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Research Article Design of a Fuzzy Zone Control Chart for Improving the Process Variation Monitoring Capability

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Abstract

Control charts have become one of the most commonly used tools to monitor process variations in manufacturing. Traditional zone control charts cut the chart into a number of crisp zones and the appropriate zone box score is calculated according to the observations plotted in the related zone and these scores are then used to produce a cumulative value to monitor of the mean shift of process variation. However, two problems frequently occur when using zone control charts. First, too great a difference in the zone box scores of the observed values around both sides of the zone boundary, even if the two observed values are very close. Second, while the score of any observed value in the same zone is the same, in real life process variations are a matter of degree and thus the values should be different within each zone. However, when using a traditional zone control chart, when two sampling statistics values fall in the same zone, the recorded scores will be equal and thus the difference between the two observations is not reflected. This study uses fuzzy zones instead of crisp ones to describe the monitored zones and a fuzzy rule to construct the corresponding fuzzy zone control chart. The results of this study show that fuzzy zone control chart proposed in this study can achieve better performance against earlier literature regarding the monitoring of process variations.

Key words: Control chart, fuzzy set theory, fuzzy rule, SN, LN, MN

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Data Availability: All relevant data are within the paper and its supporting information files.

INTRODUCTION

Due to differences in equipment, materials, environment, operators and other factors, no two products can be manufactured that are exactly the same. In quality management, such variations are distinguished into random and non-random ones. Statistical Process Control (SPC) analyzes the quality characteristics data to determine changes in processes or the occurrence of non-random variations, in order to correct the related processes before inadequate products are produced. Control charts are one of the major SPC tools used for in this purpose¹.

The run sum control chart was first proposed by Roberts² and discussed in depth by Reynolds³, who stated that it is similar to a zone control chart with the only difference being the way that the appropriate zone box score is calculated. Cheng⁴ noted that the appropriate zone box scores of both zone control chart and run sum control charts are integers, meaning that the cumulative value of the appropriate zone box score is also an integer. The minimum increment of the zone box score is 1 and thus it is difficult to obtain appropriate monitoring capability by adjusting the threshold value for short-run process full distance control charts (i.e., a short run \bar{x} -R chart)^{5,6}. One way of using the average number control chart to control the defect rate or move beyond a specified ratio is to use an acceptance control chart^{7,8,9}. In this method, a multivariate control chart simultaneously monitors more than two quality characteristics and a number of study has examined applications of statistical technologies in control charts^{10,11,12}.

The zone control chart examined in this study was first proposed by Jaehn¹³. When a process is in the in-control state and the mean number \bar{x} of the statistical samples being monitored is assumed of the normal distribution $N(\mu_0, \frac{\sigma_0^2}{\sigma_0})$ the

zone control chart will use the control chart's center line μ_0 and variance σ_0^2/n to divide the chart into eight zones and will then specify a zone box score S for each zone based on where the sampling observation \bar{x} falls in the zone box score. The relationships between the zones and their corresponding zone box scores are as shown in Fig. 1. The numbers in the circles of the two fall points as shown in the figure are the appropriate cumulative zone box scores. The process is in the in-control state when the observed value \bar{x} falls between zone A and H, similar to the general average number control chart (\bar{x} chart) and warnings will be given if it is beyond the control limit. In addition to monitor small shifts, after each sampling, the zone control chart will calculate the zone box score of the continuous observed values. When the cumulative value of the zone box score is greater than or equal to the set threshold value, the control chart will produce warnings to monitor the abnormal phenomena occurring in the process.

Cheng⁴ pointed out that one disadvantage of the zone control chart is that the specified zone box scores are all integers and thus the accumulated value of the zone box score is also to be an integer. Since the minimal increment is 1, it is not easy to adjust the threshold value to achieve a good monitoring capability of the non-random variation. Therefore, the traditional zone control chart proposed by Jaehn¹³, two common problems arise based on the calculated values of \bar{x} . First, if the zone box scores of the observed values around the boundary differ too greatly, for example, when the \bar{x} values are 1.99 and 2.01, the recorded fall points of the appropriate zone box scores using the zone control chart approach will be 2 and 4, respectively, since the zones do not overlap. Therefore, even if the two observed values are very close, if they fall around both sides of the zone boundary, the specified zone box score will differ significantly. Second, the



Fig. 1: Relationship between zones and the appropriate zone box score (using $\bar{x} \sim N(0,1)$ as an example)

inability to distinguish the two observation values which are quite different but fall in the same zone. For example, if the sample average numbers for \bar{x} are 1.05 and 1.99, the recorded zone box scores using the zone control chart method will both be 2 and thus the difference between the two will not be reflected. Therefore, specifying the zone box score can actually influence the effectiveness of using the zone control chart to monitor the mean shift.

