.0355	0.8522	0.8405
4.93	0.0214	0.0261	0.9749	0.9690	0.0251	0.0309	0.8556	0.8444

Table 3: Mixture Lindley distributions at the parameters $\theta_1 = 1$, $\theta_2 = 1$, $\alpha = 2$ and $p = 0.1$

x	f (x)	F (x)	R (x)	h (x)
0.07	0.3688	0.0252	0.9748	0.3783
0.19	0.3916	0.0709	0.9291	0.4215
0.34	0.4064	0.1309	0.8691	0.4374
0.65	0.4033	0.2574	0.7426	0.5431
0.83	0.3879	0.3287	0.6713	0.5778
0.99	0.3692	0.3893	0.6107	0.6046
1.32	0.3227	0.5037	0.4963	0.6502
1.57	0.2851	0.5796	0.4204	0.6782
1.75	0.2585	0.6286	0.3714	0.6960
1.97	0.2274	0.6819	0.3181	0.7149
2.19	0.1985	0.7287	0.2713	0.7317
2.45	0.1676	0.7763	0.2237	0.7492
2.77	0.1423	0.8245	0.1817	0.7505
2.98	0.1162	0.8508	0.1492	0.7788
3.17	0.1012	0.8714	0.1286	0.7896
3.49	0.0799	0.9003	0.0997	0.8014
3.86	0.0602	0.9261	0.0739	0.8146
4.13	0.0488	0.9407	0.0593	0.8229
4.77	0.0292	0.9652	0.0348	0.8391
4.93	0.0256	0.9696	0.0304	0.8421

Table 4: Mixture Lindley distributions at the parameters $\theta_1 = 1$, $\theta_2 = 1$, $\alpha = 2$ and $p = 0.5$

x	f (x)	F (x)	R (x)	h (x)
0.07	0.4265	0.0295	0.9705	0.4395
0.19	0.4362	0.0813	0.9187	0.4748
0.34	0.4377	0.1470	0.8530	0.5132
0.65	0.4154	0.2800	0.7200	0.5770
0.83	0.3928	0.3528	0.6472	0.6069
0.99	0.3694	0.4138	0.5862	0.6302
1.32	0.3170	0.5271	0.4729	0.6768
1.57	0.2772	0.6014	0.3986	0.6955
1.75	0.2498	0.6488	0.3512	0.7113
1.97	0.2183	0.7002	0.2978	0.7285
2.19	0.1896	0.7451	0.2549	0.7435
2.45	0.1592	0.7903	0.2097	0.7598
2.77	0.1273	0.8360	0.1640	0.7769
2.98	0.1094	0.8609	0.1391	0.7869
3.17	0.0951	0.8803	0.1197	0.7952
3.49	0.0748	0.9074	0.0926	0.8078
3.86	0.0562	0.9314	0.0686	0.8206
4.13	0.0454	0.9451	0.0549	0.8288
4.77	0.0271	0.9679	0.0321	0.8463
4.93	0.0237	0.9719	0.0281	0.8494

Table 5: Mixture Lindley distributions at the parameters $\theta_1 = 1$, $\theta_2 = 1$, $\alpha = 2$ and $p = 0.3$

x	f (x)	F (x)	R (x)	h (x)
0.07	0.3977	0.0274	0.9726	0.4089
0.19	0.4139	0.0761	0.9239	0.4479
0.34	0.4221	0.1389	0.8611	0.4902
0.65	0.4094	0.2687	0.7313	0.5598
0.83	0.3903	0.3408	0.6592	0.5921
0.99	0.3693	0.4016	0.5984	0.6171
1.32	0.3199	0.5154	0.4846	0.6601
1.57	0.2812	0.5905	0.4095	0.6867
1.75	0.2541	0.6387	0.3613	0.7033
1.97	0.2229	0.6911	0.3089	0.7216
2.19	0.1940	0.7369	0.2631	0.7374
2.45	0.1635	0.7833	0.2167	0.7545
2.77	0.1311	0.8303	0.1697	0.7725
2.98	0.1128	0.8557	0.1443	0.7817
3.17	0.0982	0.8759	0.1241	0.7913
3.49	0.0773	0.9039	0.0961	0.8044
3.86	0.0582	0.9288	0.0712	0.8174
4.13	0.0471	0.9429	0.0571	0.8249
4.77	0.0282	0.9666	0.0334	0.8443
4.93	0.0247	0.9708	0.0292	0.8459

