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Research Article
Modeling of Extreme Rainfall Recurrence Time by Using Point Process Models

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Abstract

Background and Objective: Period of extreme rainfall recurrence in certain location becomes a very interesting thing to be studied. In agriculture, uncertain emergence affects many things. The calendar of cropping pattern is closely related to the prediction of recurrence time of extreme rainfall. This study aimed to construct the mathematical model to find the time of extreme rainfall recurrence through point process modeling. **Methodology:** The assumption used in this study was the waiting time until the subsequent occurrence of subsequent rain follows the inverse Gaussian and the log normal distribution. To estimate the parameters model, the moment method was used. **Results:** The time of recurrence of subsequent extreme rainfall can be determined by calculating the length of waiting time since the last extreme rain event. **Conclusion:** The expected waiting time until the subsequent occurrence of the next rain, does not depends on the time difference since the appearance of the extreme last rainfall.

Key words: Point process modeling, extreme rainfall recurrence, inverse gaussian distribution, log normal distribution, drought

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Competing Interest: The authors have declared that no competing interest exists.

Data Availability: All relevant data are within the paper and its supporting information files.

INTRODUCTION

Indonesia as an archipelago country became one of the most countries vulnerable to the negative impacts of climate change. As a country extremely vulnerable to climate change, Indonesia must adapt to these various environmental pressures. Indonesia has also become a front runner in the international limelight due to the country's eagerness to reduce greenhouse emissions by roughly 26%[1].

Due to global climate change, the territory of Indonesia is projected to experience changes in patterns and intensity rainfall that will increase the risk of flooding and drought in the season dry. This has an impact, such as prolonged drought, floods and the increasing frequency of extreme climatic events that affect health and community livelihoods. Risks of impact future losses will be suppressed if the projection of the weather and extreme climate can be done[2]. Extreme rainfall projection assessment in Indonesia expected can provide an overview of the expected extreme rainfall values. Forecasting was done to anticipate the adverse effects of extreme rainfall events. An important entity for the analysis of duration (include minimum inter event time)[3], used to capture duration dependence is hazard rate which gives the instantaneous exit probability conditional on survival[4,5].

Research on modeling of rain phenomenon has been studied by previous researchers[4,6-9]. Several special investigators studied the determination of time-lapse distributions between events, i.e., by Aldersley *et al.*[2] and McShane *et al.*[7]. Although, various models of renewal have been studied by previous researchers, it has not been determined which model can best be used in the forecast of future extreme rainfall events. In this study, examined two types of time distribution between extreme rain events i.e., inverse Gaussian and log normal. These two distributions are usually used to describe non-negative positively skewed data. This corresponds to the phenomenon of extreme rain events where the data has extreme skewness properties. This study aims to construct the mathematical model to find the time of extreme rainfall recurrence through point process models.

MATERIALS AND METHODS

The study area was South Sulawesi during the period January-August, 2017. South Sulawesi province is located at 0°12'- 8° South latitude and 116° 48'-122°36' East longitude. The total area is 62,482.54 km^2. Administratively, the boundary of South Sulawesi province: North side with Central Sulawesi province, West side with Makassar Strait, East side with Bone Bay and South side with Flores Sea. The total area, 62,482.54 km^2 (42% of the entire island of Sulawesi and 4.1% of the entire area of Indonesia). South Sulawesi has a strategic location in Eastern Indonesia enabling South Sulawesi to function as a service center for both Eastern Indonesia and international scale.

Variables and analytical procedures: The forecast analysis of the time of occurrence of extreme rainfall was done through the determination of waiting time until the occurrence of the next extreme rain. The assumption used in the process is the waiting time until the subsequent occurrence of subsequent rain follows the inverse Gaussian and the log normal model. To estimate the model parameters, the moment method was used.

RESULTS AND DISCUSSION

In this section it was expected to display the time lapse of the upcoming extreme rain by calculating the estimated waiting time until the next rainfall occurs using conditional probabilities. The concept of conditional probability has been introduced by previous researchers, including Gigerenzer and Hoffrage[10], Sanfilippo *et al.*[11] and Ferraes[12]. A review of the recurrence time in various phenomena has been done by some previous researchers using various time distribution models between events, such as Weibull, Pareto, Brownian Passage model and Normal log[13]. In the study it was shown that all types of time distribution between events used to model reappearance time gave the same result i.e., $\hat{\tau}$ (the waiting time until the occurrence of the next extreme rain events) does not depend on the time difference t_0 since the last extreme rainfall event[12].

Consider R_t as the time of the last extreme rain event at one location, t_0 is the time difference since the last extreme rain event. Furthermore, the probability of occurrence of extreme precipitation events as follows: If the time lapse t_0 since the last extreme event will occur at intervals (t_0, $t_0+\Delta t_0$). Suppose τ the waiting time until the occurrence of the next extreme rain events. This interval is a random variable $T\geq 0$ the distribution $F(t_0) = P(T\leq t_0)$.