Since fuzzy theory was first proposed by Zadeh¹⁴, a large number of fuzzy items with unclear boundaries in nature have been described using fuzzy sets. More closely related to the aims of the current study, Karwowski and Evans¹⁵ proposed three reasons to apply fuzzy theory to production management, while Guiffrida and Nagi¹⁶ reviewed studies on the application of fuzzy theory to quality control. With regard to the application of fuzzy theory to control charts, Bradshaw¹⁷ included the idea of various grades of defects and modified an acceptance control chart based on fuzzy theory. Wang and Raz¹⁸ noted that it is not appropriate in most cases to classify product quality into qualified and unqualified using a defect rate control chart and instead there should be a number of grades of guality between valuable and worthless items. Such quality grades can be described using the linguistic variables of the fuzzy grade. On the other hand, Kanagawa et al.¹⁹ developed a new linguistic control chart to monitor process average numbers and variations based on the premise of the known distribution of semantic data. In addition, Laviolette et al.²⁰ changed the certain classes used in Marcucci²¹ to fuzzy ones and built corresponding control charts using fuzzy theory methods. Finally, there are also a number of articles that apply fuzzy theory to control charts²²⁻³¹.

The present study aims to describe the original zones using fuzzy sets and produce the specified zone box scores of the observed values before inferring the monitoring mechanism to address the two common problems that arise with traditional zone control charts, which are: (1) Large differences in the zone box scores of the observed values around the boundary and (2) Identical scores for any observed values in the same zone with different levels of process variation, leading to the same recorded zone box scores and thus not reflecting the difference between the two. The fuzzy zone control chart proposed in this study is expected to more accurately monitor real quality characteristics during process control, thus reducing the costs associated with unqualified products.

MATERIALS AND METHODS

This study aims to describe the original zones using fuzzy sets to produce the specified zone box scores of the observed

values and thus derive the appropriate monitoring mechanism to address some of the weaknesses of traditional zone control charts. The proposed fuzzy zone control chart can more accurately reflect the actual situations being observed and the details of the method are described in the following subsections.

Traditional zone control charts: To illustrate a traditional zone control chart, if the process is assumed to be in the in-control state and the sample average number \bar{x} is assumed to have a normal distribution of $N(\mu_0, \frac{\sigma_0^2}{n})$ and the relationship between the zones and their corresponding zone box scores is as shown in Table 1 with each zone of the \bar{x} observed value having a corresponding zone box score S. Figure 1 illustrates an example of \bar{x} at N (0,1) and (S1, S2, S3, S4, S5, S6, S7 and S8) as (8, 4, 2, 0, 0, 2, 4 and 8).

To monitor the mean shift and consider the direction of the shift, this study uses the centerline μ_0 to separate the zones. After each sampling, the zone control chart is used to calculate the accumulated zone box score T of the continuous observed value. When the process begins, the initial value of T is set at 0. The rules for the accumulated zone box score are as follows:

• When the next observed value and the previous observed value are on the same side of the centerline, the zone box scores are added up to T:

$$T_i = T_{i-1} + S$$
, if $\overline{x}_{i-1}, \overline{x}_i \ge \mu_0$ or $\overline{x}_{i-1}, \overline{x}_i \le \mu_0$ i = 1, 2, 3,....

 When the next observed value and the previous observed value are on different sides of the centerline, after T returns to zero, the zone box scores of the most recent observed values are added up to T:

 $\begin{array}{l} T_{i} = 0 + S, \ if \ (\overline{x}_{i:-1} {>} \mu_{0} \ and \ \overline{x}_{i} {<} \mu_{0}) \ or \\ (\overline{x}_{i:-1} {<} \mu_{0} \ and \ \overline{x}_{i} {>} \mu_{0}) \ i = 1, \ 2, \ 3, \ldots \end{array}$

