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# Research Article
# Profit Analysis of a Serial-parallel System under Partial and Complete Failures

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## Abstract
**Background and Objectives:** The industrial and manufacturing systems comprise of large complex subsystems arranged in series-parallel whose failure is costly. Through mathematical modelling of such systems, the maximum profit level in which the system can attain can be identified and the corresponding subsystem that enable the maximum profit in order to lay emphasis on its preventive maintenance as well as the most critical subsystem leading to drop in profit can also be identified. **Materials and Methods:** Through the transition diagrams, systems of first order differential difference equations are developed and solved recursively to obtain the steady-state availability, busy period of repair men and profit function. Profit matrices for each subsystem have been developed to provide various performance values for different combinations of failure and repair rates of all subsystems. **Results:** Mathematical models of availability, busy period of remain man due to partial and complete failure as well as profit function for the systems have been developed using the probabilistic approach. Through these mathematical models impact of each subsystem parameters on the system's profit has been analyzed through simulation and profit matrices. Through profit matrices and value of correlation coefficient, the most critical system leading to drop in profit is identified. **Conclusion:** Mathematical models of the system are developed in the form of availability, busy period of repairman due to partial and complete failure and as well as profit function. Profit generated are presented in the form of matrices (or tables). The effects of failure and repair rates of all the subsystems are presented in the form of profit matrices. It is evident from the profit matrices that as failure/repair rates increases, the profit tend to decrease/increase. From the value of correlation coefficient presented in the study, it is evident sub-system E is the most critical whose failure is catastrophic to the system. Mathematical models developed in this paper are vital to plant management for proper maintenance analysis and safety of the system as a whole. The models will also assist plant management to avoid an incorrect reliability, availability and profit assessment and leading to inadequate maintenance decision making, which cause unnecessary expenditures and reduction of safety standards.

Key words: Profit, mathematical modelling, availability, partial failure, complete failure, series-parallel

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Data Availability: All relevant data are within the paper and its supporting information files.

## INTRODUCTION

Many industrial and manufacturing systems arrange in series parallel are liable to instant failure. The system operation must ceased at failure of any of its subsystem which results incomplete failure. Similarly, subsystems containing units are also liable to partial due to failure of some units or complete failure due to failure of all units which retard the system operation, system performance and leads to high maintenance cost. Most of the industrial and manufacturing systems consist of large complex subsystems arranged either in series, parallel, parallel-series or series-parallel. Examples of these systems are feeding, crushing, refining, steam generation, evaporation, crystallization, fertilizer plant, crystallization unit of a sugar plant and piston manufacturing plant, etc[1-4]. Series-parallel systems consist of subsystems connected in series where each subsystem consists of units arranged in either parallel or k-out-of-n. Failure of any one of the subsystems leads to the failure of the system called complete failure whereas the failure of a unit in the subsystems is called partial failure. These systems are used in industries, power stations, manufacturing, production and telecommunications[5-7]. Due to the importance of series-parallel systems in various industries, determinations of their availability and profit have become an increasingly important issue. System availability represents the percentage of time the system is available to users. Failure is an unavoidable phenomenon which can be dangerous and costly and bring about less production and profit. The importance in promoting, sustaining industries, manufacturing systems and economy through reliability measurement leading to increase in production output, less maintenance cost as well as generated revenue has become vital to the growth industrial and manufacturing systems. Reliability, availability and profit of a system may be enhanced using highly reliable structural design of the system or subsystem of higher reliability. Reliability, availability and profit are some of the most important factors in any successful industry and manufacturing settings proper maintenance planning plays a role in achieving high system reliability, availability and production output. It is therefore important to keep the equipments/systems always available. Availability and profit of an industrial system are becoming an increasingly important issue. Where the availability of a system increases, the related profit will also increase. Most of these industrial and manufacturing systems were modelled to obtain their availability using Markov birth-death process with the assumption that the failure and repair rates of each subsystem follow exponential distribution[8,9]. These systems are exposed to different types of failures such as common cause, partial, human and complete failure[10]. Several method in studying the behavior, maintenance and performance of these exit[1,11,12]. Such performance are measured in terms of reliability, availability, mean time to failure, mean time between failure and mean time to repair[13,14]. Increase in the reliability and availability of such systems or their subsystems, the production output and associated profit will also increase. Increase in production leads to the increase of profit[15]. Studies carried to enhance the availability of such system through failure dependencies have shown that redundancy optimization of components in the subsystems and availability of repair teams in the event of failure occurrence are some of the technique employed in enhancing the availability of the systems. However, within such systems, failure dependency can exit leading to increase in the number of repair teams, associated cost of maintenance and repair teams[16]. A large volume of literature exists on the issue of predicting performance evaluation of various systems configured as series-parallel system. It is clear that best quality leads to higher productivity as well as generated revenue. However system configuration or design can play a vital role increasing the productivity and generated profit[4].

Existing literatures either ignores the importance of profit on both industrial growth, employment, increase in volume of business, etc. Most literatures laid emphasis of availability and performance evaluation of the systems alone without paying much attention to the generated revenue. More sophisticated models of series parallel systems should be developed to assist in reducing risk of a complete breakdown, operating costs, prolonging the overall availability and the generated revenue (profit) as well.

