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# Evolution of Regional City Size Distribution in an Indian State: Kerala (1951-2001) Using Markovian Approach

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ABSTRACT
The purpose of this study is to analyze the evolution of city size distribution over time for a regional urban system. The evolution of cities and towns in Indian state is studied: Kerala for the period of 1951-2001. The cross-sectional distribution of urban population growth process is modeled as a first order stationary Markov chain. Based on the methodology two main results are obtained. First, intra-distribution movements have taken place in the ranking of cities and towns during the period of 1951-2001. Secondly, this evolution has not been homogeneous, with a divergent pattern of growth for the entire data relating to the regional city size distribution. The research framework proposed and utilized here is able to portray the movements of cities and towns of a regional urban system and the framework can be used to evaluate regional urban policies for the economic development of the region.

**Key words:** Urban system, city-size distribution, markov chain, transition probability matrix, ergodic probability distribution

## INTRODUCTION

Urban growth is an economic phenomenon inextricably linked with the process of urbanization. The size distribution of cities is the result of the patterns of urbanization which result in city growth and city creation. Based on the pioneering work of Auerbach (1913), a substantial literature has arisen on city size distribution. Zipf (1949) proposed that the distribution of city sizes takes the special form of Pareto Distribution. Rosen and Resnick (1980) documented this regularity in the 1980's for a wide range of countries. Soo (2007) has updated this study using modern data and more sophisticated econometric techniques. The major findings of this literature are the robustness of this phenomenon both over time and across countries. But this approach documents does not provide information on city mobility within the size distribution. This deficiency has been corrected based on the approach proposed by Quah (1993).

This method has been applied to the city size distribution of various countries by Eaton and Eckstein (1997), Black and Henderson (2003), Lanaspa *et al.* (2003), Anderson and Ge (2005), Le Gallo and Chasco (2008) and Xu and Zhu (2009). Kumar and Subbarayan (2014), studied the intra-distribution movements of a regional city size distribution of an Indian State: Andhra Pradesh (1951-2001). In this study, the evolution of city size distributions analyze are over time in a regional urban system. i.e., for an Indian state viz., Kerala. The urban system has constant flux with changing groups and city migration across existing and newly created groups. The long term transition across all groups reveals striking system structure.

## URBAN SCENARIO IN KERALA

**Kerala and its demographic profile:** Kerala is one of the 28 states of India and covers an area of 38,863 square kilometers. It is the twenty-first largest state in India by area

Table 1: A Demographic profile of Kerala and India based on 2001 Census

| Events | Kerala | India |
|---|---|---|
| Total population (millions) | 31.84 | 1028.73 |
| Decadal population growth (%) | 9.43 | 21.54 |
| Population density (per sq. km.) | 819.00 | 324.00 |
| Sex ratio | 1058.00 | 933.00 |
| Literacy rate (%) | 90.86 | 64.84 |
| Urban to total population (%) | 25.97 | 27.70 |

Table 2: Population of Kerala by sex and residence 2001

| Kerala | Male | Female | Total population | Sex ratio |
|---|---|---|---|---|
| Urban | 40,17,332 | 42,49,593 | 82,66,925 | 1058 |
| Rural | 1,14,51,282 | 121,23,167 | 2,35,74,449 | 1059 |
| Total | 1,54,68,614 | 1,63,72,760 | 3,18,41,374 | 1058 |

Table 3: Kerala-city size distribution (No. of cities/towns)

| | Class I | II | III | IV | V | VI | |
|---|---|---|---|---|---|---|---|
| Census year | >1,00,000 | 50,000-1,00,000 | 20,000-50,000 | 10,000-20,000 | 5,000-10,000 | <5,000 | Total |
| 1951 | 4 | 3 | 10 | 21 | 6 | 1 | 45 |
| 1961 | 4 | 4 | 22 | 17 | 4 | 1 | 52 |
| 1971 | 5 | 8 | 32 | 11 | 3 | 1 | 60 |
| 1981 | 6 | 8 | 55 | 14 | 4 | 1 | 88 |
| 1991 | 9 | 17 | 69 | 34 | 10 | 1 | 140 |
| 2001 | 10 | 24 | 72 | 37 | 15 | 1 | 159 |

and 12th most populous state. Kerala lies in between the high Western Ghats on the east and the Arabian Sea on the west, the width of the state varies from 35-120 km. According to the geographical features the state can be divided into hills, valleys, midland plains and coastal belt. Kerala is the 9th largest contributor to Indian GDP. A demographic profile of Kerala based on 2001 census is given in Table 1.