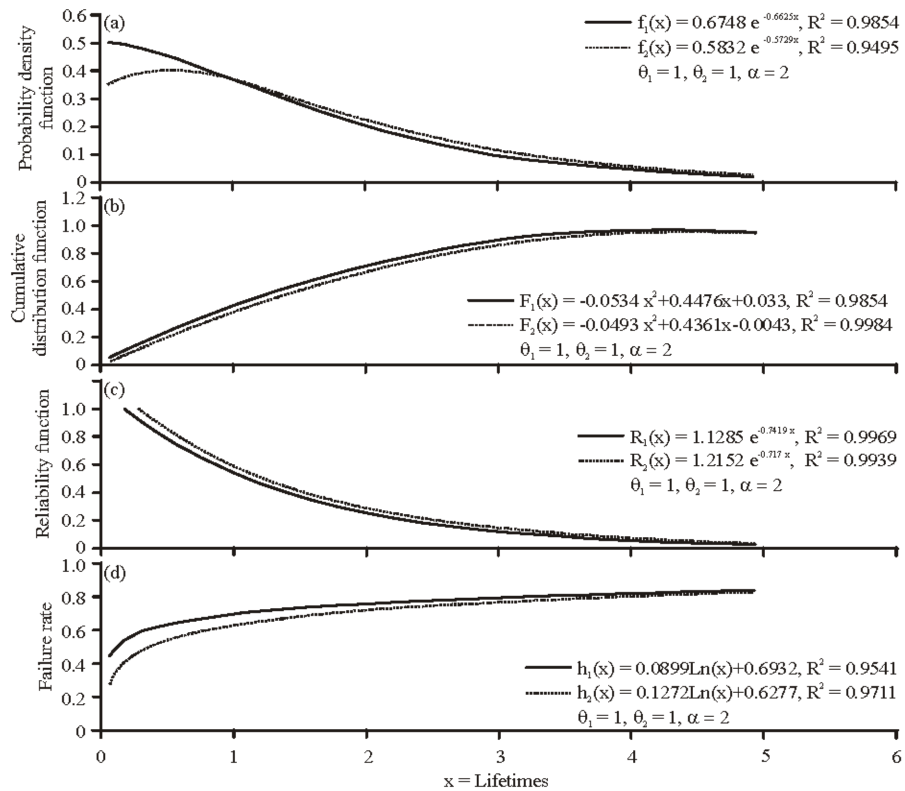

Fig. 1(a-d): (a) Probability density functions, (b) Cumulative distribution functions, (c) Reliability functions and (d) Failure rates of two Lindley distributions