To achieve the goal of high system reliability, availability and profit, it is necessary that the subsystems that constituted the entire system should remain operative for a longer period[17-19]. Mathematical modelling of such systems may prove beneficial by analyzing the performance of the system/subsystems through reliability, availability as well as generated profit and the degree of identification of the most critical subsystem that result in low reliability, availability and profit between the subsystems[20-22]. Through this mathematical model, the optimal profit level in which the profit is maximum can be identified and the corresponding subsystem that enable the maximum profit in order to lay emphasis on its preventive maintenance as well as the most critical subsystem leading to drop in profit.

The needs of this research is motivated by the work of the authors above due to the fact that profit optimization is affected by failure of some subsystems, system design as well as (of units/subsystems) and identification of the most critical subsystem is not captured which may affect industrial growth, employment, increase in volume of business, etc.:

- The objective is develop a mathematical model availability, minor and catastrophic failure relationship model based on profit
- To explore the impact of failure on profit
- To explore the impact of repair on profit
- To determine the most critical unit/subsystem and parameter in the relation to profit/cost optimization
- To identify the subsystem with maximum profit and the most critical subsystem as far as maintenance is concerned and required immediate attention

## MATERIALS AND METHODS

The system consists of five dissimilar subsystems which are:

- **Subsystem A:** Single units in series whose failure cause complete failure of the entire system
- **Subsystem B:** Consists of 2 active parallel units. Failure of one unit, the system will work in reduced capacity. Complete failure occurs when both units failed
- **Subsystem C:** Consisting of 4 units in which 2 are required in operation while the remaining 2 on standby. Failure of the system occurs when all the three units have failed
- **Subsystem D:** A single unit in series whose failure cause complete failure of the entire system
- **Subsystem E:** A single unit in series whose failure cause complete failure of the entire system

**Notations:**

○ Indicate the system is in full working state
□ Indicate the system is in failed state
⬭ Indicate the system in reduced capacity state

- A, B, C, D, E represent full working state of subsystem
- B2 denote that the subsystem B is working in reduced capacity
- C1 denote subsystem is working on standby unit
- a, b, c, d, e represent failed state of subsystem
- $\beta_1$, $\beta_2$, $\beta_3$, $\beta_4$, $\beta_5$ represent failure rates of subsystems A, B, C
- $\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$, $\alpha_5$ represent repair rates of subsystems A, B, C
- $P_0(t)$, $P_1(t)$, $P_2(t)$: Probability of the system working with full capacity at time t
- $P_3(t)$, $P_4(t)$, $P_5(t)$: Probability of the system in reduced capacity state
- $P_6(t)$ to $P_{28}(t)$: Probability of the system in failed state
- $P_i'(t)$, i = 0, 1, 2,..., 28: Represents the derivatives with respect to time t
- $A_v(\infty)$: Steady state availability of the system
- $B_{P1}(\infty)$: Busy period probability of repairman due to type I failure
- $B_{P2}(\infty)$: Busy period probability of repairman due to type II failure
- $P_F(\infty)$: Profit function
- $C_0$: Total revenue generated
- $C_1$: Cost due to partial failure
- $C_2$: Cost due to complete failure

**Models formulation:** The following differential difference equations associated with the transition diagram in Fig. 1 and 2 of the system are formed using Markov birth-death process:

$$\left(\frac{d}{dt}+\sum_{i=1}^{5}\beta_i\right)P_0(t)=\alpha_1P_6(t)+\alpha_2P_3(t)+\alpha_3P_1(t)+\alpha_4P_7(t)+\alpha_5P_8(t) \tag{1}$$

$$\left(\frac{d}{dt}+\sum_{i=1}^{5}\beta_i\right)P_1(t)+\alpha_3P_1(t)=\alpha_1P_9(t)+\alpha_2P_4(t)+\alpha_3P_2(t)+\alpha_4P_{10}(t)+\alpha_5P_{11}(t)+\beta_3P_0(t) \tag{2}$$

$$\left(\frac{d}{dt}+\sum_{i=1}^{5}\beta_i\right)P_2(t)+\alpha_3P_2(t)=\alpha_1P_{12}(t)+\alpha_2P_5(t)+\alpha_3P_{13}(t)+\alpha_4P_{14}(t)+\alpha_5P_{15}(t)+\beta_3P_1(t) \tag{3}$$

$$\left(\frac{d}{dt}+\sum_{i=1}^{5}\beta_i\right)P_3(t)+\alpha_2P_3(t)=\alpha_1P_{16}(t)+\alpha_2P_{17}(t)+\alpha_3P_4(t)+\alpha_4P_{18}(t)+\alpha_5P_{19}(t)+\beta_2P_0(t) \tag{4}$$

$$\left(\frac{d}{dt}+\sum_{i=1}^{5}\beta_i\right)P_4(t)+\alpha_2P_4(t)=\alpha_1P_{20}(t)+\alpha_2P_{21}(t)+\alpha_3P_5(t)+\alpha_4P_{22}(t)+\alpha_5P_{23}(t)+\beta_2P_1(t) \quad (5)$$

$$\left(\frac{d}{dt}+\sum_{i=1}^{5}\beta_i\right)P_5(t)+\alpha_3P_5(t)=\alpha_1P_{24}(t)+\alpha_2P_{25}(t)+\alpha_3P_{26}(t)+\alpha_4P_{27}(t)+\alpha_5P_{28}(t)+\beta_3P_4(t) \quad (6)$$

