Journal of Applied Sciences

ISSN 1812-5654

science alert

ANSInet
an open access publisher
http://ansinet.com

OPEN ACCESS Journal of Applied Sciences

ISSN 1812-5654
DOI: 10.3923/jas.2017.90.96

Research Article
Optimized GM(1,1) Model Based on the Modified Initial Condition

[1]Mahdi Hassan Madhi and [2]Norizan Mohamed

[1]Department of Statistics, Faculty of Science, Sebha University, Sebha, Libya
[2]School of Informatics and Applied Mathematics, University Malaysia Terengganu (UMT), 21300 Kuala Terengganu, Terengganu, Malaysia

Abstract
Background and Objective: The grey model GM(1,1) has been widely applied in various areas. However, the performance of the traditional GM(1,1) model usually indicates certain limitations which may affect the applicability directly or otherwise of the model and its prediction precision. Therefore, the improvement of GM(1,1) model is an important issue. The current study aimed to improve the prediction accuracy of GM(1,1) model. Specifically, in order to improve the prediction accuracy of GM(1,1) model, it is necessary to consider improving the initial condition in the response function of the model. Therefore, the rationale behind this study is to come up with a new approach to improve prediction accuracy of GM(1,1) model through an optimization of the initial condition. **Methodology:** In this study, the new modified GM(1,1) model is proposed by optimizing the initial condition. The new initial condition consists of the first item and the last item of a sequence generated by applying the first-order accumulative generation operator on the sequence of raw data. Weighted coefficients of the first item and the last item in the combination as the initial condition are derived from a method of minimizing the error summation of the square. **Results:** In this study, the numerical results show that the modified GM(1,1) model gives a better prediction performance when compared with the traditional GM(1,1) model. **Conclusion:** The result also shows the efficiency and effectiveness of the new approach.

Key words: GM(1,1) model, initial condition, prediction accuracy

Received: August 03, 2016 Accepted: October 01, 2016 Published: January 15, 2017

Citation: Mahdi Hassan Madhi and Norizan Mohamed, 2017. Optimized GM(1,1) model based on the modified initial condition. J. Applied Sci., 17: 90-96.

Corresponding Author: Mahdi Hassan Madhi, Department of Statistics, Faculty of Science, Sebha University, Sebha, Libya

Competing Interest: The authors have declared that no competing interest exists.

Data Availability: All relevant data are within the paper and its supporting information files.

INTRODUCTION

Grey systems theory was first proposed by Ju-Long[1] as the system model of small samples and incomplete information to build a grey model for prediction and decision-making[2]. It has been widely applied in various research elds and has achieved good prediction accuracy which motivated many researchers focus on the development of the theory and utilized it in a various fields of research, such as engineering science, natural science, social science, industry, agriculture, tourist flow, etc.