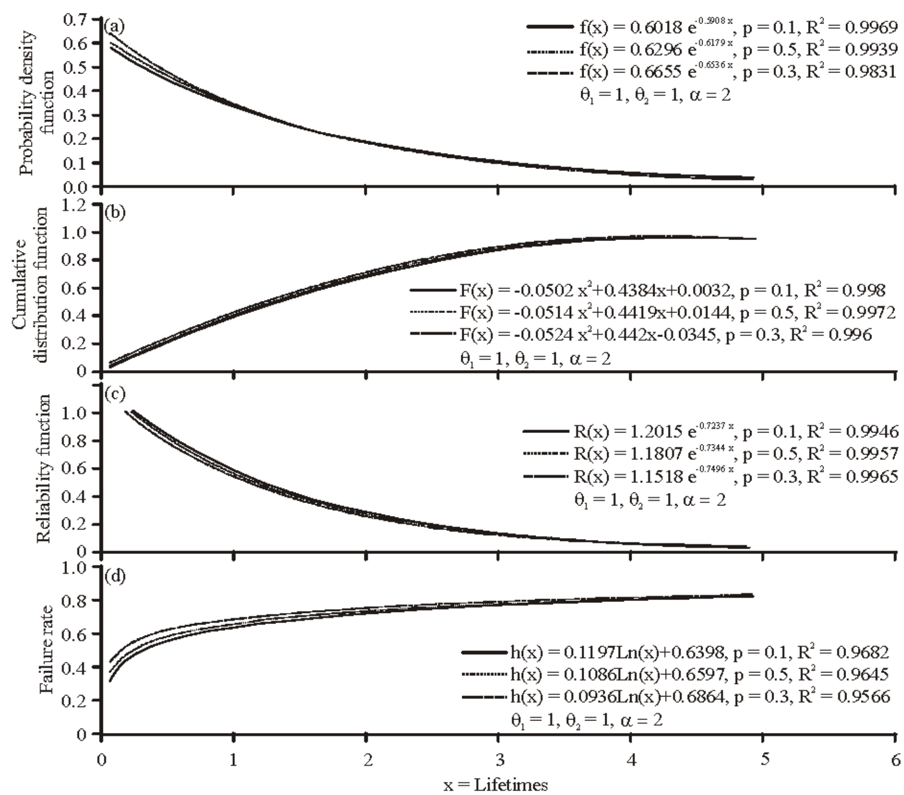

Fig. 2(a-d): (a) Probability density functions, (b) Cumulative distribution functions, (c) Reliability functions and (d) Failure rates of mixture Lindley distributions

Table 6: Tests of normality (For reliability R and failure rate h)

Groups	Shapiro-Wilk Sig.
R	
Group 1	0.031
Group 2	0.027
Group 3	0.020
h	
Group 1	0.050
Group 2	0.059
Group 3	0.088

Table 7: Kruskal Wallis test (For effecting mixing parameter p on R and h)

Groups	df	Asymp. Sig.
R	2	0.930
h	2	0.703

Table 8: Kruskal Wallis test (For reliabilities R, R_1, R_2 and failure h, h_1, h_2)

Groups	df	Asymp. Sig.
R at p = 0.1, R_1, R_2	2	0.927
R at p = 0.5, R_1, R_2	2	0.927
R at p = 0.3, R_1, R_2	2	0.922
h at p = 0.1, h_1, h_2	2	0.545
h at p = 0.5, h_1, h_2	2	0.569
h at p = 0.3, h_1, h_2	2	0.535

parameter and obtaining the probability density function, cumulative distribution function, reliability function and failure rate for the mixture Lindley distribution.

Nath and Bhattacharjee (2012) used in their study the pattern mixture modeling, while Wasinart *et al.* (2013) presented discussion on mixture of Weibull hazard rate but mixture Logit model was applied by Nugraha (2011). Alharpy and Ibrahim (2013) performed parametric tests under Weibull distribution, Zaman *et al.* (2005) used Chi-square mixture of gamma distribution while, MSR method used in reliability evaluation was applied by Yu *et al.* (2013). In this study the mixture Lindley distribution with four parameters was obtained from combining two Lindley distributions and gives us accuracy results furthermore, the applied method in this study can be implemented on more than two Lindley distributions of the components in the electronic system.

On the other hand, Awikunprasert and Sittiprapaporn (2013) performed their study on cancer patients, while Pan *et al.* (2014) carried out the study on heat dissipation and thermo-mechanical reliability study. Reshid and Majid (2011) have done study on a gas fueled cogenerated power plant, but in this study a statistical method was applied on the lifetime of electronic system contains 20 electronic unit.

Effect of the mixing parameter on the reliability R and failure rate h of mixture Lindley distribution: Suppose three groups, group 1, 2 and 3 represent the values of R and h at p = 0.1, p = 0.5 and p = 0.3, respectively, such that the size of each group is 20 then perform Shapiro-Wilk tests of normality on the three groups for each reliability and failure rate of mixture Lindley distribution, the results were shown in Table 6.