$$\frac{dP_6(t)}{dt}=-\alpha_1P_6(t)+\beta_1P_0(t) \quad (7)$$

$$\frac{dP_7(t)}{dt}=-\alpha_4P_7(t)+\beta_4P_0(t) \quad (8)$$

$$\frac{dP_8(t)}{dt}=-\alpha_5P_8(t)+\beta_5P_0(t) \quad (9)$$

$$\frac{dP_9(t)}{dt}=-\alpha_1P_9(t)+\beta_1P_1(t) \quad (10)$$

$$\frac{dP_{10}(t)}{dt}=-\alpha_4P_{10}(t)+\beta_4P_1(t) \quad (11)$$

$$\frac{dP_{11}(t)}{dt}=-\alpha_5P_{11}(t)+\beta_5P_1(t) \quad (12)$$

$$\frac{dP_{12}(t)}{dt}=-\alpha_1P_{12}(t)+\beta_1P_2(t) \quad (13)$$

$$\frac{dP_{13}(t)}{dt}=-\alpha_3P_{13}(t)+\beta_3P_2(t) \quad (14)$$

$$\frac{dP_{14}(t)}{dt}=-\alpha_4P_{14}(t)+\beta_4P_2(t) \quad (15)$$

$$\frac{dP_{15}(t)}{dt}=-\alpha_5P_{15}(t)+\beta_5P_2(t) \quad (16)$$

$$\frac{dP_{16}(t)}{dt}=-\alpha_1P_{16}(t)+\beta_1P_3(t) \quad (17)$$

$$\frac{dP_{17}(t)}{dt}=-\alpha_2P_{17}(t)+\beta_2P_3(t) \quad (18)$$

$$\frac{dP_{18}(t)}{dt}=-\alpha_4P_{18}(t)+\beta_4P_3(t) \quad (19)$$

$$\frac{dP_{19}(t)}{dt}=-\alpha_5P_{19}(t)+\beta_5P_3(t) \quad (20)$$

$$\frac{dP_{20}(t)}{dt}=-\alpha_1P_{20}(t)+\beta_1P_4(t) \quad (21)$$

$$\frac{dP_{21}(t)}{dt}=-\alpha_2P_{21}(t)+\beta_2P_4(t) \quad (22)$$

$$\frac{dP_{22}(t)}{dt}=-\alpha_4P_{22}(t)+\beta_4P_4(t) \quad (23)$$

$$\frac{dP_{23}(t)}{dt}=-\alpha_5P_{23}(t)+\beta_5P_4(t) \quad (24)$$

$$\frac{dP_{24}(t)}{dt}=-\alpha_1P_{24}(t)+\beta_1P_5(t) \quad (25)$$

$$\frac{dP_{25}(t)}{dt}=-\alpha_2P_{25}(t)+\beta_2P_5(t) \quad (26)$$

$$\frac{dP_{26}(t)}{dt}=-\alpha_3P_{26}(t)+\beta_3P_5(t) \quad (27)$$

$$\frac{dP_{27}(t)}{dt}=-\alpha_4P_{27}(t)+\beta_4P_5(t) \quad (28)$$

$$\frac{dP_{28}(t)}{dt}=-\alpha_5P_{28}(t)+\beta_5P_5(t) \quad (29)$$

With initial condition:

$$P_i(t)=\begin{cases}1, & i=0\\ 0, & i\neq 0\end{cases} \quad (30)$$

**Steady state availability of the system:** In the steady state, the derivatives of states probabilities are set equal to 0.

Let $P_0(\infty)$ be the probability of full working state when the system is new and is determine using the condition normalizing:

$$P_0(\infty)+P_1(\infty)+P_2(\infty)+...+P_{28}(\infty) = 1 \quad (31)$$

Setting $\frac{d}{dt}$ as $t\to\infty$ in Eq. 1-29 and solving them recursively using Eq. 31, it obtained the steady state probabilities given in Table 1:

$$\Delta_1 = (y_1+y_2+y_3+y_4+y_5) \quad (32)$$

$$\Delta_2=\left(1+y_3+y_3^2+y_2y_3^2\right) \quad (33)$$

$$\Delta_3 = (y_1+y_2+y_4+y_5) \quad (34)$$

$$\Delta_4=\left(y_2+y_2y_3^2\right) \quad (35)$$

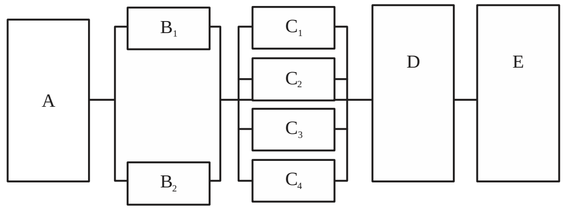


Fig. 1: Reliability block diagram of the system

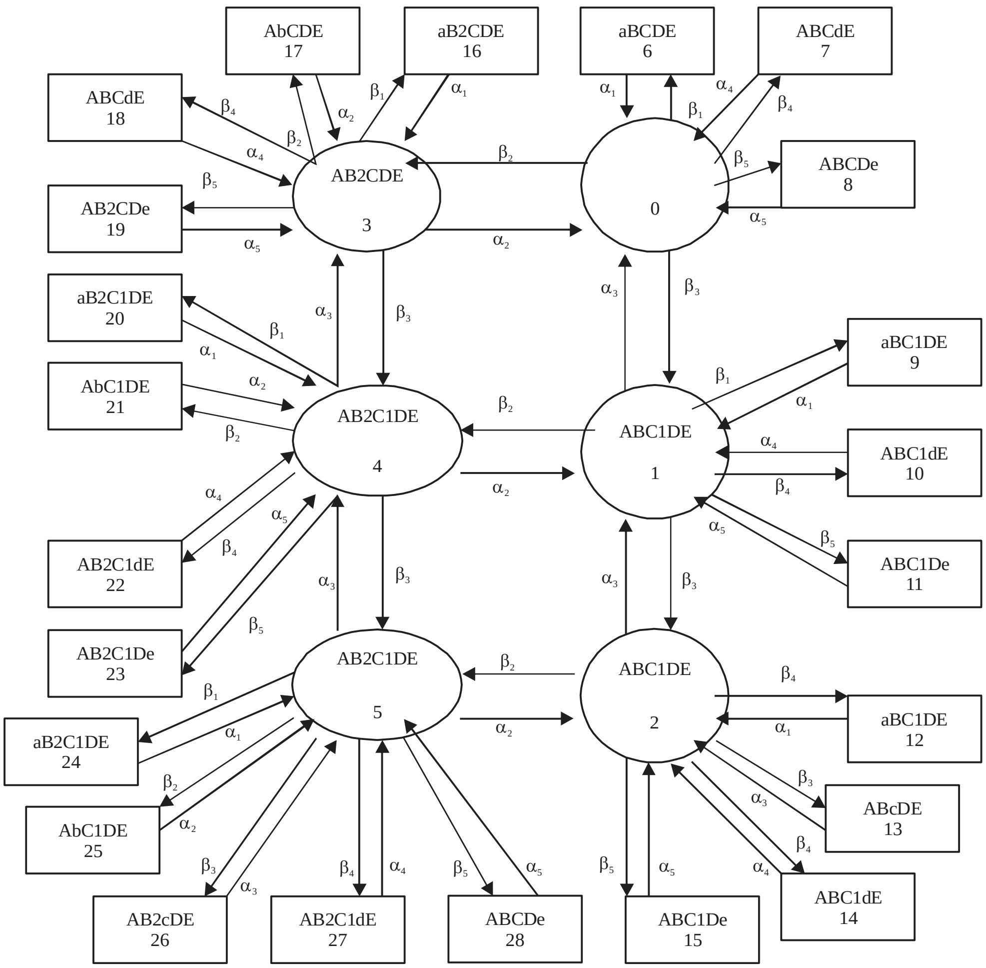


Fig. 2: Transition diagram of the system

The steady-state availability, busy period probability due to type I and II failure and profit function of the system are given below:

$$A_V(\infty) = \sum_{k=0}^{5} P_k(\infty) = \frac{(1+y_2)(1+y_3+y_3^2)}{1+\Delta_1\Delta_2+\Delta_3\Delta_4} \tag{36}$$

$$B_{P1}(\infty) = \sum_{k=1}^{5} P_k(\infty) = \frac{y_2+y_3+y_3^2+y_2y_3+y_2y_3^2}{1+\Delta_1\Delta_2+\Delta_3\Delta_4} \tag{37}$$

$$B_{P2}(\infty) = \sum_{k=6}^{28} P_k(\infty) = \frac{(1+y_3+y_3^2)\left((y_1+y_4+y_5)+y_2\Delta_3\right)+y_3^2(1+y_2)}{1+\Delta_1\Delta_2+\Delta_3\Delta_4} \tag{38}$$

$$P_F(\infty) = C_0\,A_V(\infty) - C_1\,B_{P1}(\infty) - C_2\,B_{P2}(\infty) \tag{39}$$

## RESULTS AND DISCUSSION

Using the failure and repair rates values of Aggarwal *et al.*[8], the matrices consisting profit are computed.

Table 1: Solutions of states probabilities

| | | |
|---|---|---|
| $P_0(\infty)=\frac{1}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{10}(\infty)=\frac{y_3y_4}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{20}(\infty)=\frac{y_1y_2y_3}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ |
| $P_1(\infty)=\frac{y_3}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{11}(\infty)=\frac{y_3y_5}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{21}(\infty)=\frac{y_2^2y_3}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ |
| $P_2(\infty)=\frac{y_3^2}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{12}(\infty)=\frac{y_1y_3^2}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{22}(\infty)=\frac{y_2y_3y_4}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ |
| $P_3(\infty)=\frac{y_2}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{13}(\infty)=\frac{y_3^3}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{23}(\infty)=\frac{y_2y_3y_5}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ |
| $P_4(\infty)=\frac{y_2y_3}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{14}(\infty)=\frac{y_3^2y_4}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{24}(\infty)=\frac{y_1y_2y_3^2}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ |
| $P_5(\infty)=\frac{y_2y_3^2}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{15}(\infty)=\frac{y_3^2y_5}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{25}(\infty)=\frac{y_2^2y_3^2}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ |
| $P_6(\infty)=\frac{y_1}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{16}(\infty)=\frac{y_1y_2}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{26}(\infty)=\frac{y_2y_3^3}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ |
| $P_7(\infty)=\frac{y_4}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{17}(\infty)=\frac{y_2^2}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{27}(\infty)=\frac{y_2y_3^2y_4}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ |
| $P_8(\infty)=\frac{y_5}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{18}(\infty)=\frac{y_2y_4}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{28}(\infty)=\frac{y_2y_3^2y_5}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ |
| $P_9(\infty)=\frac{y_1y_3}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | $P_{19}(\infty)=\frac{y_2y_5}{1+\Delta_1\Delta_2+\Delta_3\Delta_4}$ | |