The GM(1,1) model gives a vital role in grey forecasting, grey programming and grey control[2]. It can be observed from the procedure of GM(1,1) model construction that, the model is neither a differential equation, nor a difference equation[3]. But an approximate model that possessed the characteristics of both differential equation and difference equation. It is inevitable for the models to results in error-free practical applications. Therefore, many researchers paid much attention on the improvement and optimization research of GM(1,1) model during the last two decades. They have improved the model via the following aspects: (i) Background value, (ii) Initial value selection and (iii) Gray differential equation. For example, from the study that carried out by Chang *et al*.[4], an optimization approach was adopted that combined the grey model to improve the modeling error of grey prediction. This study considered each background value at a discrete point as an independent parameter. Yaoguo *et al*.[5] aimed in their study to display a method for grey model improvement using the nth item of $X^{(1)}$ as the starting condition of the grey differential model to increase prediction precision. Yinao *et al*.[6] presented a GM(1,1) direct modelling method with a step by step optimizing grey derivative's whitened values to the unequal time interval sequence modelling. They also proved that the new method still has the same characteristics of linear transformation consistency as the old method. Dai and Chen[7] asserted that the background value in grey model GM(1,1) is an important factor of precision and adaptability. The scholars proposed that the traditional background is replaced by a new reconstruction approach to the background value of GM(1,1) based on Gauss-Legendre formula. Xie and Liu[8] discussed the influence of different fitting points and proposed optimization model fitting point contribute to the model. Sun and Wei[9] applied another improved approach of grey derivative in the direct GM(1,1), which raised the modelling precision once again. The new model has been efficiently proven to have the property of exponent, coefficient and translation of constant superposition. The model is not only suitable for the low growth sequence but also the high growth sequence. In addition, it is appropriate for the non-homogeneous exponential sequence. Another study by Wang *et al*.[10] put forward a new approach for optimization of the initial condition in the time response function for traditional GM(1,1) model and derived the optimal weights of the first item and the last item of $X^{(1)}$ by the least square method. Zhou *et al*.[11] showed a new approach for optimization of the white differential equation based on the original grey differential equation. The process started from the original grey differential equations, through examining the relationship between the raw data $X_{(k)}^{(0)}$ and the derivative of its 1-AGO. A new white differential equation which is equal to the original grey differential equation was constructed. Meanwhile, the new GM(1,1) model which is closer to the changes of data was obtained. Yao and Wang[12] used an improved GM(1,1) model based on background value to forecast the electricity consumption in Eastern China society. Jong and Liu[13] proposed a novel approach to improve prediction accuracy of grey power models including GM(1, 1) and grey verhulst model. The modified new models were proposed by optimizing the initial condition and model parameters. Li and Xie[14] aimed in their study to remedy the defects about the applications of the traditional grey model and buffer operators in medium and long-term forecasting, a variable weights buffer grey model was proposed. The proposed model integrated the variable weights buffer operator with the background value optimized GM(1,1) model to implement dynamic preprocessing of original data. Xiaofei and Renfang[15] proposed the novel background value of GM(1,1) obtained from the combinative interpolation optimization idea. They used the improved grey model for forecasting in malignant tumors. Chen and Li[16] proposed a new modified GM(1,1) model based on an optimum weighted combination with different initial value.

Grey models improvement of the initial condition may increase the prediction precision in some practical applications. However, some spaces for improvement of GM(1,1) model prediction precision still exist. What has been done so far is to use the first term of $X^{(1)}$ as the initial condition of the grey differential model. Repeatedly, it has occurred in practical applications that we do not adequately utilize new information contained in terms other than the first term of $X^{(1)}$.

The overall purpose of this study is to propose a novel approach to improve prediction accuracy of GM(1,1) model by optimizing the initial condition and compare empirically with traditional GM(1,1) model in terms of the measurement criteria for the forecasting performance. The new initial condition consists of the first item and the last item of a sequence generated by applying the first-order accumulative generation operator on the sequence of raw data. Weighted coefficients of the first item and the last item in the combination as the initial condition are derived from a method of minimizing error summation of the square.

MATERIALS AND METHODS

Traditional GM(1,1) model: Ju-Long[1] has been assured that the raw data must be taken in the sequential time period and as few as four. In addition, the Grey Model (GM) is the core of grey system theory and the GM(1,1) is one of the most widely used grey forecasting model, where the symbol GM(1,1) stands for "First order grey model in one variable". The construction process of GM(1,1) model is described as follows[3]:

Step 1: Denote the original data sequence:

$$x^{(0)} = \{x^{(0)}_{(1)}, x^{(0)}_{(2)}, ..., x^{(0)}_{(n)}\} \quad n \geq 4$$

Step 2: Use Accumulative Generation Operator (AGO) to form a new data series:

$$x^{(1)} = \{x^{(1)}_{(1)}, x^{(1)}_{(2)}, ..., x^{(1)}_{(n)}\} \quad n \geq 4$$

where, $x^{(1)}_{(1)} = x^{(0)}_{(1)}$ and:

$$x^{(1)}_{(k)} = \sum_{i=1}^{k} x^{(0)}_{(i)}, \; k = 2,3,...,n$$

Step 3: Calculate background values $z^{(1)}$:

$$z^{(1)}_{(k)} = 0.5x^{(1)}_{(k-1)} + 0.5x^{(1)}_{(k)}, \; k = 2,3,...,n$$

Then, the following equation:

$$x^{(0)}_{(k)} + az^{(1)}_{(k)} = b$$

is a grey differential equation, also called model GM(1,1).