Furthermore:

$$R(t_0) = 1\text{-}F(t_0) = P(T>t_0) \tag{1}$$

as extreme rainfall system reliability with $F(t_0)$ is probability that extreme rainfall system is broken off until time t_0 and $R(t_0)$ represent probability of extreme rainfall system function at t_0. Conditional probability:

$$F(\tau \mid T > t_0) = \frac{P(t_0 < T \leq t_0 + \Delta t_0)}{P(T > t_0)} \tag{2}$$

is probability that extreme rainfall system will fail until time τ. It's clear that:

$$F(\tau|T>\tau) = 0 \quad \text{for} \quad T<t_0$$

Thus:

$$F(\tau \mid T > t_0) = \frac{F(\tau) - F(t_0)}{1 - F(t_0)}, \tau > t_0 \tag{3}$$

The differential of Eq. 3 to τ is defined in Eq. 4:

$$\begin{aligned}\frac{\partial}{\partial \tau}\left(F(\tau \mid Tt_0)\right) &= F(\tau \mid T \geq t_0) \\ &= \frac{f(\tau)}{1 - F(t_0)}; \quad \tau > t_0\end{aligned} \tag{4}$$

where, $\int f(\tau|T \geq t_0)d\tau$ is probability that extreme rainfall system will occur in time interval (t_0, $t_0+\Delta t_0$) with assumption that there is no extreme rainfall until time t_0. Here described the distribution of waiting time model until the appearance of extreme rainfall next using inverse Gaussian model and Gamma model.

Inverse Gaussian model: Let the probability density function of inverse Gaussian of random variable T is defined as:

$$f(\tau) = \sqrt{\frac{\lambda}{2\pi}} x^{-1/2} \exp\left(-\frac{\lambda(x-\mu)^2}{2\mu^2 x}\right) \tag{5}$$

Then mean:

$$E[T] = \mu \text{ and } \mathrm{Var}[T] = \frac{\mu^3}{\lambda} \tag{6}$$

Cumulative distribution for random variable inverse Gaussian as follows:

$$F(\tau) = \int_0^\tau \sqrt{\frac{\lambda}{2\pi}} \tau^{-1/2} \exp\left(-\frac{\lambda(x-\mu)^2}{2\mu^2 \tau}\right) dx \tag{7}$$

So, based on Eq. 4, we have:

$$f(\tau \mid T \geq t_0) = \frac{\sqrt{\frac{\lambda}{2\pi}} \tau^{-1/2} \exp\left(-\frac{\lambda(\tau-\mu)^2}{2\mu^2 \tau}\right)}{1 - \int_0^\tau \sqrt{\frac{\lambda}{2\pi}} \tau^{-1/2} \exp\left(-\frac{\lambda(\tau-\mu)^2}{2\mu^2 \tau}\right) d\tau} \tag{8}$$

Since the cumulative distribution function for the random variable Gaussian inverse distributed in Eq. 7 is a constant C. Thus, the probability density function of extreme rainfall occurrence[14] can be written as follows:

$$f(\tau \mid T \geq t_0 = W \sqrt{\frac{\lambda}{2\pi}} \tau^{-1/2} \exp\left(-\frac{\lambda(\tau-\mu)^2}{2\mu^2 \tau}\right) \tag{9}$$

With:

$$\omega = \frac{1}{1-C}$$

Furthermore, the waiting time τ until the occurrence of the next extreme rainfall that maximizes the conditional probability density function for extreme rainfall in Eq. 9 is:

$$\frac{\partial f(\tau \mid T \geq t_0)}{\partial \tau} = -\frac{1}{4} \frac{\sqrt{(2)} \exp\left(-\frac{1}{2}\frac{\lambda(-\tau+\mu)^2}{\mu^2 \tau}\right) \lambda\left(3\mu^2\tau - \mu^2\lambda + \tau^2\lambda\right)}{\tau^5 \sqrt{\frac{\lambda}{\pi\tau^3}} \pi\mu^2} \tag{10}$$

The critical point for the Eq. 10 is the solution of:

$$-\frac{1}{4} \frac{\sqrt{(2)} \exp\left(-\frac{1}{2}\frac{\lambda(-\tau+\mu)^2}{\mu^2 \tau}\right) \lambda\left(3\mu^2\tau - \mu^2\lambda + \tau^2\lambda\right)}{\tau^5 \sqrt{\frac{\lambda}{\pi\tau^3}} \pi\mu^2} = 0$$

or:

$$3\mu^2\tau - \mu^2\lambda + \tau^2\lambda = 0 \tag{11}$$

Based on the Eq. 11, the critical point is obtained as follows:

$$\begin{aligned}\hat{\tau}_1 &= \frac{1}{2}\frac{(-3\mu + \sqrt{9\mu^2 + 4\lambda^2})\mu}{\lambda} \\ \hat{\tau}_2 &= \frac{1}{2}\frac{(-3\mu - \sqrt{9\mu^2 + 4\lambda^2})\mu}{\lambda}\end{aligned} \tag{12}$$

Since τ is positive, the solution of Eq. 11 is $\hat{\tau}_1$. Further, it checked the maximum conditional probability criteria.