$y_1=\frac{\beta_1}{\alpha_1},\ y_2=\frac{\beta_2}{\alpha_2},\ y_3=\frac{\beta_3}{\alpha_3},\ y_4=\frac{\beta_4}{\alpha_4},\ y_5=\frac{\beta_5}{\alpha_5}$

Table 2: Effect of failure and repair rates of subsystem A on profit of the system

| | $\alpha_1$ | | | | | |
|---|---|---|---|---|---|---|
| $\beta_1$ | 0.004 | 0.005 | 0.006 | 0.007 | 0.008 | 0.009 |
| 0.35 | 603.20 | 651.63 | 685.86 | 711.34 | 731.04 | 746.73 |
| 0.40 | 570.57 | 623.54 | 661.24 | 689.44 | 711.34 | 728.82 |
| 0.45 | 539.47 | 596.54 | 637.44 | 668.19 | 692.15 | 711.34 |
| 0.50 | 509.77 | 570.57 | 614.43 | 647.55 | 673.45 | 694.25 |
| 0.55 | 481.39 | 545.57 | 592.15 | 627.49 | 655.22 | 677.56 |
| 0.60 | 454.24 | 521.49 | 570.57 | 607.99 | 637.44 | 661.24 |

With constant values $\beta_2 = 0.005$, $\beta_3 = 0.001$, $\beta_4 = 0.002$, $\beta_5 = 0.004$, $\alpha_2 = 0.1$, $\alpha_3 = 0.5$, $\alpha_4 = 0.4$, $\alpha_5 = 0.4$, $C_0 = 100{,}000$, $C_1 = 10{,}000$, $C_2 = 15{,}000$

Table 3: Effect of failure and repair rates of subsystem B on profit of the system

| | $\alpha_2$ | | | | | |
|---|---|---|---|---|---|---|
| $\beta_2$ | 0.004 | 0.005 | 0.006 | 0.007 | 0.008 | 0.009 |
| 0.05 | 648.71 | 658.58 | 666.05 | 672.11 | 677.25 | 681.75 |
| 0.1 | 608.51 | 623.54 | 634.19 | 642.27 | 648.71 | 654.04 |
| 0.15 | 573.77 | 594.23 | 608.51 | 619.15 | 627.46 | 634.19 |
| 0.2 | 541.58 | 567.17 | 585.01 | 598.25 | 608.51 | 616.75 |
| 0.25 | 511.22 | 541.58 | 562.81 | 578.56 | 590.75 | 600.50 |
| 0.3 | 482.41 | 517.16 | 541.58 | 559.72 | 573.77 | 585.01 |

With constant values $\beta_1 = 0.005$, $\beta_3 = 0.001$, $\beta_4 = 0.002$, $\beta_5 = 0.004$, $\alpha_1 = 0.4$, $\alpha_3 = 0.5$, $\alpha_4 = 0.4$, $\alpha_5 = 0.4$, $C_0 = 100{,}000$, $C_1 = 10{,}000$, $C_2 = 15{,}000$

Table 2 and Fig. 3a and b presented the impact of failure and repair rates of subsystem A against the profit for different values of parameters $\alpha_1$ and $\beta_1$. The failure and repair rates of other subsystems are kept constant as can be seen at the bottom of the table. It is evident from Table 2 and Fig. 3a that the profit showed increasing pattern with respect to repair rate $\alpha_1$ and decreasing pattern with respect to failure rate $\beta_1$ from Fig. 3b. It is clear that profit is higher with the higher value of $\alpha_1$ and lower with higher value of $\beta_1$.

Table 3 and Fig. 4a and b displayed the effect of failure and repair rates of subsystem B against the profit for different values of parameters $\alpha_2$ and $\beta_2$. The failure and repair rates of other subsystems are kept constant as can be seen at the bottom of the table. It is evident from Table 3 and Fig. 4a that the profit showed increasing pattern with respect to repair rate $\alpha_2$ and decreasing pattern with respect to failure rate $\beta_2$ from Fig. 4b. It is clear that profit is higher with the higher value of $\alpha_2$ and lower with higher value of $\beta_2$.