**Basic statistics of urban population Kerala:** The basic statistics of urban and rural population according to 2001 census is given in Table 2. The total number of urban dwellers in Kerala as per the population total of census in India 2001 is 82,66,925. Males number is 40,17,332 while Females total number is 42,49,593. The total number of urban dwellers in the country is 28,53,54,954 consisting of 15,01,35,894 males and 135,219,060 females. The percentage of urban population to total population in the country works out to be 27.78% as against the ratio of 25.97% in Kerala. Kerala stands 12th in terms of its urban population.

**Urban size under indian census:** India has very rich source of information for urban studies. The census volumes, both at the national and state and district levels, provide a mine of information for rural and urban places for period of 100 years. It is also main source of information for temporal studies focusing in the recent past. The census periods covered are: 1951, 1961, 1971, 1981, 1991 and 2001.

Urban population by size classification is based on the following:

| Class | Population |
|---|---|
| I | > 1,00,000 |
| II | 50,000-1,00,000 |
| III | 20,000-50,000 |
| IV | 10,000-20,000 |
| V | 5,000-10,000 |
| VI | < 5,000 |

The number of cities/towns for each census year under six classes is given in Table 3. It is important to note that the process of urbanization in Kerala primarily consists of Class II, III, IV and V cities/towns. The share of these three classes has been growing up systematically during 1951-2001. It is also observed that the share of smaller towns' viz., Class VI is negligible. It is also observed that the share of Class I cities has considerably increased during 1991 and 2001.

## MATERIALS AND METHODS

**Based on Markov Chain:** Formally, denote $F_t$ the cross-sectional distribution of city size distribution at time 't' of the region. Define a set of K for different size classes which provide a discrete approximation of the population distribution. Initially assumed that the frequency of the distribution follows a first-order stationary Markov process. In this case, the evolution of city size distribution is represented by a transition probability matrix M, in which element (i, j) indicates the probability that a region, that was in class 'i' at time 't' ends up in class 'j' in the following period.

Formally, the (K,1) vector $F_t$, indicating the frequency of the region in each class at time t, is described by the following equation:

$$F_{t+1} = MF_t \quad (1)$$

where, M is the (K,K) transition probability matrix representing the transition between the two distributions as follows:

$$M = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1K} \\ p_{21} & p_{22} & \cdots & p_{2K} \\ & & & \\ P_{K1} & P_{K2} & \cdots & P_{KK} \end{bmatrix} \quad (2)$$

where, each element:

$$p_{ij} \geq 0, \sum_{j=1}^{K} p_{ij} = 1$$

The stationary transition probabilities $p_{ij}$ capture the probability that a region in class t-1 ends up in class j in t. The elements of M can be estimated from the observed frequencies in the changes of class from one period to another. Thus, following Hamilton (1994), the maximum likelihood estimator of $p_{ij}$ is given as:

$$\hat{p}_{ij} = \frac{n_{ij}}{n_i} \quad (3)$$

where, $n_{ij}$ is the total number of cities/towns moving from class 'i' in decade t-1 to class 'j' in the immediate following decade t over all the transitions and $n_i$ is the total sum of cities/towns ever in 'i' over the transitions.

Defining $M_{i,i+1}$ as the actual transition matrix from period i to i+1, matrix M is estimated by computing the average of $M_{i,i+1}$ for all periods with respect to the data. If the transition probabilities are stationary, that is, if the probabilities between two classes are time-invariant, then:

$$F_{t+s} = M^s F_t \quad (4)$$

In this framework, it is possible to determine the ergodic distribution (also called the long-term, long-run, equilibrium a steady state distribution) of $F_t$, characterized when s tend toward infinity in Eq. 4, that is to say, once the changes represented by matrix M are repeated an arbitrary number of times. Such a distribution exists if the Markov chain is regular, that is, if and only if, for some m, $M^m$ has no zero entries. In this case, the TPM converges to a limiting matrix M* of rank 1. The existence of an ergodic distribution, F*, is then characterized by:

$$F^*M = F^* \quad (5)$$

where, the vector F* describes the future distribution of the regional city size distribution if the movements observed in the reference period. Each now of $M^t$ trends to the limit distribution at $t \to \infty$.