The values of the significance level in the shows that the values of reliability was taken from a population does not follow a normal distribution, while the values of failure rate was taken from a population with a normal distribution therefore, Kruskal Wallis test was applied to determine the effecting of missing parameter p on both reliability and failure rate of mixture Lindley distribution, the results shown in Table 7.

From the previous result it is clear that the value of the significance level for each of the two groups reliability and failure rate greater than 0.05 and as a result the values of the mixing parameter p does not have an impact on both the reliability and failure rate. On the other hand, by using SPSS program partial correlation between the reliability and failure rate of mixture Lindley distribution with mixing parameter p was obtained and equal to -0.992. This means that there is a strong inverse correlation between R and h, this is completely logical and good result.

Comparing the reliability and failure rate in both of two Lindley distributions and mixture Lindley distribution: In this section, the reliability R of mixture Lindley distribution at different values of mixing parameter p and reliabilities R_1, R_2 of two Lindley distributions were compared by Kruskal Wallis test. Similarly, failure rate h with different values of p and failure rates h_1, h_2 were compared by the same test, the results are shown in Table 8.

It is clear that the levels of significance in the previous table are greater than 0.05 in each comparison and this means that there are no significant differences between reliability and failure rate in both distributions.

CONCLUSION

The previous numerical results show that the estimated parameters of the mixture Lindley distribution with different values of mixing parameter were good estimators because there are no significant differences for each of the reliability and failure rate in two Lindley distributions of two components in the electronic unit and mixture Lindley distribution with different values of mixing parameter. In addition, the reliability and failure rate in the mixture Lindley distribution are not affected by the different values of the parameter missing, as there is a strong inverse relationship between each of the reliability and failure rate and this means that the mixture Lindley distribution achieved a good results. The method used in this study to deduce the mixture Lindley distribution by the combining two Lindley distribution enable the researchers to apply it in the field of specialization with other distributions in order to deal with one mixture distribution, instead of two or more distributions and obtain the reliability and failure rate of the electronic system which contains any number of electronic units which each unit include different numbers of electronic components.

REFERENCES

Alharpy, A.M. and N.A. Ibrahim, 2013. Parametric tests for partly interval-censored failure time data under Weibull distribution via multiple imputation. J. Applied Sci., 13: 621-626.

Awikunprasert, C. and W. Sittiprapaporn, 2013. Validity and reliability of Thai hospice quality of life index-revised version for cancer patients. Asian J. Sci. Res., 6: 819-823.

Nath, D.C. and A. Bhattacharjee, 2012. Pattern mixture modeling: An application in anti diabetes drug therapy on serum creatinine in type 2 diabetes patients. Asian J. Math. Stat., 5: 71-81.

Nugraha, J., 2011. Mixed logit model on multivariate binary response using maximum likelihood estimator and generalized estimating equations. Asian J. Math. Stat., 4: 113-127.

Pan, K., Y. Guo, G. Ren and H. Lin, 2014. Heat dissipation and thermo-mechanical reliability study for multi-chip module high power LED integrated packing with through silicon vias. Inform. Technol. J., 13: 1316-1322.

Reshid, M.N. and M.A.A. Majid, 2011. A multi-state reliability model for a gas fueled cogenerated power plant. J. Applied Sci., 11: 1945-1951.

Wasinart, S., W. Bodhisuwan, P. Zeephongsekul and A. Thongtheeraparp, 2013. A mixture of Weibul hazard rate with power variance function frailty. J. Applied Sci., 13: 103-110.

Yu, J., H.L. Zhong, W.S. Tang, T.T. Wang, Z.G. Li and L. Qiao-Chan, 2013. Applied research on CMSR method used in reliability evaluation of the CNC machine tools. J. Applied Sci., 13: 5284-5286.

Zaman, M.R., M.K. Roy and N. Akhter, 2005. Chi-square mixture of gamma distribution. J. Applied Sci., 5: 1632-1635.