Step 4: Establish the whitened equation:

$$\frac{dx^{(1)}_{(k)}}{dt} + ax^{(1)}_{(k)} = b$$

where, a and b are referred to as the development coefficient and grey action quantity, respectively.

Step 5: Solve the whitened equation by using the least square method and the predicted values, it can be obtained as:

$$\begin{cases} \hat{x}^{(1)}_{(k)} = \left(x^{(0)}_{(1)} - \dfrac{b}{a}\right)e^{-a(k-1)} + \dfrac{b}{a} \\ \hat{x}^{(0)}_{(k)} = \hat{x}^{(1)}_{(k)} - \hat{x}^{(1)}_{(k-1)} = (1-e^{a})(x^{(0)}_{(1)} - \dfrac{b}{a})e^{-a(k-1)} \end{cases}$$

Where:

$$[a, b]^T = \left(B^T B\right)^{-1} B^T Y$$

$$Y = [x^{(0)}_{(1)}, x^{(0)}_{(2)}, ..., x^{(0)}_{(n)}]^T$$

$$B = \begin{bmatrix} -z^{(1)}_{(2)} & 1 \\ -z^{(1)}_{(3)} & 1 \\ \vdots & \vdots \\ -z^{(1)}_{(n)} & 1 \end{bmatrix}$$

Through the above expression, it can observe that the initial condition in time response function of the traditional GM(1,1) model is the first item in a sequence generated by applying the 1-AGO to $X^{(0)}$. This type of initial condition in time response function cannot take advantage of new pieces of information in the raw data. Furthermore, the principle of new information priority cannot be expressed by this type of initial condition. For this purpose, this study proposed a new initial condition that expressed this principle well in time response function.

Optimization of initial condition: From the construction procedure of the traditional GM(1,1) model we can find that the general solution of the whitened equation can be expressed as follows:

$$x_{(t)}^{(1)} = ce^{-at} + \frac{b}{a}$$

where, c is a constant, parameters a and b can be derived from the least square method.

From the expression of the general solution, if we let t = 1 and t = n, respectively, then we can obtain the following Eq. 1 and 2:

$$x_{(1)}^{(1)} = ce^{-a} + \frac{b}{a} \quad (1)$$

$$x_{(n)}^{(1)} = ce^{-an} + \frac{b}{a} \quad (2)$$

We also assume another parameter β, where, β∈[0, 1] is called the initial condition parameter. Then we multiply (1-β) and β at the both sides of Eq. 1 and 2, respectively, then Eq. 3 and 4 are derived as follows:

$$(1-\beta)x_{(1)}^{(1)} = (1-\beta)ce^{-a} + (1-\beta)\frac{b}{a} \quad (3)$$

$$\beta x_{(n)}^{(1)} = \beta ce^{-an} + \beta\frac{b}{a} \quad (4)$$

From Eq. 3 and 4, we have:

$$(1-\beta)x_{(1)}^{(1)} + \beta x_{(n)}^{(1)} = (1-\beta)ce^{-a} + (1-\beta)\frac{b}{a} + \beta ce^{-an} + \beta\frac{b}{a}$$

And the constant c can be expressed as follows:

$$c = \frac{(1-\beta)x_{(1)}^{(1)} + \beta x_{(n)}^{(1)} - \frac{b}{a}}{(1-\beta)e^{-a} + \beta e^{-an}} \quad (5)$$

We can find the estimate of c through minimizing the error summation of square in terms of the time response function. Construct a function f(c), yielding:

$$f(c) = \sum_{k=1}^{n}(\hat{x}_{(k)}^{(1)} - x_{(k)}^{(1)})^2$$

where, $\hat{x}_{(k)}^{(1)} = ce^{-ak} + \frac{b}{a}$ representing the predicted values of $x_{(k)}^{(1)}$. Hence:

$$f(c) = \sum_{k=1}^{n}(ce^{-ak} + \frac{b}{a} - x_{(k)}^{(1)})^2$$

let $\frac{\partial f(c)}{\partial c} = 0$, then:

$$2\sum_{k=1}^{n}(ce^{-ak} + \frac{b}{a} - x_{(k)}^{(1)})e^{-ak} = 0$$

$$c = \frac{\sum_{k=1}^{n}\left[(x_{(k)}^{(1)} - \frac{b}{a})e^{-ak}\right]}{\sum_{k=1}^{n}e^{-2ak}} \quad (6)$$