**Division of cities and towns in Kerala state:** Different class sizes are presented the city size distribution of Kerala (1951-2001). The use of Markov chain requires making the space of states discrete, in order to manage a finite number of categories. Based on Eaton and Eckstein (1997) approach, six states are defined and related with the average population for each census period.

These states are as follows:

- Less than 0.30 of the mean
- Between 0.30 and 0.50 of the mean
- Between 0.50 and 0.75 of the mean
- Between 0.75 and 1.00 of the mean
- Between the mean and twice the mean
- More than twice the mean

By using method different transition matrices and average transition matrix are formed.

## RESULTS

**Formation of transition probability matrices and the average transition probability matrix:** The transition probability matrix computed for the census periods 1951-61, 1961-71, 1971-81, 1981-91 and 1991-2001 are given in Table 4. The average transition probability matrix estimated based on the above transition probability matrices is presented in Table 5.

Table 4: Transition probability matrix

| | Cell's upper endpoint | | | | | |
|---|---|---|---|---|---|---|
| Time period | 0.3 | 0.5 | 0.75 | 1 | 2 | α |
| **$M_{(1,2)}$ = 1951-1961** | | | | | | |
| 0.3 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.5 | 0.1429 | 0.8571 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.75 | 0.0000 | 0.0000 | 0.7692 | 0.0769 | 0.1538 | 0.0000 |
| 1 | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 |
| 2 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 |
| α | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.1667 | 0.8333 |
| **$M_{(2,3)}$ = 1961-1971** | | | | | | |
| 0.3 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.5 | 0.2000 | 0.8000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.75 | 0.0000 | 0.3846 | 0.6154 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2 | 0.0000 | 0.0000 | 0.0000 | 0.3333 | 0.6667 | 0.0000 |
| α | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 |
| **$M_{(3,4)}$ = 1971-1981** | | | | | | |
| 0.3 | 0.8000 | 0.2000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.5 | 0.0000 | 0.8500 | 0.1500 | 0.0000 | 0.0000 | 0.0000 |
| 0.75 | 0.0000 | 0.0000 | 0.9375 | 0.0625 | 0.0000 | 0.0000 |
| 1 | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.0000 | 0.0000 |
| 2 | 0.0000 | 0.0000 | 0.0000 | 0.3750 | 0.6250 | 0.0000 |
| α | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 |
| **$M_{(4,5)}$ = 1981-1991** | | | | | | |
| 0.3 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.5 | 0.2308 | 0.7692 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.75 | 0.0000 | 0.6000 | 0.4000 | 0.0000 | 0.0000 | 0.0000 |
| 1 | 0.0000 | 0.0000 | 0.2143 | 0.7857 | 0.0000 | 0.0000 |
| 2 | 0.0000 | 0.0000 | 0.0000 | 0.2500 | 0.6250 | 0.1250 |
| α | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 |
| **$M_{(5,6)}$ = 1991-2001** | | | | | | |
| 0.3 | 1.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.5 | 0.0000 | 0.8718 | 0.1282 | 0.0000 | 0.0000 | 0.0000 |
| 0.75 | 0.0000 | 0.0000 | 0.8857 | 0.1143 | 0.0000 | 0.0000 |
| 1 | 0.0000 | 0.0000 | 0.0000 | 0.7059 | 0.2941 | 0.0000 |
| 2 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.9286 | 0.0714 |
| α | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 |

Table 5: Average Transition probability matrix (1951-2001)

| | 2001 | | | | | |
|---|---|---|---|---|---|---|
| Time period | 0.3 | 0.5 | 0.75 | 1 | 2 | α |
| **M = 1951** | | | | | | |
| 0.3 | 0.9600 | 0.0400 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 0.5 | 0.1150 | 0.8300 | 0.0560 | 0.0000 | 0.0000 | 0.0000 |
| 0.75 | 0.0000 | 0.1960 | 0.7220 | 0.0510 | 0.0310 | 0.0000 |
| 1 | 0.0000 | 0.0000 | 0.4430 | 0.4980 | 0.0590 | 0.0000 |
| 2 | 0.0000 | 0.0000 | 0.0000 | 0.1920 | 0.7690 | 0.0390 |
| α | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0330 | 0.9670 |

