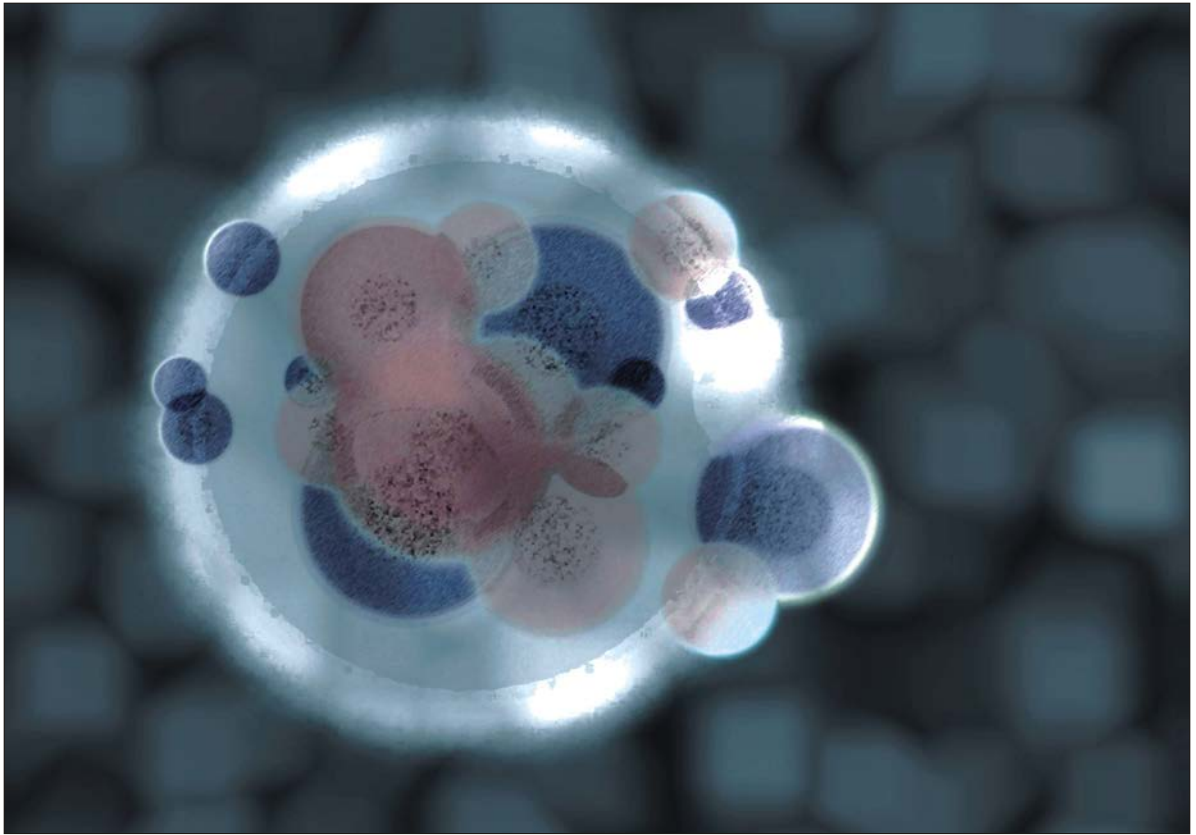




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Comparison of CAR and VAR Biomass Models

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Abstract: CAR and VAR are two basic biomass models, most widely used in research and applications. Re-sampling and sign test were employed in this paper to compare these two models for their parameters stabilities and their predictions.

Key words: CAR and VAR biomass models, allometric, stability of parameters

As the first forest productivity, forest biomass plays an important role in studying forest ecological systems and the relationship of forests climate and always receives more attentions from ecologists and forest researchers in the world. The developed countries have listed forest biomass investigation as one important component of forest monitoring, thus resulting in the research on biomass models. Since the notion of "full tree utilization" was first suggested in Finland and Swede in 1964. Many biomass models were thus provided since then, and in general they fall into three categories: linear models, non-linear models and poly-nominal models. Both linear and nonlinear models can also be divided into single-variate models or multivariate models according to the number of independent variables. The nonlinear models are used most extensively, among which allometric models, CAR and VAR are two most widely-used biomass models.