From Eq. 5 and 6 we can obtain the initial condition parameter β as follows:

$$\beta = \frac{e^{-a}\sum_{k=1}^{n}[(x_{(k)}^{(1)} - \frac{b}{a})e^{-ak}] - (x_{(1)}^{(1)} - \frac{b}{a})\sum_{k=1}^{n}e^{-2ak}}{(x_{(n)}^{(1)} - x_{(1)}^{(1)})\sum_{k=1}^{n}e^{-2ak} - (e^{-an} - e^{-a})\sum_{k=1}^{n}[(x_{(k)}^{(1)} - \frac{b}{a})e^{-ak}]}$$

Then the time response function of the optimized GM(1,1) model is given by:

$$\hat{x}_{(t)}^{(1)} = \left(\frac{(1-\beta)x_{(1)}^{(1)} + \beta x_{(n)}^{(1)} - \frac{b}{a}}{(1-\beta)e^{-a} + \beta e^{-an}}\right)e^{-at} + \frac{b}{a} \quad (7)$$

And the restored value of raw data is given as follows:

$$\hat{x}_{(t)}^{(0)} = \hat{x}_{(t)}^{(1)} - \hat{x}_{(t-1)}^{(1)} = (1-e^{a})\left(\frac{(1-\beta)x_{(1)}^{(1)} + \beta x_{(n)}^{(1)} - \frac{b}{a}}{(1-\beta)e^{-a} + \beta e^{-an}}\right)e^{-at}$$

Evaluative accuracy of forecasting models: In order to examine the accuracy of performance of the modified grey model and the traditional GM(1,1) model in this study, the prediction performance is evaluated according to the Mean Absolute Percentage Error (MAPE) defined by:

$$\text{MAPE} = \frac{1}{n}\sum_{k=1}^{n}\frac{\left|x_{(k)}^{(0)} - \hat{x}_{(k)}^{(0)}\right|}{x_{(k)}^{(0)}} \times 100\%$$

RESULTS

To demonstrate the effectiveness of the proposed method, let's consider the following data sequence generated by $f(t) = e^{0.3t}$, t = 1, 2,..., 11 as an illustrating example (1.349859, 1.822119, 2.459603, 3.320117, 4.481689, 6.049647, 8.16617, 11.02318, 14.87973, 20.08554 and 27.11264). To compare with prediction performances between the traditional GM(1,1) model and the modified GM(1,1) model proposed in this study, we utilize the first eight data in the sequence of simulation data (in-sample data) to structure the traditional GM(1,1) model and the modified GM(1,1) model, respectively, i.e.:

$$X^{(0)} = (1.349859, 1.822119, 2.459603, 3.320117, 4.481689, 6.049647, 8.16617 \text{ and } 11.02318)$$

On the other hand, the last three data in the sequence of simulation data, i.e. (14.87973, 20.08554 and 27.11264) (out sample data) are used for predictive inspection.

First, the parameters are estimated and the traditional GM(1,1) model is structured as follows:

$$a = -0.2978,\ b = 1.1489,$$
$$\hat{x}^{(0)}_{(k)} = (1-e^{-0.2978})\,(x^{(0)}_{(1)} + \frac{1.1489}{0.2978})e^{0.2978(k-1)},\ k = 2,3,\ldots,8$$

Second, the parameter is derived and the modified GM(1,1) model is structured as follows:

$$a = -0.2978,\ b = 1.1489,$$
$$\hat{x}^{(0)}_{(k)} = 3.9175(1-e^{-0.2978})e^{0.2978(k-1)},\ k = 2,3,\ldots,8$$

The comparison of simulation and prediction data from the two constructed models is shown as a Fig. 1 and Table 1. The MAPE of in sample data for the traditional GM(1,1) model and the modified GM(1,1) model is 1.5195 and 0.9512%, respectively. Furthermore, the MAPE of out sample data for the traditional GM(1,1) model and the modified GM(1,1) model is 2.6112 and 1.3350%, respectively.