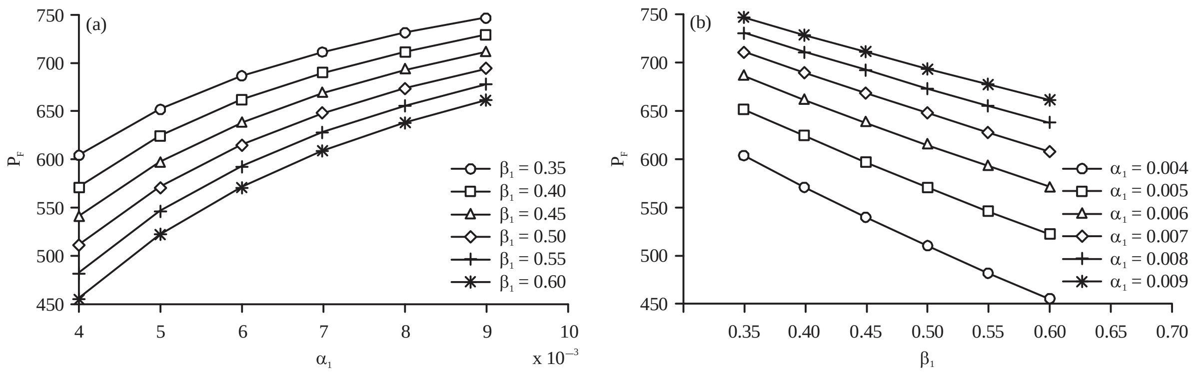


Fig. 3(a-b): Profit against (a) $\alpha_1$ and (b) $\beta_1$

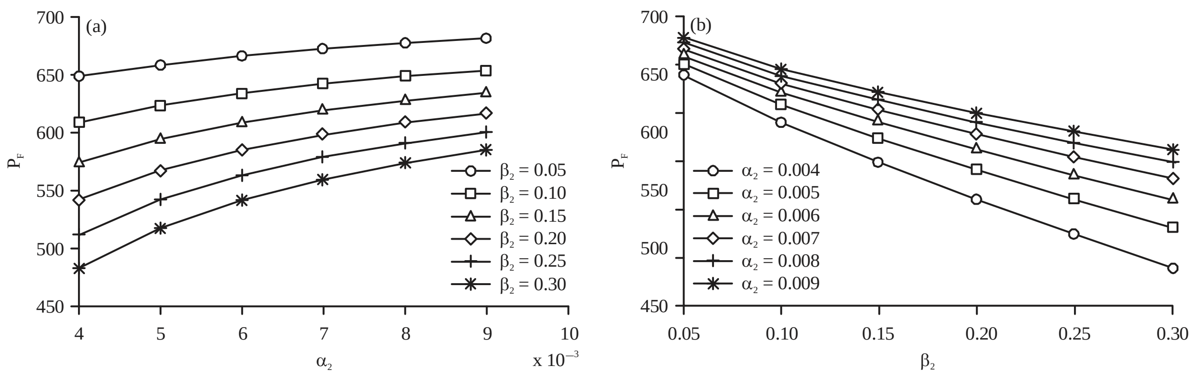


Fig. 4(a-b): Profit against (a) $\alpha_2$ and (b) $\beta_2$

Table 4: Effect of failure and repair rates of subsystem C on profit of the system

| | $\alpha_3$ | | | | | |
|---|---|---|---|---|---|---|
| $\beta_3$ | 0.0005 | 0.001 | 0.0015 | 0.002 | 0.0025 | 0.003 |
| 0.4 | 477.97 | 694.09 | 817.70 | 898.02 | 954.61 | 996.79 |
| 0.5 | 413.63 | 623.54 | 750.79 | 836.37 | 898.02 | 944.66 |
| 0.6 | 364.57 | 566.05 | 694.09 | 782.80 | 848.00 | 898.02 |
| 0.7 | 325.91 | 518.29 | 645.40 | 735.76 | 803.37 | 855.93 |
| 0.8 | 294.67 | 477.97 | 603.12 | 694.09 | 763.27 | 817.70 |
| 0.9 | 268.89 | 443.48 | 566.05 | 656.92 | 727.02 | 782.80 |

With constant values $\beta_1 = 0.005$, $\beta_2 = 0.005$, $\beta_4 = 0.002$, $\beta_5 = 0.004$, $\alpha_1 = 0.4$, $\alpha_2 = 0.1$, $\alpha_4 = 0.4$, $\alpha_5 = 0.4$, $C_0 = 100{,}000$, $C_1 = 10{,}000$, $C_2 = 15{,}000$

Results from Table 4 and Fig. 5a, b presented the impact of failure and repair rates of subsystem C against the profit for different values of parameters $\alpha_3$ and $\beta_3$. The failure and repair rates of other subsystems are kept constant as can be seen at the bottom of the table. It is evident from Table 4 and Fig. 5a that the profit shows increasing pattern with respect to repair rate $\alpha_3$ and decreasing pattern with respect to failure rate $\beta_3$ from Fig. 5b. It is cleared that profit is higher with the higher value of $\alpha_3$ and lower with higher value of $\beta_3$.

It is evident from Table 5 and Fig. 6a and b that the profit increases and decreases with increase in the values of parameters $\alpha_4$ and $\beta_4$. The failure and repair rates of other subsystems are kept constant as can be seen at the bottom of the table. It is evident from Table 5 and Fig. 6a that the profit showed increasing pattern with respect to repair rate $\alpha_4$ and decreasing pattern with respect to failure rate $\beta_4$ from Fig. 6b. It is cleared that profit is higher with the higher value of $\alpha_4$ and lower with higher value of $\beta_4$.

Table 6 and Fig. 7a and b presented the impact of failure and repair rates of subsystem A against the profit for different values of parameters $\alpha_5$ and $\beta_5$. The failure and repair rates of other subsystems are kept constant as can be seen at the bottom of the table. It is evident from Table 6 and Fig. 7a that the profit showed increasing pattern with respect to repair