View the Eq. 11 for interval [0, ∞). Due to f'(τ)>0 all τ at [0, $\hat{\tau}_1$), then based on the monotonous theorem, f increased on [0, $\hat{\tau}_1$). Due to f'(τ)>0 all τ in ($\hat{\tau}_1$, ∞), then f decreased on [$\hat{\tau}_1$, ∞). So it can be concluded that f($\hat{\tau}_1$) maximum.

Using Eq. 11, it is possible to predict the occurrence of an upcoming extreme rainfall for the Gaussian inverse model. In the theory of density function of conditional probability, it is known that the predicted time of occurrence of extreme rainfall that will come through the determination of waiting time depends on the elapse of time (elapsed time) t_0. However, Eq. 12 shows that the expected waiting time until the occurrence of extreme rainfall in the future $\hat{\tau}$ does not depend on the time difference t_0 since the last extreme rainfall event.

Gamma model: Let probability density function of Gamma of random variable T is:

$$f(\tau) = \frac{\beta^\alpha}{\Gamma(\alpha)} \tau^{\alpha-1} e^{-\beta\tau}, \quad \tau > 0 \tag{13}$$

Mean of T is $E[T] = \frac{\alpha}{\beta}$ and variance of T is $Var[T] = \frac{\alpha}{\beta^2}$. Cumulative distribution of random variable Gamma is:

$$F(\tau) = \int_0^\tau \frac{\beta^\alpha \tau^{\alpha-1} e^{-\beta\tau}}{\Gamma(\alpha)} \tag{14}$$

So, based on Eq. 4 and 14, we have:

$$f(\tau \mid T \geq t_0) = \frac{\dfrac{\beta^\alpha \tau^{\alpha-1} e^{-\beta\tau}}{\Gamma(\alpha)}}{1 - \displaystyle\int_0^\tau \frac{\beta^\alpha \tau^{\alpha-1} e^{-\beta\tau}}{\Gamma(\alpha)}} \tag{15}$$

Since the cumulative distribution function for the Gamma distributed random variable in Eq. 14 is a constant C. Thus, the probability density function of extreme rainfall recurrence can be written as follows:

$$f(\tau \mid T \geq t_0) = W \frac{\beta^\alpha \tau^{\alpha-1} e^{-\beta\tau}}{\Gamma(\alpha)} \tag{16}$$

with:

$$\omega = \frac{1}{1-C}$$

Furthermore, the waiting time until the occurrence of the next extreme rainfall that maximizes conditional probability density function for extreme rainfall in Eq. 15 is:

$$\frac{\partial f(\tau \mid T \geq t_0)}{\partial \tau} = \frac{\beta^\alpha \tau^{\alpha-1} \exp(-\beta\tau)}{\Gamma(\alpha)\tau} - \frac{\beta^\alpha \tau^{\alpha-1} \beta \exp(-\beta\tau)}{\Gamma(\alpha)} \tag{17}$$

The critical points for Eq. 17 is solution of:

$$\frac{\beta^\alpha \tau^{\alpha-1} \exp(-\beta\tau)}{\Gamma(\alpha)\tau} - \frac{\beta^\alpha \tau^{\alpha-1} \beta \exp(-\beta\tau)}{\Gamma(\alpha)} = 0$$

or:

$$-\frac{\beta^\alpha \tau^{\alpha-2} \exp(-\beta\tau)(\tau\beta - \alpha + 1)}{\Gamma(\alpha)} = 0 \tag{18}$$

The above equation is true if:

$$\tau\beta - \alpha + 1 = 0 \tag{19}$$

Based on Eq. 19, the critical point is obtained as follows:

$$\hat{\tau} = \frac{\alpha - 1}{\beta} \tag{20}$$

If $\alpha < 1$, then maximum occurs outside the definition range and for this case the conditional probability density for the occurrence of extreme rainfall is a monotonous function down to zero for $t_0 \to \infty$. However, Eq. 20 shows that the expected waiting time until the occurrence of extreme rainfall in the future $\hat{\tau}$ does not depend on the time difference t_0 since the last extreme rainfall event.

CONCLUSION

Analysis of the occurrence of extreme rainfall is very important to determine the period of recurrence of extreme rain. Although many researchers have conducted research in this field, it has not found the most appropriate distribution to state the time between extreme rain events. In this study we have discussed the time between extreme rain events that distributed inverse Gaussian and Gamma. The results obtained show that the expected waiting time until the occurrence of extreme rainfall in the future $\hat{\tau}$ does not depend on the time difference t_0 since the last extreme rainfall event.

SIGNIFICANCE STATEMENTS

This study provides another way to determine the approximate time of recurrence of extreme rainfall using the concept of conditional probability. Based on the characteristics of extreme rainfall, this study provides significant input to the possibility of appropriate distribution for the time between two extreme events. Based on the formula obtained, both models (inverse Gaussian and Gamma) show that the chances of occurrence of the subsequent rainfall extremes do not depend on the time difference since the appearance of the last extreme.

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