DISCUSSION

This study discusses the forecasting performance of the new approach in more details. Figure 1 shows that both the traditional GM(1,1) model and the modified GM(1,1) model have good prediction performance if parameter t happened to be much smaller in the function $f(t) = e^{0.3t}$. However, with the increment of parameter t, prediction errors from the two types of GM(1,1) model are gradually increasing. However, from Fig. 1 we can find that prediction performance of the modified GM(1,1) model is far better than that from the traditional GM(1,1) model.

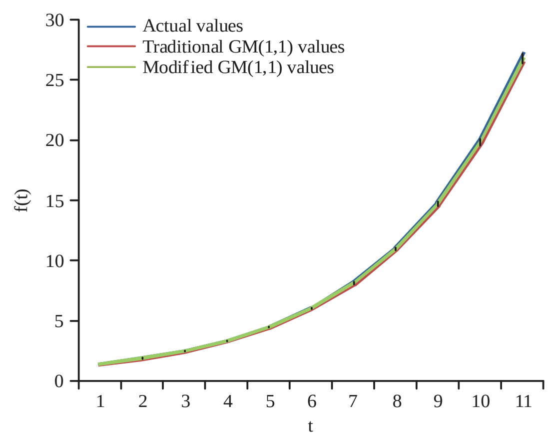

Fig. 1: Comparison of prediction performance of the traditional GM(1,1) model and the modified one

Table 1: Comparison of prediction performance of the traditional GM(1,1) model and the modified one

Original data		GM(1,1) model			
		Traditional		Modified	
K	Actual values	Model values	Absolute relative error (%)	Model values	Absolute relative error (%)
1	1.3499				
2	1.8221	1.8064	0.8594	1.8984	4.1854
3	2.4596	2.4330	1.0809	2.4649	0.2154
4	3.3201	3.2769	1.3006	3.3199	0.0072
5	4.4816	4.4135	1.5188	4.4714	0.2283
6	6.0497	5.9444	1.7407	6.0223	0.4531
7	8.1662	8.0062	1.9589	8.1111	0.6741
8	11.0232	10.7832	2.1769	10.9245	0.8950
MAPE			1.5195		0.9512
9*	14.8797	14.5235	2.3942	14.7138	1.1151
10*	20.0855	19.5610	2.6114	19.8173	1.3352
11*	27.1126	26.3458	2.8281	26.6911	1.5547
MAPE			2.6112		1.3350

*Forecasting value

Although, the first absolute relative error of the modified GM(1,1) model is larger, that is the oldest information. Table 1 shows that along with the augment of parameter t, the absolute relative error of the traditional GM(1,1) model and modified GM(1,1) model increases gradually, while the increasing speed in traditional GM(1,1) model is quicker than the modified GM(1,1) model distinctively that indicates the ameliorating effect of the latter. The simulation and prediction error of the modified GM(1,1) model is fairly small that its absolute relative error is less than the former model greatly. Furthermore, Table 1 provides a means of evaluating how well the prediction values tracking the function $f(t) = e^{0.3t}$. For absolute relative errors, it is desirable to have an absolute relative error as close to zero as possible. Table 1 also displays that the absolute relative errors from the modified GM(1,1) model are much closer to zero than those from the traditional GM(1,1) model. Actual values and the fitted values of two compared models are presented in Table 1. Table 1 shows the modified GM(1,1) model has smaller MAPE (0.9512%) compared with the traditional GM(1,1) (1.5195%), respectively in-sample data. This denotes that modified GM(1,1) model can reduce the fitted error of a traditional GM(1,1) model. From a short-term forecasting viewpoint, the mean absolute percentage error (MAPE) of the modified GM(1,1) model is 1.335% lower than the traditional GM(1,1) model, which implies that the modified GM(1,1) model reaches the objective of minimizing the forecasting error and has high accurate forecasting power. From the comparison analysis of Fig. 1 and Table 1, it is obvious that the modified GM(1,1) model obtain a better prediction performance. Notably, the results of this study are similar and an alternative work with the findings of some previous studies[10,13]. Furthermore, the results of the modified GM(1,1) model are an extension of work done by studies[3,17,5] when β takes zero, a half and one, respectively.