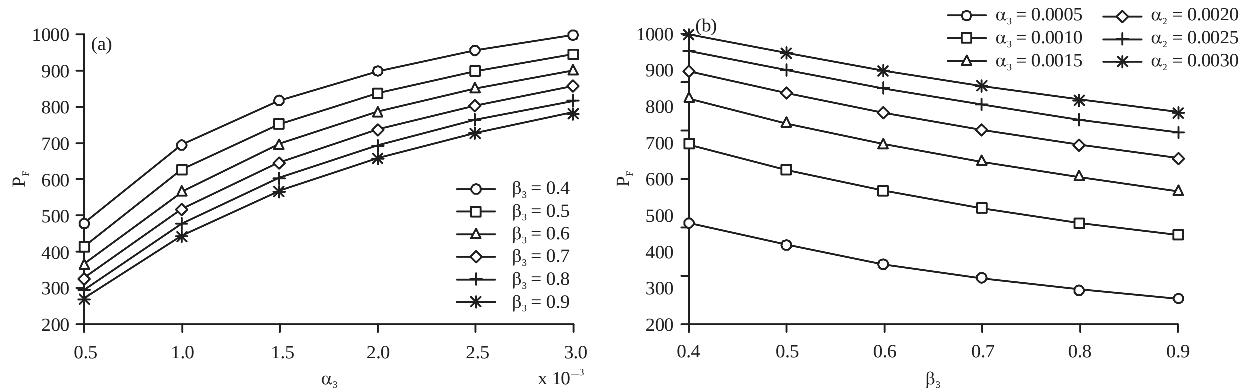


Fig. 5(a-b): Profit against (a) $\alpha_3$ and (b) $\beta_3$

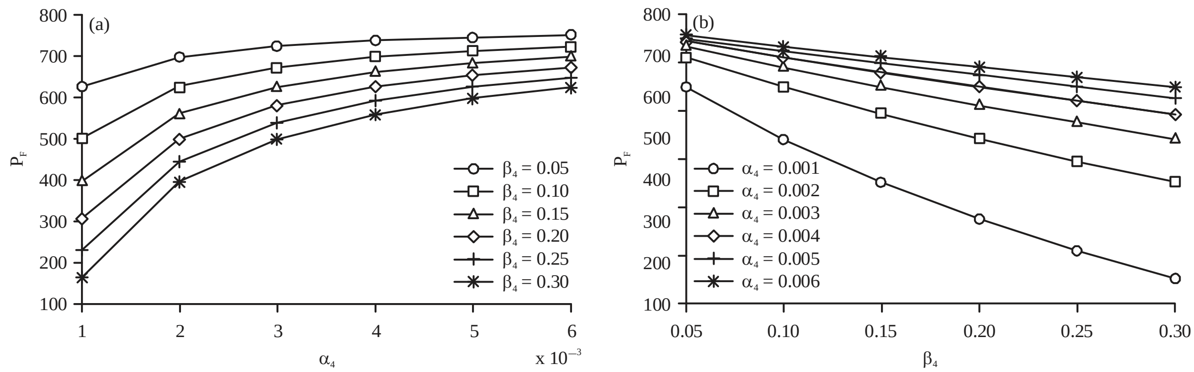


Fig. 6(a-b): Profit against (a) $\alpha_4$ and (b) $\beta_4$

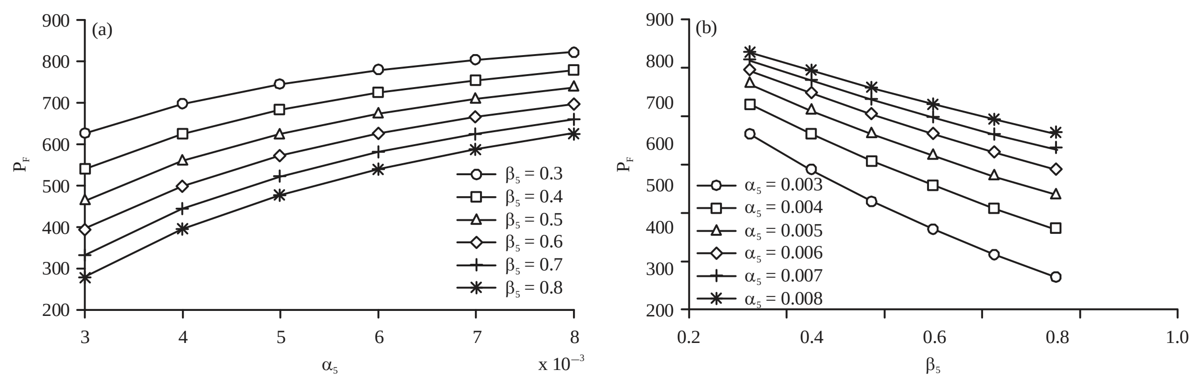


Fig. 7(a-b): Profit against (a) $\alpha_5$ and (b) $\beta_5$

Table 5: Effect of failure and repair rates of subsystem D on profit of the system

| | $\alpha_4$ | | | | | |
|---|---|---|---|---|---|---|
| $\beta_4$ | 0.001 | 0.002 | 0.003 | 0.004 | 0.005 | 0.006 |
| 0.05 | 623.54 | 695.94 | 721.78 | 735.04 | 743.11 | 748.54 |
| 0.1 | 498.27 | 623.54 | 670.99 | 695.94 | 711.34 | 721.78 |
| 0.15 | 393.66 | 557.96 | 623.54 | 658.83 | 680.87 | 695.94 |
| 0.2 | 305.00 | 498.27 | 579.12 | 623.54 | 651.63 | 670.99 |
| 0.25 | 228.89 | 443.71 | 537.44 | 589.96 | 623.54 | 646.87 |
| 0.3 | 162.85 | 393.66 | 498.27 | 557.96 | 596.54 | 623.54 |

With constant values $\beta_1 = 0.005$, $\beta_2 = 0.005$, $\beta_3 = 0.001$, $\beta_5 = 0.004$, $\alpha_1 = 0.4$, $\alpha_2 = 0.1$, $\alpha_3 = 0.5$, $\alpha_5 = 0.4$, $C_0 = 100{,}000$, $C_1 = 10{,}000$, $C_2 = 15{,}000$