Table 6: Average of ergodic probabilities for cities and towns in Kerala

| Cell's upper point | Frequency | Probability |
|---|---|---|
| 0.3 | 10 | 0.6630 |
| 0.5 | 18 | 0.2310 |
| 0.75 | 6 | 0.0650 |
| 1 | 2 | 0.0120 |
| 2 | 4 | 0.0140 |
| ∞ | 5 | 0.0150 |

**Movements of cities/towns during 1951-2001:** First, the persistence which is given by the values in the diagonal, is greater for smaller towns and larger cities. In other words, the probability of moving out of the initial state is lower for the smaller towns and larger cities. Secondly, the possibility of parallel growth has to be rejected, since several values in the diagonal are significantly different from one which clearly indicates that the size distribution of cities and towns in the region have undergone significant intra-distribution movements during 1951-2001. Thirdly, medium, sized towns in Class IV had the smallest probability of persistence.

**Ergodic probabilities for cities and towns of Kerala:** Using the average transition probability matrix 'M' ergodic probability distribution is computed and the same is given in Table 6. Concentration of frequencies around 1 would imply convergence to the mean. The results show no such convergence for the entire data relating to the regional city size distribution for Kerala state. It is significant noted that about 75% of the cities and towns will be below the average and the balance 25% will be above the average in the subsequent decades.

## DISCUSSION

The long-run analysis of the size distribution of cities and towns in a region is an interesting exercise and helps us to know what might be the future behaviour. In this study, analysis have been carried out for a region in an Indian state: Kerala during the period 1951-2001. Intra-distribution movements have taken place in the ranking of cities and towns during the period 1951-2001. Kumar and Subbarayan (2014) have stated that the persistence is greater for Class I and II cities and Class VI towns. In the case of regional city size distribution for Kerala the persistence is greater for Class I cities and Class VI towns. But in the case of medium sized cities and towns in both the region (Andhra Pradesh and Kerala) have a tendency to move up in their size hierarchy.

This proves the major role played by medium sized cities in the process of urbanization. The tendency to move up by these medium sized cities is more for Kerala than for Andhra Pradesh. The findings of the study support to great extent the results obtained by Kumar and Subbarayan (2014) in respect of intra-distribution dynamics of cities and towns in Andhra Pradesh. In general terms, movements up are slower than movements down for all class of cities and towns in both the regions. The research framework proposed and utilized in this study is able to portray the movements of cities and towns of a regional urban system and the framework can be used to evaluate regional urban policies for the economic development of the region.

## REFERENCES

Anderson, G. and Y. Ge, 2005. The size distribution of Chinese cities. Region. Sci. Urban Econ., 35: 756-776.

Auerbach, F., 1913. Das gesetz der bevolkerungskonzentration. Petermann's Geographische Mitteilungen, 59: 74-76.

Black, D. and V. Henderson, 2003. Urban evolution in the USA. J. Econ. Geogr., 3: 343-373

Eaton, J. and Z. Eckstein, 1997. Cities and growth: Theory and evidence from France and Japan. Regional Sci. Urban Econ., 27: 443-474.

Hamilton, J.D., 1994. Time Series Analysis. Princeton University Press, Princeton, USA., ISBN: 9780691042893, Pages: 799.

Kumar, G. and A. Subbarayan, 2014. The temporal dynamics of regional city size distribution: Andhra Pradesh (1951-2001). J. Math. Stat., 10: 221-230.

Lanaspa, L., F. Pueyo and F. Sanz, 2003. The evolution of Spanish urban structure during the twentieth century. Urban Stud., 40: 567-580.

Le Gallo, J. and C. Chasco, 2008. Spatial analysis of urban growth in Spain, 1900-2001. Empirical Econ., 34: 59-80.

Quah, D., 1993. Empirical cross-section dynamics in economic growth. Eur. Econ. Rev., 37: 426-434.

Rosen, K.T. and M. Resnick, 1980. The size distribution of cities: An examination of the Pareto law and primacy. J. Urban Econ., 8: 165-186.

Soo, K.T., 2007. Zipf's law and urban growth in Malaysia. Urban Stud., 44: 1-14.

Xu, Z. and N. Zhu, 2009. City size distribution in China: Are large cities dominant? Urban Stud., 46: 2159-2185.

Zipf, G.K., 1949. Human Behavior and the Principle of Leeast Effort. Addison-Wesley, Cambridge.