CONCLUSION AND FUTURE RECOMMENDATIONS

In order to overcome the drawbacks facing GM(1,1) model, this study proposed a new approach to improve traditional GM(1,1) model. The new optimization approach of the initial condition comprised the first item and the last item of a sequence generated by applying the first-order accumulative generation operator on the sequence of raw data. It can conclude that:

- The optimized initial condition reflected the principle of new information priority emphasized on in grey systems theory
- The modified GM(1,1) model is an extension of the previous optimized GM(1,1) model and an alternative work of some previous studies
- The numerical results indicate that the modified GM(1,1) model can improve the prediction accuracy of GM(1,1) significantly

Finally, more experiments on other data using modified GM(1,1) model are future topics for analyzing limited time series data.

ACKNOWLEDGMENTS

We would like to be thankful Ministry of Higher Education in Libya for financial support, Universiti Malaysia Terengganu (UMT) for their support.

REFERENCES

1. Ju-Long, D., 1982. Control problems of grey systems. Syst. Control Lett., 11: 288-294.
2. Deng, J.L., 1989. Introduction to grey system theory. J. Grey Syst., 1: 1-24.
3. Liu, S. and J.Y.L. Forrest, 2010. Grey Systems: Theory and Applications. Springer, New York, USA., ISBN-13: 9783642161575, Pages: 379.
4. Chang, T.C., H.T. Chang and M.L. You, 1999. Inverse approach to find an optimum α for grey prediction model. Proceedings of the IEEE International Conference on Systems, Man and Cybernetics, Volume 3, October 12-15, 1999, Tokyo, pp: 309-313.
5. Yaoguo, D., L. Sifeng and C. Kejia, 2004. The GM models that x(n) be taken as initial value. Kybernetes, 33: 247-254.
6. Yinao, W., C. Zhijie, G. Zhiqiang and C. Mianyun, 2004. A generalization of the GM(1,1) direct modeling method with a step by step optimizing grey derivative's whiten values and its applications. Kybernetes, 33: 382-389.
7. Dai, W. and Y. Chen, 2007. Research of GM (1,1) background value based on gauss-legendre quadrature and its application. Proceedings of the IEEE International Conference on Integration Technology, March 20-24, 2007, Shenzhen, China, pp: 100-102.
8. Xie, N.M. and S.F. Liu, 2009. Discrete grey forecasting model and its optimization. Applied Math. Modell., 33: 1173-1186.
9. Sun, Y.N. and Y. Wei, 2009. Optimization of grey derivative in GM (1,1) based on the discrete exponential sequence. Proceedings of the International Symposium on Information Processing, August 21-23, 2009, China, pp: 313-315.
10. Wang, Y., Y. Dang, Y. Li and S. Liu, 2010. An approach to increase prediction precision of GM(1,1) model based on optimization of the initial condition. Expert Syst. Appl. Sci., 37: 5640-5644.

11. Zhou, R., R.G. Li and Y. Chen, 2011. The optimized white differential equation based on the original grey differential equation. Int. J. Educ. Manage. Eng., 1: 44-50.
12. Yao, M. and X. Wang, 2014. Electricity Consumption Forecasting Based on a Class of New GM (1,1) Model. In: Mechatronics and Automatic Control Systems, Wang, W. (Ed.). Springer, New York, ISBN: 9783319012735, pp: 947-953.
13. Jong, Y. and S. Liu, 2014. Grey power models based on optimization of initial condition and model parameters. Grey Syst.: Theory Applic., 4: 370-382.
14. Li, W. and H. Xie, 2014. Geometrical variable weights buffer GM (1,1) model and its application in forecasting of China's energy consumption. J. Applied Math., Vol. 2014. 10.1155/2014/131432.
15. Xiaofei, L. and H. Renfang, 2014. Grey forecasting model based on interpolation optimization in malignant tumor. SHS Web Conf., Vol. 6. 10.1051/shsconf/20140603003.
16. Chen, Q. and J. Li, 2015. Research on Optimum Weighted Combination GM (1,1) Model with Different Initial Value. In: Advanced Intelligent Computing Theories and Applications, Huang, D.S. and K. Han (Eds.). Springer, New York, ISBN: 9783319220536, pp: 354-362.
17. Wang, Y., J. Tang and W. Cao, 2012. Grey prediction model-based food security early warning prediction. Grey Syst.: Theory Applic., 2: 13-23.