Table 1: Relationship between zone range and the corresponding zone box scores

Zone	Range	Zone box score
A	$\overline{X} < \mu_0 - 3 \cdot \sigma_0 \big/ \sqrt{n}$	S1
В	$\mu_0 - 3 \cdot \sigma_0 \big/ \sqrt{n} \leq \overline{X} < \mu_0 - 2 \cdot \sigma_0 \big/ \sqrt{n}$	S2
С	$\mu_0 - 2 \cdot \sigma_0 \big/ \sqrt{n} \leq \overline{X} < \mu_0 - \sigma_0 \big/ \sqrt{n}$	S3
D	$\mu_0 - \sigma_0 \big/ \sqrt{n} \leq \overline{X} \leq \mu_0$	S4
E	$\mu_0 \leq \overline{X} \leq \mu_0 + 1 \cdot \sigma_0 \big/ \sqrt{n}$	S5
F	$\mu_0 + 1 \cdot \sigma_0 \big/ \sqrt{n} < \overline{X} \leq \mu_0 + 2 \cdot \sigma_0 \big/ \sqrt{n}$	S6
G	$\mu_0 + 2 \cdot \sigma_0 \big/ \sqrt{n} < \overline{X} \leq \mu_0 + 3 \cdot \sigma_0 \big/ \sqrt{n}$	S7
Н	$\mu_0 + 3 \cdot \sigma_0 \big/ \sqrt{n} < \overline{X}$	S8

 When the accumulated zone box scores are bigger or equal to the threshold value T_U, the control chart will generate warnings to monitor the process mean shift, and also to identify the causes of these phenomena.

When Jaehn¹³ proposed the zone control chart the appropriate zone box scores (S1, S2, S3, S4, S5, S6, S7 and S8) were set as (8, 4, 2, 1, 1, 2, 4 and 8) and the threshold value T_{U} was set at 8. Figure 2 presents an example of a control chart, in which the numbers in the circles of the observed values are the accumulated appropriate zone box scores. Since the accumulated zone box scores of the No. 7 and 11 observed values are bigger or equal to the threshold value of 8, the control chart will produce warnings. The causes of this should then be discovered in order to return the process to the in-control state.

With regard to the improvement and application of the zone control chart as mentioned previously, when using the above \bar{x} zone control chart users may encounter the following two problems: (1) Too great a difference in the zone box scores of the observed values around both sides of the zone boundary and (2) The recorded zone box scores are the same, making it impossible to tell the difference between the two.

Fuzzy theory applications: This section describes the fuzzy zone control chart method used in this study to overcome the two problems mentioned above. The basic assumptions of the proposed fuzzy zone control chart method are as follows:

- A fixed sample number n in each sampling
- No inspection errors
- Sample average number \overline{x} is assumed to have a normal distribution:

$$N(\mu_0, \frac{\sigma_0^2}{n})$$

where, μ_0 and σ_0 are known.

Linguistic variables used to describe the zones of the zone control chart: This study uses μ_0 as the centerline and

linguistic variables to describe the zones of the zone control chart, which are called shifts from the centerline. There are seven zones as follows: Large negative shift (LN), medium negative shift (MN), small negative shift (SN), no shift (NO), small positive shift (SP), medium positive shift (MP) and large positive shift (LP). Each zone can be represented by a fuzzy set. Figure 3 gives examples of a triangular fuzzy number and a trapezoidal fuzzy number. The membership function number

of each fuzzy set can be obtained by the direct specifying method or the probability of being close or far from the center of the zone. The number of zones is subject to the needs of the real situation being assessed.

Using fuzzy rules to determine the zone box scores of the observed values: Fuzzy rules are used to determine the relationship between the zones of the observed values and the specified zone box scores and to obtain definite specified zone box scores through the processes pf fuzzy inference and defuzzification. Using the above seven zones as an example, the rules used are as shown below, based on the type I³² singletoninference rule:

- R_1 : If \bar{x} is LN, then y is w_1
- R_2 : If \bar{x} is MN, then y is w_2
- R_3 : If \bar{x} is SN, then y is w_3
- R_4 : If \bar{x} is NO, then y is w_4
- R_5 : If \overline{x} is SP, then y is w_5
- R_6 : If \overline{x} is MP, then y is w_6
- R_7 : If \bar{x} is LP, then y is w_7

When the level of satisfaction of the observed value \overline{X} with rule R_i is α_i (degree of membership), then the specified zone box score can be represented by the center of gravity method:

$$S = \frac{\sum_{i=1}^{n} w_i \alpha_i}{\sum_{i=1}^{n} \alpha_i}$$

Based on the appropriate zone box score of the observed values and after each sampling the accumulated zone box score T of the continuous observed values is calculated. When the process begins, the initial value of T is set at 0. When the accumulated appropriate zone box score is bigger than or equal to the threshold value T_U , the control chart will generate warnings to monitor the process mean shift and identify the causes of these phenomena.