Allometric Models: The allometric models reflect the harmonious growth among forest tree components by use of power or logarithmic relationship among them. The early studies on relative growth were focused on live bodies (Geron and Ruark, 1988).

Assume that the relative growth rate of some dimensional attribute of a forest tree, say Y , is a constant ratio of the relative growth rate of some other dimensional attribute of a

forest tree, say X , that is $\frac{dY}{Ydt} = b \frac{dX}{Xdt}$, where the parameter b is the constant allometric ratio, and thus the relationship between Y and X at any size of the forest tree can be obtained by integration:

$$\ln Y + k_1 = b \ln X + k_2$$

or

$$Y = e^{k_2 - k_1} X^b$$

where k_1, k_2 are constants of integration.

and substituting $a = e^{k_2 - k_1}$, yields

$$Y = aX^b, \quad [1]$$

or

$$\ln Y = \ln a + b \ln X, \quad [2]$$

the power function commonly used as an allometric model. Because the parameter b in equation [1] or [2] represents the constant allometric ratio of Y to X , so [1] is called CAR model. A value of $b > 1$ indicates the positive allometry (the Y is growing faster than X , $b < 1$ represents the negative allometry (the Y is growing slower than X , and a value of $b = 1$ indicates the equal relative growth.

Kittredge (1944) firstly applied the CAR model to tree data and successfully estimated the foliar weights. After that time, many researchers applied this model to estimate the weights of other components of forest trees. Ruark *et al.* (1987) developed a model that do not assume a constant allometric ratio, and they suppose that the allometric ratio varies as function of the X dimension, as say $b + cX$, then

$$\frac{dY}{Ydt} = (b + cX) \frac{dX}{Xdt}$$

The corresponding allometric function can be obtained by integration:

$$\ln Y + k_1 = b \ln X + cX + k_2$$

or

$$Y = e^{k_2 - k_1} X^{b+cX}$$

where k_1, k_2 are constants of integration.

and substituting $a = e^{k_2 - k_1}$, yields

$$Y = aX^{b+cX}, \quad [3]$$

or

$$\ln Y = \ln a + b \ln X + cX, \quad [4]$$

the resulting equation [3] or [4] differs from equation [1] or [2] by an addition term, cX , that provides for a variable allometric ratio, thus the ratio is called variable allometric ratio and [3] or [4] is called VAR model.

Arising of Problems: The CAR and VAR models have been used widely until today because they could reflect the harmonious growth relationship between different components of forest trees. Meantime, there are so many evaluations on these two models by different researchers but without the same results.

Ruark *et al.* (1987) compared the CAR and VAR models for estimating *Populus tremuloides* biomass and resulted that the VAR is superior to CAR model for estimating the biomass of bolebarks, branches, leaves or twigs, but the CAR model is superior to the VAR model for estimating bolewood biomass. It was because of less freedom of the VAR model than the CAR. Ruark *et al.* (1987) also pointed out that both CAR and VAR model tracked the data for small trees well, but the CAR model estimates were severely biased for trees larger than 20cm dbh, and he thought this was because the CAR is less flexible than the VAR model.

Geron and Ruark (1988) compared CAR and VAR models for predicting foliar biomass of six tree species. The results are that the VAR model is superior to the CAR model for Radiata pine, White spruce, Balsam fir and Aspen, but the CAR model is superior to the VAR for Red maple and Loblolly pine.

Liu (1992) found that, for larch, the CAR model overestimated dried stem or dried bark biomass of larger size but underestimated those of middle or small size, and that the CAR model's estimates of twig or foliar biomass were biased while the VAR ones were almost unbiased.