Table 6: Effect of failure and rate rates of subsystem E on profit of the system

| | $\alpha_4$ | | | | | |
|---|---|---|---|---|---|---|
| $\beta_4$ | 0.003 | 0.004 | 0.005 | 0.006 | 0.007 | 0.008 |
| 0.3 | 623.54 | 695.94 | 721.78 | 735.04 | 743.11 | 748.54 |
| 0.4 | 498.27 | 623.54 | 670.99 | 695.94 | 711.34 | 721.78 |
| 0.5 | 393.66 | 557.96 | 623.54 | 658.83 | 680.87 | 695.94 |
| 0.6 | 305.00 | 498.27 | 579.12 | 623.54 | 651.63 | 670.99 |
| 0.7 | 228.89 | 443.71 | 537.44 | 589.96 | 623.54 | 646.87 |
| 0.8 | 162.85 | 393.66 | 498.27 | 557.96 | 596.54 | 623.54 |

With constant values $\beta_1 = 0.005$, $\beta_2 = 0.005$, $\beta_3 = 0.001$, $\beta_4 = 0.002$, $\alpha_1 = 0.4$, $\alpha_2 = 0.1$, $\alpha_3 = 0.5$, $\alpha_4 = 0.4$, $C_0 = 100{,}000$, $C_1 = 10{,}000$, $C_2 = 15{,}000$

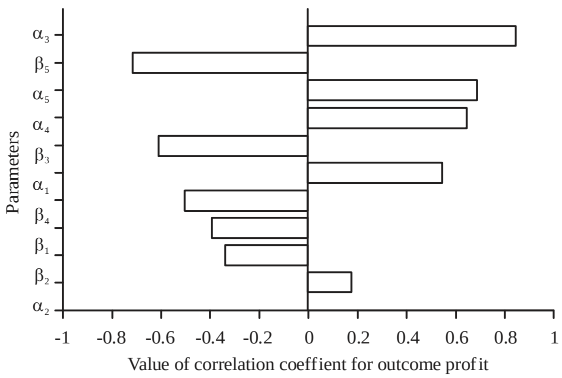


Fig. 8: Tornado plot of profit

rate $\alpha_5$ and decreasing pattern with respect to failure rate $\beta_5$ from Fig. 7b. It is clear that profit is higher with the higher value of $\alpha_5$ and lower with higher value of $\beta_5$.

It conducted sensitivity analysis on parameters to measure their statistical influence on $P_F(\infty)$. To do that each $\beta_1$ to $\beta_4$ and $\alpha_1$, $\alpha_3$, $\alpha_4$ and $\alpha_5$ are uniformly distributed while $\beta_5$ and $\alpha_2$ follows triangular distribution on [0 1] and it drawn 1000 samples from this distribution using Latin Hypercube sampling. This gives a matrix with 1000 rows 5 columns. Each row of the matrix represent a unique parameter set. For each of these sets, we simulated the model outcomes $P_F(\infty)$. It then performed sensitivity analysis on the model outcomes using Partial Rank Correlation Coefficients (PRCC). Figure 8 depicted the tornado plots of the results. From the Fig. 8, it is evident that $\alpha_3$ is the most sensitive parameter affecting all the outcomes ($P_F(\infty)$). Increase in $\alpha_3$ will lead to increase in the outcomes. From the Fig. 8, it is evident that $\beta_5$ is the most sensitive, while $\beta_2$ is the least sensitive for all the outcomes. Increasing the value of $\beta_5$ will decrease each one of the outcomes more significantly than the others. These means that in order to have high values of profit $P_F(\infty)$, it is necessary to consider the combinations of high values of $\alpha_3$ together with low values of $\beta_5$. This means that subsystem C and E should be given more care in terms of maintenance.

## CONCLUSION AND FUTURE STUDY

In this research, it constructed a series-parallel system configurations consisting of five subsystems to study the cost analysis of the system. Explicit expressions for steady-state availability, busy period and profit function for the system are derived. Numerical results presented have shown the effect of both failure and repair rates on profit. From the analysis, it is evident that profit can be enhancing through:

- Proper maintenance planting to avoid the occurrence of catastrophic failure
- Maintaining the system availability at the highest order
- Adding more fault tolerant redundant units/subsystem

Mathematical models of the system are developed in the form of availability, busy period of repairman due to partial and complete failure and as well as profit function. Profit generated are presented in the form of matrices (or tables). The effects of failure and repair rates of all the subsystems are presented in the form of profit matrices. It is evident from the profit matrices that as failure/repair rates increases, the profit tend to decrease/increase. From the value of correlation coefficient presented in the study, it is evident subsystem E is the most critical whose failure is catastrophic to the system.

With modifications and assumptions, the model in this paper will plant management to avoid an incorrect reliability assessment and consequent erroneous decision making, which may lead to unnecessary expenditures. The present work can extended to incorporate failure dependency, condition monitoring to enable management in determining the optimal maintenance/ replacement time.

## SIGNIFICANCE STATEMENT

The study discovers the existence of correlation between the parameters of the system with the outcome profit using tornado 2019. Through this the most critical parameter or subsystem that needs more attention in terms of condition monitoring, preventive or corrective maintenance than the rest is discovered. This would assist the researchers to uncover reliable systems for efficiency, long term survival, growth as well as profit maximization. The study will assist the researchers to uncover the critical areas of profit maximization of series parallel system exposes to different types of failures such as common cause, electrical, mechanical, installation, partial, human and catastrophic failures, that many researchers were not explore. This will pave way to new theory on profit maximization of series parallel system exposes to different types of failure.

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