Zone control chart performance evaluation indicators: The threshold value T_u will influence the effectiveness of the fuzzy zone control chart, subject to the in-control Average Run Length (ARL). The out-of-control ARL is used to assess the performance of the control chart. The smaller the value is, the better the performance will be. When a non-random variation occurs during a real process, in the case of a fixed in-control state ARL, a number of samples are required for shifts of different sizes to get the signal for an out-of-control state



Fig. 2: Example of a zone control chart (using $\bar{x} \sim N(0,1)$ as an example)



Fig. 3: Examples using a triangular fuzzy number and trapezoidal fuzzy number, LN: Large negative, MN: Medium negative shift, SN: Small negative shift, NO: No shift, SP: Small positive shift, MP: Medium positive shift and LP: Large positive shift

when using the control chart. The signal is generated when the accumulated zone box score of the observed values is bigger than or equal to the threshold value T_{U} .

RESULTS AND DISCUSSION

To compare the performance of the traditional zone control chart and the fuzzy zone control chart proposed in this study, this study uses the zone control chart proposed by Jaehn¹³ with the appropriate zone box scores (S1, S2, S3, S4, S5, S6, S7 and S8) set at (8, 4, 2, 0, 0, 2, 4 and 8) and the threshold value T_{U} set at 8. For the fuzzy zone control chart, this study uses the linguistic variable introduced in the previous section to describe the various zones. Regarding the fuzzy rules, the common rules include type I (singleton

inference rule), type II (linguisticinference rule) and type III (linear inference rule)³². Based on the aims of this study, the type linference rule is used to determine the appropriate zone box scores of the observed value zones.

Simulation: Since the threshold value T_U can affect the performance of the fuzzy zone control charts, it is used for the simulation of the zone control chart. In order to satisfy the general hypotheses of the standard normal distribution, the simulation method is used to generate 10,000 samples of data, in line with the in-control state process. The average in-control ARL of the simulated data of the 10,000 samples, obtained using the traditional zone control chart proposed by Jaehn¹³, is 38.61. This is then used as the ARL for the fuzzy zone control chart to determine the simulated zone control chart's threshold value T_{μ} of 5.48. The comparison standard is established in the same way. Consequently, using the above mentioned simulation method, the ARL of the 10,000 samples in the in-control process, in the case of a threshold value T_{μ} of 5.48 determined using the fuzzy zone control chart, is equal to the in-control ARL of 38.61 obtained using the zone control chart proposed by Jaehn¹³.

For the case of a process in a state of non-random variation, namely, a mean shift occurring due to non-random reasons, this study simulated 10,000 samples based on different shifts of 0.5, 1.0, 1.5, 2.0, 2.5 and 3.0 times the standard deviation, in order to compare the traditional zone control chart and the simulated zone control chart in terms of the out-of-control ARL. The corresponding mean shifts are as shown in the first column of Table 2.

Assessment of the performance of the fuzzy control chart:

In the case of a non-random variation, the out-of-control ARL is used to compare the performance of the traditional zone

Table 2: Comparison of out-of-control ARL values of the two control chart	S
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Shift size	Traditional zone	Fuzzy zone
(Negative/positive)	control chart	control char
0.5	16.066	15.751
1.0	6.612	6.068
1.5	3.970	3.398
2.0	2.790	2.393
2.5	2.093	1.850
3.0	1.637	1.532

control chart with that of the simulated zone control chart, and the smaller the value is, the better the performance is. In the case of a given ARL for an in-control state, the performance refers to the number of samples required to get an out-of-control signal when the real process has a non-random variation, with the accumulated zone box score rule obtained as described in an earlier section. An out-of-control signal will occur when the accumulated zone box score of the traditional zone control chart is bigger than 8 and the accumulated zone box score of the proposed fuzzy zone control chart is bigger than or equal to the threshold value T_U of 5.48. In the case of a process with non-random variations, the detection capability of the out-of-control signals can serve as the comparison standard for the control chart performance.

Using linguistic variables to describe the zones of the zone

control chart: In this illustration of the of the fuzzy zone control chart approach, the basic assumptions are identical hose described above.

Figure 3 illustrates the use of a triangular fuzzy number and a trapezoidal fuzzy number. The membership function number of each fuzzy setcan be obtained by the direct specifying method. The zone number is determined by real needs. For the needs of the traditional zone control chart, this study uses seven zones. With μ_0 as the centerline (the centerlines of the zones in this study are -3.5, -2.5, -1.5, 0, 1.5, 2.5, 3.5), linguistic variables are used to describe each zone in the chart. From left to right, the seven zones are: Large negative shift (LN), medium negative shift (MN), small negative shift (SN), no shift (NO), small positive shift (SP), medium positive shift (MP) and large positive shift (LP). Each zone can be represented by a fuzzy set. For example, if the central point of no shift (NO) on the x-axis is 0, the distance is between negative and positive 1.5. If the center of the small positive shift (SP) on the x-axis is 1.5, the distance i between 0.5 and 2.5. The centers and distances on the x-axis are as shown in Fig. 3.