From all the papers mentioned above, it was easily found that the CAR and VAR model were compared only from their abilities to fit the data, that is, mainly from three criteria, coefficient of determination (R^2), residual standard error (S) and residuals analyses. And it is commonly considered that a fitted model would be better if it has a higher R^2 , a lower S and a unbiased residuals distribution. However, it should be

noted that whether a model fits the data well or not is affected not only by the model forms, but also by the fitting data, which explains that why a model may fit a data set well but fit another one badly and thus explains that why different researchers could draw different conclusions on comparing CAR and VAR models. So it is difficult to clearly decide which one, the CAR or VAR model, is superior.

It is well known that the ultimate aim of fitting a model is for prediction, and it should be emphasized that a better model should provide accurate estimations or predictions, not just to give a better fit. So this study compares these two models emphatically for their parameter stabilities and their predictions with a biomass data set of 99 planted larch trees to evaluate the CAR model and the VAR model more accurately. The CAR and VAR model forms for tree stem, branch and leaf biomass are as following:

	CAR model	VAR model
Tree stem	$W_i = b_0(S_i^2 H_i)^{b_1} + \varepsilon_i$	$W_i = b_0(D_i^2 H_i)^{b_1} e^{b_2(D_i^2 H_i)} + \varepsilon_i$
Branch or leaf	$W_i = b_0(D_i)^{b_1} e^{b_2 D_i} + \varepsilon_i$	$W_i = b_0(D_i)^{b_1} e^{b_2 D_i} + \varepsilon_i$

where W is biomass of tree stem, branch, or leaf, D is diameter at 1.3 m height, H is total height, ε is error, i is tree, e is exponential, b_0 , b_1 , b_2 are parameters.

Model comparison: Parameter stability is an important factor to be considered when to construct a model. The more stable the parameters, the more accurate the parameter estimation and thus the model prediction (Zeide, 1993). Resampling and sign test were employed to compare CAR and VAR models for their predictions. The resampling is one of most widely used methods of testing models and two main goals could be obtained with it, one is to test the stabilities of parameters, and another is to test model's predictive ability. The operative steps are as following:

Resampling(sample organization): 30 sets of random data were produced with computer, with each set composed of 50 random data and the range of them from 1 to 99, and the 99 planted larch trees were coded from number 1 to number 99. Then draw each set of samples from the 99 trees according to each set of random data, thus forming 30 sets, each one with 50 samples.

Model fitting: The CAR and VAR models were fitted respectively for tree stem, branch or leaf biomass with each

of 30 sample sets, and 30 sets of parameters estimates and standard error of estimate of CAR model or VAR model for tree stem, branch, and leaf biomass were obtained (not listed here).

Model comparison

Parameter stability: The parameter variations of the CAR model and the VAR model with 30 sets of samples were described in terms of bias, variance(var), and coefficient of variation(c%) (Table 1).

In Table 1, parameter mean $\bar{b} = \frac{1}{n} \sum_{i=1}^n \hat{b}_i$, where $\hat{b}_i$ is the i th estimate of parameter b_0 or b_1 , or b_2 of CAR model or VAR model, $i = 1, \dots, n$, n is the number of repetition and $n = 30$. Bias

is defined as bias = $E\hat{b} - b$ and is calculated as bias = $\bar{\hat{b}} - b$.

Variance of parameters, var = $E[(\hat{b} - E\hat{b})^2]$ and is calculated

as $\text{var} = \frac{1}{n} \sum_{i=1}^n (\hat{b}_i - \bar{\hat{b}})^2$. The coefficient of variation, $c\% = \frac{\sqrt{\text{var}}}{\bar{\hat{b}}} * 100$.

Here b is considered as the population parameter and estimated by fitting the CAR or VAR model with all the pooled data of all the 99 trees.

Table 1 showed that biases, variances or variations of CAR models for stem, branch or leaf biomass were all smaller than those of VAR models, which indicated that the parameters of CAR models were more stable than those of VAR models or that the parameters of the latter were more variable or more flexible.

Model prediction: The sign test, followed by mean and variance of the coefficient of determination, was employed to decide which model, the CAR or VAR, can provide better predictions and thus is superior. The operation of sign test was as followed:

First, we define that, for one sample, the CAR model is superior to VAR model if the standard error of prediction

$s = \sqrt{\frac{\sum_{i=1}^n (\hat{w}_i - w_i)^2}{n}}$, where W is measured biomass and $\hat{w}$ is predicted biomass of the former model is less than

Table 1: Descriptive Statistics of Parameter Variations of CAR and VAR models

Component	Statistics	CAR models		VAR models		
		b_0	b_1	b_0	b_1	b_2
Stem	$\bar{\hat{b}}$	0.0243	0.9573	0.0131	1.0692	-0.00001
	bias	0.0025	-0.0051	-0.0032	0.0717	-9.01E-06
	var	0.00006	0.0013	0.000096	0.0136	1.76E-10
	c%	31.88	3.77	74.79	10.91	-132.66
Branch	$\bar{\hat{b}}$	0.0060	2.9501	0.4524	2.0412	0.0912
	bias	0.0032	-0.0038	0.2887	0.9950	0.0086
	var	7.82E-05	0.1541	0.7348	0.2392	0.0026
	c%	147.38	13.31	189.48	83.99	55.91
Leaf	$\bar{\hat{b}}$	0.0205	1.9113	0.1335	1.7057	0.0548
	bias	0.0014	0.0858	0.0938	0.2486	0.0370
	var	0.0002	0.1025	0.0558	1.5369	0.0030
	c%	68.99	16.75	176.94	72.68	99.95

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Table 2: Comparison of CAR and VAR models for biomass predictions

Component	Model	Superiority numbers	Mean R^2	variance of R^2
Stem	CAR	22	0.9938	1.789E-07
	VAR	8	0.9922	6.539E-06
Branch	CAR	15	0.9419	3.185E-05
	VAR	15	0.9362	0.00027
Leaf	CAR	23	0.8959	6.556E-06
	VAR	7	0.8863	0.00033

that of the latter.

Secondly, the stem, branch and leaf biomass of each tree of 99 trees were predicted respectively with the resulted CAR model or VAR model fitted with each of 30 sample sets. Then the standard error of prediction (S) were calculated to decide the superiority of the CAR or VAR model. For stem biomass prediction, the CAR model had 22 superiority numbers while the VAR had 8 ones, for branch, both had the same superiority numbers (that is 15), and for leaf, the CAR model had 23 ones while the VAR had 7 ones (Table 2).

Thirdly, it could be concluded from the theory of sign test that when the larger number of 22 and 8, the number 22, is larger than the critical value of $20.96 = [(n+1) / 2 + 0.98 * \sqrt{n+1}]$, where $n=30$, the CAR model is superior to the VAR for the stem biomass predictions at the significance level of 0.05 ($\alpha=0.05$). Similarly, the CAR model is also superior to the VAR for the leaf biomass predictions because the number 23 is greater than the critical value 20.96. In addition, Table 2 also showed that the mean coefficients of determination of the CAR models were all larger than those of VAR models for stem, branch or leaf biomass while the variances were all smaller, which indicated that the CAR models provided more accurate predictions than the VAR models.

The VAR model is more flexible than the CAR, thus providing

a better fit than the CAR, but the parameters are more easily influenced by the fitting data and are more variable than the CAR model.

The CAR model is more stable and can provide better predictions than the VAR model.

The CAR model is recommended here as the first choice of biomass models for research or application.

References

- Geron, C.D. and G.A. Ruark, 1988. Comparison of constant and variable allometric ratios for predicting foliar biomass of various tree genera. *Can. J. For. Res.*, 18: 1298-1304.
- Kittredge, J., 1944. Estimation of the amount of foliage of trees and stands. *J. For.*, 42: 905-912.
- Liu, Z.G., 1992. Research on biomass and productivity of *Larix principis-rupprechtii* plantations. *J. Beijing For. Univ.*, 14(Suppl.1): 114-123.
- Ruark, G.A., G.L. Martin and J.G. Bockheim, 1987. Comparison of constant and variable allometric ratios for estimating populus tremuloides biomass. *For. Sci.*, 33: 294-300.
- Zeide, B., 1993. Analysis of growth equations. *For. Sci.*, 39: 594-616.