Using fuzzy rules to determine the appropriate zone box scores of the observed values: Fuzzy rules are used to

represent the relationship between the zones of the observed values and the specified zone box scores and to obtain definite values to specify the zone box scores through the fuzzy inference and defuzzification process. Using the above seven zones as an example, the rules are as shown below, based on the type Isingletoninference rule:

- R_1 : If \overline{x} is LN, then y is 8
- R_2 : If \bar{x} is MN, then y is 4
- R_3 : If \overline{x} is SN, then y is 2
- R_4 : If \overline{x} is NO, then y is 0
- R_5 : If \overline{x} is SP, then y is 2
- R_6 : If \overline{x} is MP, then y is 4
- R_7 : If \overline{x} is LP, then y is 8

If the level of satisfaction of observed value \bar{x} with rule R_i is α_i , then the specified zone box score can be represented by the center of gravity as:

$$\mathbf{S} = \frac{\sum_{i=1}^{n} \mathbf{w}_{i} \boldsymbol{\alpha}_{i}}{\sum_{i=1}^{n} \boldsymbol{\alpha}_{i}}$$

For example, when the observation value $\overline{x} = 2.5$, then the specified zone box score:

$$S = \frac{(0.0 \times 2.0 + 1.0 \times 4.0)}{(0.0 + 1.0)} = 4.0$$

Hence, using the fuzzy zone control chart and the above-mentioned linguistic variable to describe the zones, this study produced simulated 10,000 samples. Based on these, the determined threshold value T_U is 5.48. Type Isingletoninference rules are then used to determine the relationships among the observed value zone box scores, the observed values and the zone box scores. The results are obtained using the relevant Matlab applications and shown in Fig. 4.

Based on the specified zone box scores of the observed values inferred above, in order to monitor the mean shift and consider the direction of the shift, the accumulated zone box score T of the continuous observed values are calculated. When the process begins, the initial value of T is set at 0 and the rules used for the accumulated zone box score are the same as those used with the traditional zone chart.

When the accumulated zone box scores are bigger than or equal to the threshold value $T_{\rm U}$ of 5.48, the control chart will generate warnings to monitor the process mean shift and identify the causes of this phenomenon.

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Fig. 4: Relationships among the observed values and the appropriate zone box score

Performance of fuzzy zone control chart and traditional zone control chart: Table 2 shows that when the shifts are negative and positive 0.5, the ARL values of the traditional zone control chart and the fuzzy zone control chart are 16.066 and 15.751, respectively. When the shifts are negative and positive 1.0, the ARL values of the traditional zone control chart and the proposed one are 6.612 and 6.068, respectively. The ARL values of the traditional zone control chart and the proposed one are 6.612 and 6.068, respectively. The ARL values of the traditional zone control chart and the fuzzy zone control chart of other shifts caused by non-random variations can be seen in Table 2. The data suggested that the fuzzy zone control chart is better than the traditional zone control chart in terms of monitoring capability for processes in a state of non-random variation.

The traditional zone control chart is divided into a number of crisp zones, meaning that it is unable to effectively monitor the production process. In contrast and as show by the experimental data reported above, the fuzzy zone control chart presented in this study can accurately monitor the real quality characteristics, thus reducing the amount of unqualified products and the related costs.

CONCLUSION

This study discussed the application of a traditional zone control chart and its disadvantages when directly specifying the crisp zone box scores. Fuzzy theory was used to define a continuous zone instead of crisp zones to monitor the sampling statistics values and to design afuzzy zone control chart for more effective monitoring of the mean shift.

The fuzzy zone control chart proposed in this work can overcome two common problems that arise with traditional zone control charts, namely the large differences in the specified zone box scores of sampled observations around both sides of the zone boundary and the inability to distinguish the two observation values which are quite different but fall in the same zone. The experimental data suggest that the proposed fuzzy zone control chart can more accurately monitor the occurrence of process variations, thus reducing unqualified products, cutting additional costs and better meeting the real needs of manufacturers.

With regard to the occurrence of a mean shift in a real process, the proposed fuzzy zone control chart can help researchers in the production management field to more accurately describe the application of fuzzy theory in a control chart. In practice, the quality department of a factory could build a specific fuzzy zone control chart according to its specific environment by following the steps presented in this study and further improve the performance of the control chart with the use of computers, thus further reducing the amount of unqualified products.

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